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Effects of volumetric allocation on heave response of semisubmersible in deep sea

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The configuration of semisubmersibles consisting of pontoons and columns and their corresponding heave motion response in incident progressive waves are examined. The purpose of the present study is to provide a theoretical approach to estimating the effects of volumetric allocation on natural period and response amplitude operator (RAO) in heave motion. We conclude that the amplitude of heave motion response can be considerably suppressed by appropriately adjusting volumetric allocation so that the natural heave period keeps away from the range of wave energy. The theoretical formulae are found in good agreement with the corresponding computational results by WAMIT.

semisubmersible, heave, natural period, RAO, WAMIT

1 Introduction

With ever-growing needs for oil and gas resources, the ocean engineering has been paid much attention to by world engineering community. In the South China Sea known as the second Persia Bay, there reserves a large amount of oil and natural gas. China has started to stride forward from coastal regions with depth under 300 m to offshore regions as deep as 3000 m. Ongoing worldwide trends in deepwater oil and gas production call for in-depth research in the interaction of fluid and structures as a basis of platform design. Major types of platforms such as tension leg platforms (TLP), semisubmersibles and spar platforms have been successfully used in deepwater oil and gas exploitation during the last two decades^[1].

Floating structures may experience resonant motions, which should be avoided as much as possible under installation, operation and survival conditions. In particular, the heave motion response of a floating structure should be kept adequately low to guarantee the safety of risers and umbilical pipes as most important components in the equipment of oil production. Therefore, we should make every effort to minimize vertical motion of floating structures.

The natural period in heave motion can be effectively enhanced simply by adding structural mass, which, however, seems to be infeasible due to a number of restrictions in the design. Haslum and Faltinsen^[2] proposed that heave response to wave frequency is reduced by increasing system damping. As an example, Tao^[3] considered the suppression of heave resonant response by altering hull shapes with larger damping. A new large floating platform was designed by Srinivasan et al.^[4,5]. They tried to control its heave motion by applying the concept of both hydrodynamic added mass and separated-flow damping intelligently. Do and Pan^[6] developed a nonlinear controller for an active heave compensation system, which may reduce heave motion of a vessel connected to the riser. Chen et al.^[7] demonstrated that the location or draft of heave plates exerted a significant effect on the heave motion of a semisubmersible. Rho et al.^[8,9] considered both heave and pitch motions of a spar platform with damping plate at its bottom. Experiments were carried out in a wave tank for scaled models with/without damping plate and showed that the

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spar platform with damping plate exhibited better performance than classic spar in heave motion at resonance. Bjarte-Larsson and Falnes^[10] conducted an experiment on latching control, which may result in a significant increase or decrease in heave response. Perry et al.^[11] presented a novel optimization strategy capable of carrying out the preliminary design of floating offshore structures using genetic algorithm. Clauss and Birk^[12] proposed a hydrodynamic shape optimization procedure applicable to various types of offshore structures with improved seakeeping qualities.

In a word, efficient and benign motion behaviors of floating structures interacting with surrounding fluid are indispensable requirements in many engineering design. The design of deep-water platforms often borrows methods in ship building area^[13,14]. In order to obtain suitable shape and configuration of floating structures, people usually would like to use numerical simulation, optimization, or directly resort to commercial software. On the other hand, theoretical analysis is relatively few.

The primary purpose of the current research is to provide a theoretical approach to estimating the effects of volumetric allocations between columns and pontoons on natural period and response amplitude operator (RAO for short) in heave motion of semisubmersibles. The well-known hydrodynamic software package called WAMIT based on potential theory is applied to verify theoretical formulae. The calculation shows that heave motion can indeed be considerably reduced merely by appropriately adjusting volumetric allocation. The progress in this respect can be regarded as a foundation for conceptual design and structure optimization.

