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Elastic deformation of soft membrane with finite thickness induced by a sessile liquid droplet

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ABSTRACT

In this paper, the role of vertical component of surface tension of a droplet on the elastic deformation of a finite-thickness flexible membrane was theoretically analyzed using Hankel transformation. The vertical displacement at the surface was derived and can be reduced to Lester's or Rusanov's solutions when the thickness is infinite. Moreover, some simulations of the effect of a liquid droplet on a membrane with a finite thickness were made. The numerical results showed that there exists a saturated membrane thickness of the order of millimeter, when the thickness of a membrane is larger than such a value, the membrane can be regarded as a half-infinite body. Further numerical calculations for soft membrane whose thickness is far below the saturated thickness were made. By comparison between the maximum vertical displacement of an ultrathin soft membrane and a half-infinite body, we found that Lester's or Rusanov's solutions for a half-infinite body cannot correctly describe such cases. In other words, the thickness of a soft membrane has great effect on the surface deformation of the ultrathin membrane induced by a liquid droplet.

1. Introduction

Although materials such as silicon and glass have been widely used in micro/nanoelectromechanical systems (MEMS/NEMS), their intrinsic stiffness and surface chemistry limited some applications in areas ranging from microfluidic device technology to nanofabrication [1]. However, soft materials overcome a lot of the limitations of silicon. One typical soft material is poly(dimethylsiloxane) (PDMS), which is an optically transparent, soft elastomer [2]. Besides, it also has some other advantages such as good biocompatibility, nontoxicity and easy fabrication. Therefore, it has been widely used as a base material in microfluidic devices [2–11]. In many microfluidic devices, the thickness of PDMS membrane is only on the order of several tens micrometers or even smaller. When there is a liquid droplet sessile on the surface, the membrane may deform due to Laplace pressure acting on the wetting area and the vertical or normal components of liquid–vapor interface tension – $\gamma_\perp$ [12], which acts on the contact line and cannot be balanced by the forces on the contact line using classical or modified Young's equations considering line tension [13,14].

Since PDMS is a soft material with a Young's modulus of a few MPa or lower [15], the maximum height of ridge can be estimated by Shanahan et al. using dimensional analysis as follows [16]:

$$h_{\max} \approx h^* = \frac{\gamma_{lv}\sin\theta}{G} = \frac{\gamma_\perp}{G}, \tag{1}$$

where θ is Young's contact angle, γ_{lv} is liquid–vapor interface tension, and G is the shear modulus of the soft membrane. Eq. (1) was suitable for semi-infinite materials. Then, the maximum vertical displacement may be up to several hundred nanometers or even larger [17], which is so large that it may affect the performance of microfluidic devices, therefore, the effect of a liquid droplet on the deformation of thin soft membrane may not be neglected. Moreover, such a deformation will induce molecular reorientation and subsequent intermolecular force change [18].

As to such a question, Lester [19] and Rusanov [20] have studied the deformation of the surface of a semi-infinite body induced by a liquid droplet theoretically. They both thought the liquid–vapor interface has a breadth δ and the liquid–vapor interface tension acts uniformly on the narrow annulus for simplification. Then Lester calculated the vertical displacement in the annulus and gave the formula for variation of contact angle. Rusanov considered that there is another force $\gamma_{lv}(\cos\theta_1 - \cos\theta)$ acting in the horizontal direction for the actual contact angle θ_1 might differ from Young's contact angle θ, and then he gave the vertical displacement at the surface induced by the three forces. Therefore, Lester's solution can be regarded as a special case of Rusanov's when the difference of contact angles is zero. However, both Lester and Rusanov only considered the effect of a liquid droplet on the deformation of a semi-infinite body rather than a finite-thickness substrate.

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Besides, Andrade et al. studied the contact-angle-induced deformation of gels experimentally and pointed out that the deformation might be relatively large for low-modulus, high-water-content gels [21]. Some researchers also studied the deformation of a thin solid due to the droplet [22–24]. Shanahan and Carré studied the viscoelastic dissipation of wetting ridge induced by a droplet when the solid is sufficiently soft [25,26]. Shanahan made theoretical research on the influence of solid micro-deformation on contact angle equilibrium [27] as well as statics and spreading dynamics of a liquid drop on thin solid [28–30]. In their latter papers, a direct evidence for the wetting ridge obtained by using scanning interferometric microscopy was reported [31], and they estimated the maximum height of the ridge induced by a droplet [16]. Moreover, Tomasetti et al. studied viscoelastic energy dissipation due to solid deformation induced by a liquid during both wetting and dewetting processes [32].