2 Theoretical formulae of natural period and heave RAO

Let us look at a floating platform interacting with linear progressive waves. Obviously, its 6-DOF (degree of freedom) dynamic equation of motion can be expressed as follows:

$$\left(M + M_{\rm a}\right)\frac{\partial^2 x_{\rm G}}{\partial t^2} + B\frac{\partial x_{\rm G}}{\partial t} + Kx_{\rm G} = f, \qquad (1)$$

where t is time; x_G denotes the unknown motion vector (6×1) in the sequence of surge, sway, heave, roll, pitch and yaw, respectively; f is wave load obtained from direct integration of hydrodynamic pressure over the wet surfaces; M and K are mass matrix (6×6) and stiffness

matrix (6×6); M_a and B are added mass matrix (6×6) and damping matrix (6×6), respectively.

The semisubmersibles, which operate in extra deep sea, have already been developed to the sixth generation. Their pattern tends to become simpler compared to the previous configuration with four, six or eight columns and single or double pontoons. Various types of deepwater semisubmersibles with different parameter regimes exhibit different characteristics in motion performance. In this paper, a typical semisubmersible with two rectangular pontoons, the ends of which are semicircular and two/three equally-spaced circular columns on each pontoon, is analyzed. The origin of the coordinate system is located at the intersection of the vertical axis of the semisubmersible and the undisturbed free surface. The simplified model and its coordinate system are shown in Figures 1 and 2.



Figure 1 Sketch semisubmersible below waterline.



Figure 2 Schematic definition of coordinate system.

Heave is a most important response motion for floating structures, such as semisubmersibles. Then, eq. (1) in heave component can be reduced directly to

$$\left(M_{33} + A_{33}\right)\frac{\mathrm{d}^{2}\eta_{3}}{\mathrm{d}t^{2}} + B_{33}\frac{\mathrm{d}\eta_{3}}{\mathrm{d}t} + \rho gA\eta_{3} = F_{3}\left(t\right), \qquad (2)$$

where M_{33} is the mass of the structure, A_{33} and B_{33} are the added mass and radiation damping in heave, respectively, *A* the waterplane area, ρ the density of the water, g the acceleration due to gravity, η_3 represents the heave motion of the platform and $F_3(t)$ is the vertical excitation force on the platform.

2.1 Natural frequency in heave motion

According to eq. (2), the uncoupled natural circular frequency in heave can be written as

$$\omega = \sqrt{\frac{\rho g A}{M_{33} + A_{33}}}.$$
 (3)

Considering the fact that the added mass of columns in heave is small compared to that of pontoons for semisubmersibles, we can approximate eq. (3) by

$$\omega = \sqrt{\frac{\rho g A}{\rho V + \kappa \rho V_{\rm p}}},\tag{4}$$

where V is the displacement of the semisubmersible, V_p is the volume of the pontoons, κ stands for the added mass coefficient of the pontoons.

Eq. (4) is based on the assumption that the underwater construction can be subdivided into elements with different functions and the added mass of columns can be neglected in some circumstances.

Now we may further express natural frequency in heave motion in terms of volumetric fraction ϕ as follows:

$$\omega = \sqrt{\frac{g}{L}} \left(\frac{1}{1 + (1 + \kappa)\frac{\phi}{1 - \phi}} \right)^{\frac{1}{2}}, \qquad (5)$$

where *L* denotes the draught of the columns and ϕ is defined as

$$\phi = \frac{V_{\rm p}}{V},\tag{6}$$

with $0 < \phi < 1$.

Eq. (5) shows that the natural frequency in heave motion is primarily dependent on the volumetric fraction ϕ , namely, the volumetric ratio of pontoons to the total displacement of the semisubmersible, rather than its geometric distribution. In addition, the added mass coefficient κ dependent on pontoon geometry may exhibit secondary effect in this respect. The simplicity of eq. (5) seems to facilitate the selection of platform configuration in order to keep natural frequency away from the range of incident wave energy in actual marine environment.