Apart from investigations of such a problem either theoretically or experimentally, some numerical studies have also been reported [33–35].

During the latest years, Bonaccurso et al. have made some studies on the transverse effect of a sessile droplet on the bending of a flexible microcantilever [36–39]. Recently, they studied the surface deformation of several tens micrometers-thick PDMS membrane with very low modulus induced by a sessile droplet using laser scanning confocal microscopy and estimated that the breadth of surface layer is approximately 16.0 nm by comparison between their experimental data and Rusanov's theoretical solution [17].

Additionally, for the case of a liquid droplet resting or sliding on an inclined flat surface [40,41], the droplet will also make the surface deform.

In a word, there were some reports on the transverse effect of a liquid droplet on the deformation of a solid; however, to the best knowledge of the authors there was no literature on theoretically analyzing the effect of thickness of the solid on the surface deformation induced by a liquid droplet up till now. The motivation of our present work is to study such a question. We first theoretically analyzed the deformation of a finite-thickness flexible membrane using Hankel transform and then proposed a method to numerically calculate the vertical displacement at the surface. At last we made some simulations on surface deformation of PDMS membrane with different thicknesses to demonstrate how membrane's thickness affects its surface deformation induced by a liquid droplet.

2. Theoretical analysis

Now consider a small liquid droplet sitting on a deformable membrane with finite thickness h on a rigid substrate, as shown in Fig. 1. The radius of wetting area is R, the breadth of capillary layer is δ. Suppose the contact between the membrane and the substrate is frictionless for simplicity. Here cylindrical polar coordinates (r, φ, z) are used such that the origin coincides with the center of wetting area, the z-axis is perpendicular to the wetting area, r is perpendicular to z, and φ is the angular distance between a reference line and r, as shown in Fig. 2.

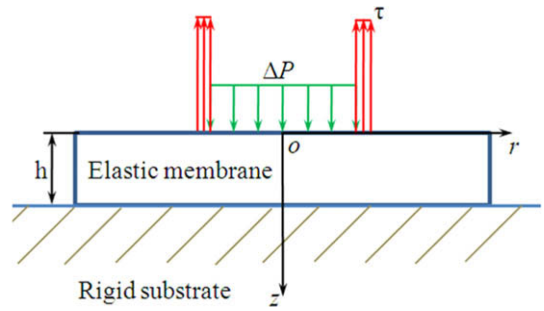

Fig. 2. Schematic diagram of coordinates and distribution of forces.

Then the components of the displacement vector and of the stress tensor will all be independent of the angle φ. The nonvanishing stresses corresponding to these coordinates (r, φ, z) are σ_r, σ_{zz}, σ_{rz} and σ_φ, which satisfy the following equilibrium equations [42]

$$\begin{cases} \dfrac{\partial \sigma_r}{\partial r} + \dfrac{\partial \sigma_{rz}}{\partial z} + \dfrac{\sigma_r - \sigma_\varphi}{r} = 0, \\ \dfrac{\partial \sigma_{rz}}{\partial r} + \dfrac{\partial \sigma_{zz}}{\partial z} + \dfrac{\sigma_{rz}}{r} = 0. \end{cases} \tag{2}$$

Similarly, the nonzero displacements corresponding to these coordinates (r, φ, z) are u_r and u_z. And due to the axisymmetry of such a problem, the linear elastic constitutive relations between the components of the stress tensor and the nonzero displacements may be written as following [43]:

$$\begin{cases} \sigma_r = \left[(\lambda + 2\mu)\dfrac{\partial}{\partial r} + \dfrac{\lambda}{r}\right] u_r + \lambda \dfrac{\partial u_z}{\partial z}, \\ \sigma_\varphi = \left[\lambda \dfrac{\partial}{\partial r} + \dfrac{\lambda + 2\mu}{r}\right] u_r + \lambda \dfrac{\partial u_z}{\partial z}, \\ \sigma_z = \lambda\left(\dfrac{\partial}{\partial r} + \dfrac{1}{r}\right) u_r + (\lambda + 2\mu)\dfrac{\partial u_z}{\partial z}, \\ \sigma_{rz} = \mu\left(\dfrac{\partial u_r}{\partial z} + \dfrac{\partial u_z}{\partial r}\right), \end{cases} \tag{3}$$

where $\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$ and $\mu = \frac{E}{2(1+\nu)}$ are the Lame constants, E and ν are Young's modulus and Poisson's ratio of the membrane, respectively.

The boundary conditions underneath the droplet and at the free surface are

$$\sigma_{zz}(r, 0) = \begin{cases} -P = -\frac{2\gamma_\perp}{R}, & r \leqslant R, \\ \tau = \frac{\gamma_\perp}{\delta}, & R \leqslant r \leqslant R_1 = R + \delta, \\ 0, & r > R_1, \end{cases} \tag{4}$$

and

$$\sigma_{rz}(r, 0) = 0. \tag{5}$$

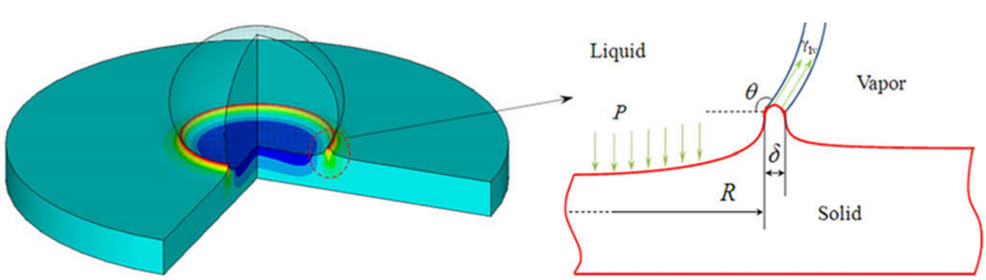

Fig. 1. Sketch of deformation of membrane induced by a water droplet.

The frictionless boundary conditions at the interface between the membrane and the substrate give

$$\sigma_{rz}(r,h) = 0 \quad \text{and} \quad u_z(r,h) = 0. \tag{6}$$

As in [19], the boundary conditions (4) were decomposed into two parts:

$$\sigma_{zz}(r,0) = P_1(r) = \begin{cases} -(P+\tau), & r \leqslant R, \\ 0, & r > R, \end{cases} \tag{7}$$

and

$$\sigma_{zz}(r,0) = P_2(r) = \begin{cases} \tau, & r \leqslant R_1, \\ 0, & r > R_1. \end{cases} \tag{8}$$

Using Hankel transformation, the displacement and stress fields in the membrane can be expressed as [44]

$$\begin{cases} u_z = \int_0^\infty \xi \left(\dfrac{d^2G}{dz^2} - \dfrac{\lambda+2\mu}{\mu}\xi^2 G \right) J_0(\xi r)d\xi, \\ u_r = \frac{\lambda+\mu}{\mu} \displaystyle\int_0^\infty \xi^2 \dfrac{dG}{dz} J_1(\xi r)d\xi, \\ \sigma_{zz} = \int_0^\infty \xi \left[(\lambda+2\mu)\dfrac{d^3G}{dz^3} - (3\lambda+4\mu)\xi^2 \dfrac{dG}{dz} \right] J_0(\xi r)d\xi, \\ \sigma_{rz} = \int_0^\infty \xi^2 \left[\lambda \dfrac{d^2G}{dz^2} + (\lambda+2\mu)\xi^2 G \right] J_1(\xi r)d\xi, \end{cases} \tag{9}$$

where J_0, J_1 are Bessel functions of orders zero and one, respectively, and $G(\xi,z) = (A_i + B_i z)\cosh(\xi z) + (C_i + D_i z)\sinh(\xi z)$, here A_i, B_i, C_i and D_i $(i = 1,2)$ are determined from the corresponding boundary conditions (5)–(7) and (5), (6), (8), respectively.