2.2 RAO in heave solution process

Assuming that the free-surface elevation at the center plane of the platform is $\zeta_a \sin \omega t$ and only the Froude-Krylov force is exerted on the structure, it now follows that^[15]

$$F_{3}(t) = \rho g \zeta_{a} \sin \omega t e^{k z_{m}} \cos\left(kB/2\right)$$
$$\cdot \left(A e^{k(z_{t}-z_{m})} - k \left(V_{p} + \frac{A_{33}}{\rho}\right)\right), \tag{7}$$

where ζ_a is the incident wave amplitude, ω is wave frequency, *k* is wave number, z_t and z_m are respectively the *z*-coordinates of the top and the geometric centre of a pontoon, *B* is the distance between the center planes of each pontoon.

Combining eqs. (2) and (7), we can obtain the motion of the semisubmersible

$$z = h\sin(\omega t - \psi), \tag{8}$$

where

$$\frac{h}{\zeta_{a}} = \frac{\rho g e^{kz_{m}} \cos(kB/2) \left(A e^{k(z_{t}-z_{m})} - k \left(V_{p} + \frac{A_{33}}{\rho}\right)\right)}{\sqrt{\left(\rho A g - \left(M + A_{33}\right)\omega^{2}\right)^{2} + \left(B_{33}\omega\right)^{2}}}, (9)$$

$$\psi = arctg \left(\frac{B_{33}\omega}{\rho g A - \left(M_{33} + A_{33}\right)\omega^{2}}\right). (10)$$

By using the fact that the added mass $A_{33} = \kappa_1 \rho V$, damping $B_{33} = c_1 \rho V \omega$ and the wave number k and the circular frequency ω satisfy the dispersion relation $\omega^2 = gk$ in deep water, now, κ_1 and c_1 are the added mass coefficient and damping coefficient of the semisubmersible, respectively. Thus, eq. (9) becomes

$$\frac{h}{\zeta_{a}} = \frac{e^{kz_{m}}\cos(kB/2)\left(e^{k(z_{1}-z_{m})}-kL\left(k_{1}+(1+k_{1})\frac{\phi}{1-\phi}\right)\right)}{\sqrt{\left(1-(1+k_{1})kL\frac{1}{1-\phi}\right)^{2}+(c_{1}kL)^{2}\left(\frac{1}{1-\phi}\right)^{2}}}.$$
(11)

If kz_m , kB/2, $k(z_t - z_m)$ are small enough we can further approximate eq. (11) by

$$\frac{h}{\zeta_{a}} = \frac{1 - kL\left(k_{1} + (1 + k_{1})\frac{\phi}{1 - \phi}\right)}{\sqrt{\left(1 - (1 + k_{1})kL\frac{1}{1 - \phi}\right)^{2} + (c_{1}kL)^{2}\left(\frac{1}{1 - \phi}\right)^{2}}} \quad (12)$$

Similarly, eq. (12) also shows that the semisubmersi-

ble motion in heave is mainly dependent on the volumetric fraction ϕ , while the added mass coefficient κ_1 and damping coefficient c_1 play secondary roles. The simplicity of eq. (12) also seems to facilitate the selection of platform configuration in order to minimize heave motion of semisubmersibles in actual marine environment.

3 Verification and discussion

The primary concern of this part is numerically to verify the foregoing theoretical formulae for natural periods and motion response RAO in heave of semisubmersibles described above. To this end, the heave responses in frequency domain are calculated by WAMIT based on potential theory and three-dimensional panel method. A number of semisubmersibles with different dimensions as shown in Figures 1 and 2 and Table 1 are considered. We have calculated natural period and RAO in heave under the condition of volumetric fraction $\phi = 0.84$, 0.82, 0.79, 0.76 and 0.57 with corresponding radii of columns 7.5, 8.0, 8.5, 9.0 and 11.0 m, by changing the height,

 Table 1
 Main parameters of the semisubmersibles

beam and length of pontoons and number of columns or columns diameter respectively when the total displacement (46128.3 m^3), draught (19.0 m), beam outside pontoons (78.68 m) and space of columns on each pontoon (70.0 m) are kept unchanged.