Substituting the boundary conditions (5)–(7) into (9), we can obtain a set of linear equations for the undetermined constants A_1, B_1, C_1 and D_1:

$$\begin{cases} (\lambda+\mu)\xi A_1 + \lambda D_1 = 0, \\ (\lambda+\mu)\xi A_1 \cosh\xi h + [\lambda \sinh\xi h + (\lambda+\mu)\xi h \cosh\xi h]B_1, \\ +(\lambda+\mu)\xi C_1 \sinh\xi h + [\lambda\cosh\xi h + (\lambda+\mu)\xi h \sinh\xi h]D_1 = 0, \\ \frac{\lambda+\mu}{\mu}\xi A_1 \cosh\xi h - \left[2\sinh\xi h - \frac{\lambda+\mu}{\mu}\xi h\cosh\xi h\right]B_1, \\ +\frac{\lambda+\mu}{\mu}\xi C_1 \sinh\xi h - \left[2\cosh\xi h - \frac{\lambda+\mu}{\mu}\xi h \sinh\xi h\right]D_1 = 0, \\ 2\mu\xi^2 B_1 - 2(\lambda+\mu)\xi^3 C_1 = \int_0^\infty rP_1(r)J_0(\xi r)dr = -\frac{R(P+\tau)}{\xi}J_1(R\xi). \end{cases} \tag{10}$$

Solving the above equations, we can get the constants with boundary conditions (5)–(7)

$$\begin{cases} A_1 = -\frac{\lambda R(P+\tau)\sinh^2\xi h}{\xi^4(\lambda+\mu)^2(2h\xi+\sinh 2\xi h)}J_1(R\xi), \\ B_1 = -\frac{R(P+\tau)\sinh 2\xi h}{2\xi^3(\lambda+\mu)(2h\xi+\sinh 2\xi h)}J_1(R\xi), \\ C_1 = \frac{R(P+\tau)[2\xi h(\lambda+\mu)+\lambda\sinh 2\xi h]}{2\xi^4(\lambda+\mu)^2(2h\xi+\sinh 2\xi h)}J_1(R\xi), \\ D_1 = \frac{R(P+\tau)\sinh^2\xi h}{\xi^3(\lambda+\mu)(2h\xi+\sinh 2\xi h)}J_1(R\xi). \end{cases} \tag{11}$$

Similarly, for uniform stress τ acting over a circle with radius R_1, we can also get the corresponding undetermined constants with boundary conditions (5), (6), (8)

$$\begin{cases} A_2 = \frac{\lambda R_1 \tau \sinh^2\xi h}{\xi^4(\lambda+\mu)^2(2h\xi+\sinh 2\xi h)}J_1(R_1\xi), \\ B_2 = \frac{R_1\tau\sinh(2h\xi)}{2\xi^3(\lambda+\mu)(2h\xi+\sinh 2\xi h)}J_1(R_1\xi), \\ C_2 = -\frac{R_1\tau[2\xi h(\lambda+\mu)+\lambda\sinh 2\xi h]}{2\xi^4(\lambda+\mu)^2(2h\xi+\sinh 2\xi h)}J_1(R_1\xi), \\ D_2 = -\frac{R_1\tau\sinh^2\xi h}{\xi^3(\lambda+\mu)(2h\xi+\sinh 2\xi h)}J_1(R_1\xi). \end{cases} \tag{12}$$

Substituting A_i, B_i, C_i and D_i $(i = 1,2)$ into (9), then we can get the distribution of displacements and stresses in the membrane by using superposition method. In fact, what is concerned is the vertical displacement at the upper surface of the membrane, and to make it agree with the actual circumstance, then it can be rewritten as:

$$\begin{aligned} U_z(r,0) &= -u_z(r,0) \\ &= \frac{4(1-\nu^2)}{E}\int_0^\infty f(\xi h)\frac{J_0(\xi r)}{\xi}[R_1\tau J_1(\xi R_1) - RP_1 J_1(\xi R)]d\xi, \end{aligned} \tag{13}$$

where $f(x) = \frac{\sinh^2 x}{2x+\sinh 2x}$ and $U_z(r,0)$ is the vertical displacement at the surface of a membrane with a finite thickness.