3.1 Natural periods of semisubmersibles

Figure 3 presents the heave natural periods obtained from the software WAMIT for the semisubmersibles of different volumetric allocations at $\phi = 0.84$ and 0.76, as shown in Table 1. It is seen that the natural periods are almost equal for the same volumetric fraction ϕ . On the other hand, those with different ϕ do exhibit different natural periods ranging from about 22 s to 28 s for $\phi =$ 0.76 (the radii of columns are 9.0 m) and 0.84 (the radii of columns are 7.5 m), respectively. Similar results can be obtained for other ϕ in Table 1. Any of the heave responses for each ϕ in Table 1 are reproduced in Figure 4. It can be seen from Figure 4 that semisubmersibles with different ϕ exhibit different natural periods, which may change from about 18, 22, 24, 26 to 28 s for $\phi=0.57$, 0.76, 0.79, 0.82 (4 and 6 columns) and 0.84.

Semi number	Length of pontoon (m)	Beam of pontoon (m)	Height of pontoon (m)	Radius of column (m)	Ratio ϕ	
1	114.07	20.12	8.81	7.5		
1a	114.07	20.69	8.54	7.5	0.84	
1b	117.03	20.12	8.54	7.5		
2	114.07	20.12	8.54	8.0	0.82	
2a	114.07	20.12	8.54	6.53		
3	114.07	20.12	8.23	8.5		
3a	114.07	19.52	8.54	8.5	0.79	
3b	110.91	20.12	8.54	8.5		
4	114.07	20.12	7.88	9.0		
4a	114.07	18.88	8.54	9.0	0.76	
4b	107.57	20.12	8.54	9.0		
5	100.0	23.0	6.05	11.0	0.57	



 $(u(u)) = 10^{-10} +$

Figure 3 Heave responses of semisubmersibles in Table 1 at volumetric fraction ϕ =0.84 and 0.76.

Figure 4 Variation of heave natural period at different volumetric fractions ϕ .

This further shows that the natural periods are very sensitive to the volumetric fraction variation. Therefore, we are able to keep natural periods away from the range of conventional surface wave periods by properly adjusting volumetric fraction ϕ . The natural periods are dependent upon volumetric fraction of pontoons to the total displacement of the semisubmersible instead of their specific geometric distribution. And added mass coefficients only play a secondary role in this regard.

In order to more clearly illustrate the variation of the natural periods in heave with volumetric ratio ϕ of semisubmersibles, both theoretical formulae and WAMIT results are plotted in Figure 5. It has been noticed that the comparison between the theoretical formulae with pontoon added mass only (solid line) and numerical calculations by WAMIT is in good agreement. However, we have found some deviation when $\phi=0.57$ or below because the added mass of columns becomes relatively important in lower ϕ region and thus eq. (4) or (5) may not be adequate under this condition.

It can be observed from Table 2 that the error of the added mass between pontoons and the whole structure is about 10% to 20% for larger ϕ , while the error may even reach to 48.5% when ϕ =0.57.



Figure 5 Comparison of theoretical (solid line and dashed line) and simulated (dots) natural period with volumetric fraction ϕ .

 Table 2
 Comparison of the added mass (kg) between pontoons considered only and the whole semisubmersible

φ	0.84	0.82	0.79	0.76	0.57
Pontoon (×10 ⁷): A1	8.282	7.693	7.195	6.696	3.441
Semisubmersible (×10 ⁷): A2	9.79	9.409	9.132	8.867	6.685
Error: (A2-A1)/A2	0.154	0.182	0.212	0.244	0.485

Consequently, the theory is compared well with calculation if the added mass of columns is included again (dashed line in Figure 5). We may observe that both theoretical curves are almost identical in high ϕ range while solid line fades away faster than dashed line in small ϕ range. The fact demonstrates that formula (5) can reasonably figure out the volumetric allocation influence on the natural period of semisubmersibles. And careful examinations and assessments of the added mass for other types of floating platform are recommended.