When the thickness of the membrane is infinite, that is $h \to \infty$, then

$$\lim_{h\to\infty} \frac{\sinh^2\xi h}{2\xi h + \sinh 2\xi h} = \frac{1}{2}, \tag{14}$$

and the corresponding solution of vertical displacement reduces to

$$U_z^\infty(r,0) = \frac{2(1-\nu^2)}{E}\int_0^\infty \frac{J_0(\xi r)}{\xi}[R_1\tau J_1(\xi R_1) - RP_1 J_1(\xi R)]d\xi, \tag{15}$$

which is the solution by Lester [19] or a special case of Rusanov's solution [20] when $\theta_1 = \theta$.

The integral $\int_0^\infty \frac{J_0(\xi r)J_1(\xi R)}{\xi}d\xi$ in (15) may be expressed as [45]:

$$\int_0^\infty \frac{J_0(\xi r)J_1(\xi R)}{\xi}d\xi = \begin{cases} {}_2F_1\left(\frac{1}{2}, -\frac{1}{2}, 1; \frac{r^2}{R^2}\right), & r \leqslant R, \\ \frac{R}{2r}\cdot {}_2F_1\left(\frac{1}{2}, \frac{1}{2}, 2; \frac{R^2}{r^2}\right), & r > R, \end{cases} \tag{16}$$

where ${}_2F_1(a,b,c;z)$ is the hypergeometric function [46], whose generalized form can be expressed as following:

$${}_pF_q(a_1,\ldots,a_p;b_1,\ldots,b_q;x) = \sum_{m=0}^\infty \frac{(a_1)_m\ldots(a_p)_m}{(b_1)_m\ldots(b_q)_m}\frac{x^m}{m!}, \tag{17}$$

where $(a)_m = a(a+1)(a+2)\cdots(a+m-1)$, $(a)_0 = 1$.

Therefore, for a deformable solid with infinite thickness, the total vertical displacement at the free surface can be written as

$$U_z^\infty(r,0) = \begin{cases} \frac{2(1-\nu^2)}{E}\left[R_1\tau\cdot {}_2F_1(\frac{1}{2},-\frac{1}{2};1;\frac{r^2}{R_1^2}) - R(P+\tau)\cdot {}_2F_1(\frac{1}{2},-\frac{1}{2};1;\frac{r^2}{R^2})\right], & r \leqslant R, \\ \frac{2(1-\nu^2)}{E}\left[R_1\tau\cdot {}_2F_1(\frac{1}{2},-\frac{1}{2};1;\frac{r^2}{R_1^2}) - \frac{R^2}{2r}(P+\tau)\cdot {}_2F_1(\frac{1}{2},\frac{1}{2};2;\frac{R^2}{r^2})\right], & R \leqslant r \leqslant R_1, \\ \frac{2(1-\nu^2)}{E}\left[\frac{R_1^2}{2r}\tau\cdot {}_2F_1(\frac{1}{2},\frac{1}{2};2;\frac{R_1^2}{r^2}) - \frac{R^2}{2r}(P+\tau)\cdot {}_2F_1(\frac{1}{2},\frac{1}{2};2;\frac{R^2}{r^2})\right], & r \geqslant R_1, \end{cases} \tag{18}$$

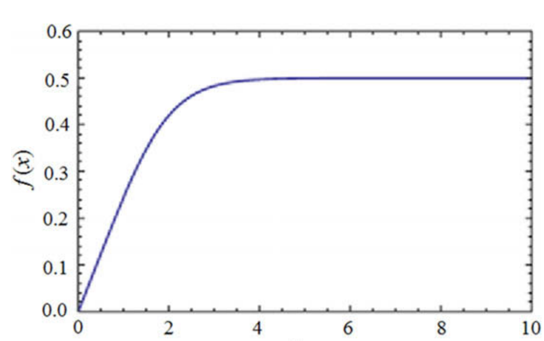

Fig. 3. Relation of $f(x) = \frac{\sinh^2 x}{2x+\sinh 2x}$ versus x.

Table 1
Value of $f(x)$ with respect to x.

x	5.0	6.0	7.0	8.0	9.0
$f(x)$	0.499501	0.49992	0.499988	0.499998	0.5

or

$$U_z^\infty(r,0)=\begin{cases}\frac{(1-\nu)\gamma_\perp}{G}\left[\frac{R_1}{\delta}\cdot{}_2F_1\left(\frac{1}{2},-\frac{1}{2};1;\frac{r^2}{R_1^2}\right)-\left(2+\frac{R}{\delta}\right)\cdot{}_2F_1\left(\frac{1}{2},-\frac{1}{2};1;\frac{r^2}{R^2}\right)\right], & r\leqslant R,\\ \frac{(1-\nu)\gamma_\perp}{G}\left[\frac{R_1}{\delta}\cdot{}_2F_1\left(\frac{1}{2},-\frac{1}{2};1;\frac{r^2}{R_1^2}\right)-\frac{R}{2r}\left(2+\frac{R}{\delta}\right)\cdot{}_2F_1\left(\frac{1}{2},\frac{1}{2};2;\frac{R^2}{r^2}\right)\right], & R\leqslant r\leqslant R_1,\\ \frac{(1-\nu)\gamma_\perp}{G}\left[\frac{R_1^2}{2\delta r}\cdot{}_2F_1\left(\frac{1}{2},\frac{1}{2};2;\frac{R_1^2}{r^2}\right)-\frac{R}{2r}\left(2+\frac{R}{\delta}\right)\cdot{}_2F_1\left(\frac{1}{2},\frac{1}{2};2;\frac{R^2}{r^2}\right)\right], & r\geqslant R_1.\end{cases}\tag{19}$$

Before studying vertical displacement at the free surface of a membrane with a finite thickness, we first plotted the value of $f(x)=\frac{\sinh^2 x}{2x+\sinh 2x}$ versus x, as shown in Fig. 3 as well as Table 1.

From the figure and table, we found that when x is larger than a certain value such as 8.0, the corresponding function value is almost equal to $\frac{1}{2}$, that is to say, for a given thickness h, the vertical displacement at the upper surface of the membrane can be approximately written as

$$U_z(r,0)=U_z^\infty(r,0)+\frac{4(1-\nu^2)}{E}\times\int_0^{x_0}\left[f(x)-\frac{1}{2}\right]\frac{R_1\tau J_1\left(\frac{R_1}{h}x\right)-R(P+\tau)J_1\left(\frac{R}{h}x\right)}{x}J_0\left(\frac{r}{h}x\right)\mathrm{d}x,\tag{20}$$

or

$$\tilde{U}_z(r,0)=\tilde{U}_z^\infty(r,0)+2(1-\nu)\times\int_0^{x_0}\left[f(x)-\frac{1}{2}\right]\frac{\frac{R_1}{\delta}J_1\left(\frac{R_1}{h}x\right)-\left(2+\frac{R}{\delta}\right)J_1\left(\frac{R}{h}x\right)}{x}J_0\left(\frac{r}{h}x\right)\mathrm{d}x,\tag{21}$$

where $\tilde{U}_z(r,0)=\frac{U_z(r,0)}{h^*}$ and $\tilde{U}_z^\infty(r,0)=\frac{U_z^\infty(r,0)}{h^*}$ are dimensionless vertical displacement at the surface of a finite-thickness membrane and semi-infinite solid, respectively. The latter part in equations (20), (21) shows the effect of membrane's thickness on the vertical displacement of the membrane. Moreover, it is easily found that the deformation is inversely proportional to Young's modulus of the membrane.

When the membrane is ultrathin, that is, $h\to 0$, then the vertical displacement (Eq. (13)) can be reduced to

$$U_z(r,0)=-u_z(r,0)=\frac{(1-\nu^2)h}{E}\int_0^\infty J_0(\xi r)[R_1\tau J_1(\xi R_1)-RP_1J_1(\xi R)]d\xi.\tag{22}$$

The integral $\int_0^\infty J_0(\xi r)J_1(\xi R)\mathrm{d}\xi$ in equation (22) can be expressed as [45]:

$$\int_0^\infty J_0(\xi r)J_1(\xi R)\mathrm{d}\xi=\begin{cases}\frac{1}{R}, & r<R,\\ \frac{1}{2R}, & r=R,\\ 0, & r>R.\end{cases}\tag{23}$$