3.2 Heave motion response

We have also noticed from Figure 3 that the heave motion RAOs for the cases with the same ϕ , are almost identical in the whole range of the incident wave periods we considered except around the heave natural period. Different damping nearby resonance period is responsible for the deviation in response amplitude.

Case 1 among the all platforms possesses the highest natural period meanwhile the relatively low RAO peak in heave, indicating that Case 1 is the most efficient structure from the point of view of increasing natural period and decreasing heave RAO as much as possible.

As an example, Figure 6 illustrates the variation of the heave RAO in terms of the volumetric ratio ϕ at incident wave period T=10s. As shown in Figure 6, good agreement is reached in high volumetric ratio ϕ range for both theoretical curves based on pontoons added mass only (solid line) and the total added mass of the platform (dashed line). Indeed, apparent deviation between two theoretical curves in small volumetric ratio



Figure 6 Comparison of the theoretical (solid line and dashed line) and calculated (dots) results of variation of heave RAO with volumetric ratio at T=10 s.

range is found owing to the neglect of the added mass of the columns and more importantly the damping of the structure.

The fact tells us that the heave motion is not only dependent on the volumetric ratio ϕ but also the added mass and damping. It further demonstrates that eq. (11) or (12) is reasonable for predicting heave RAO only in larger volumetric ratio range for platforms such as semisubmersibles.

4 Concluding remarks

In the present study, the heave motion of semisubmersibles is primarily concerned with. It seems that understanding dynamic behaviors of floating structures dependent on a great many factors such as marine environment, platform prototype and dimensions is not an easy task. However, the main point of the present study is to divide structural elements into two categories, namely, elements nearby waterline for restoring force generation such as columns and elements for the added inertial. Then we may further assume that the added mass of columns is negligibly small. As a result, two theoretical formulae have been derived, showing that the natural period and heave response are mainly dependent on the volumetric ratio of pontoon to total structure rather than specific geometric configuration. And the added mass coefficient and damping coefficient merely play secondary roles. Very likely relatively large resonant oscillation can be excited if incident wave period happens to fall in the vicinity of natural period of platforms. Thus, it becomes a handy work now to shift natural period away from the range of incident wave energy just by appropriately adjusting volumetric allocation.

The agreement of theoretical formulae with numerical calculation of dynamic behaviors of a number of semisubmersibles with different dimensions in incident

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waves by WAMIT further verifies previous argument. The results also show that the semisubmersibles with the same volumetric ratio indeed have the same natural period or behavior of RAO. The evident difference in the response amplitude of heave motion around natural periods is attributed to the difference in damping in resonance periods region. In particular, we find that the natural period is rather sensitive to the variation of volumetric ratio as $\phi \rightarrow 1$. Therefore, it is really an effective approach to improve dynamic behavior of floating structure by altering volumetric allocation. Of course, we should notice that the theoretical formulae are kept valid only when ϕ is larger than about 0.6.

As you know, conceptual selection of platform is a first and foremost step for system design to reach the goal of economy, safety and efficiency. In this phase, we need to provide a sketch of platform structure with response and other requirements given. For example, TLP, Spar, semisubmersible and their variants are among efficient platform types successfully used for deepwater oil and gas exploitation up to date. In the meanwhile, we need to describe structural details and verify dynamic response (and strength, fatigue life) with structure type given in engineering design phase. No doubt, the theoretical formulae in the present article may considerably facilitate the selection of platform configuration to minimize heave motion of semisubmersibles in the first phase.

In the end, we may summarize our conclusions in the following: As an efficient design approach to improve dynamic behavior of semisubmersibles, at least, volumetric allocation method can help engineers to select the number of columns or column diameters. Very probably, volumetric allocation concept facilitates us to envisage new variants of platform available or even completely different types of platforms. Any progress in this regard can probably be applied to deep-sea floating platform conceptual design.

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