Thus the vertical displacement at the surface of ultrathin membrane can be rewritten as below:

$$U_z(r,0)=\frac{1-\nu}{2}h^*\begin{cases}-\frac{h}{2R}\approx 0, r<R,\\ \frac{h}{2\delta}-\frac{h}{4R}\approx\frac{h}{2\delta}, r=R,\\ \frac{h}{\delta}, R<r<R_1,\\ \frac{h}{2\delta}, r=R_1,\\ 0, r>R_1.\end{cases}\tag{24}$$

Therefore, for such a case, the sessile droplet will make the region where the capillary layer is located deform sharply while the deformation outside the region is almost zero.

3. Numerical results and discussion

To demonstrate the effect of liquid–vapor interfacial tension on the deformation of a finite-thickness flexible membrane, we made some numerical simulations. Suppose the radius of wetting area is 0.2 mm, 0.5 mm and 1.0 mm, respectively (see Supplementary Material). Because it is difficult to obtain accurate value of capillary layer's breadth, we supposed it to be 1.0 nm, 10.0 nm and 40.0 nm, respectively. And then the corresponding vertical displacements at the upper surface versus the distance from the center of the droplet were obtained and here a plot for R = 0.5 mm and δ = 10.0 nm was shown in Fig. 4 as an example. For other circumstances, the plots were similar.

From Fig. 4, we can find that the surface deforms greater with increasing thickness of the membrane and when the thickness is in order of 1.0 mm, the surface deformation varies little, that is to say, there exists a saturated thickness, when a membrane's thickness is larger than such a value, it can be treated as a half-infinite body.

For membranes used in macro structures, membrane's thickness might usually be larger than several millimeters, so they can be regarded as a half-infinite body and Lester's or Rusanov's solutions are valid when considering the deformation of the surfaces induced by a liquid droplet. However, in micro/nanostructures, the thicknesses of most common used membranes are usually smaller than the saturated thickness and might be just several micrometers or even smaller, in such a case, the effect of membrane's thickness should be considered. In order to make clear how the thickness affects the surface deformation induced by a liquid droplet, we further made some numerical calculations and obtained the dimensionless vertical displacements at different points in the surface, as shown in Table 2a–c. The radius of wetting area is 0.2 mm and the thickness of capillary layer is supposed to be 1.0, 10.0 or 40.0 nm. From the three tables, it can be easily found that for a membrane with a thickness which is far smaller than the saturated one, (1) the maximum of vertical displacement increases with increasing membrane thickness; (2) the maximum of vertical displacement decreases when the thickness of capillary layer is larger; (3) when the thickness of capillary layer changes from 1.0 nm to 40.0 nm, dimensionless vertical displacements at the points which are not near the wetting ridge vary little. Additionally, relative errors between the maximum of dimensionless vertical displacement and corresponding value for a half-infinite body were calculated as follows:

$$\zeta=\frac{\tilde{U}_z-\tilde{U}_z^\infty}{\tilde{U}_z^\infty}\times 100\%,\tag{25}$$

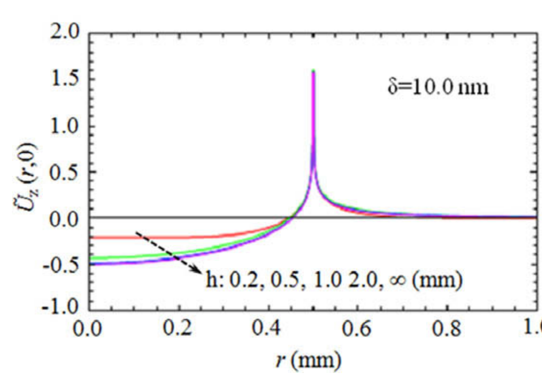

Fig. 4. Numerical results of dimensionless vertical displacement at the surface of an elastic membrane when the radius of wetting area and the breadth of capillary layer are supposed to be 0.5 mm and 10.0 nm.

Table 2
Numerical results of vertical displacements at some points in the surface when the radius of wetting area is 0.2 mm and the breadth of capillary layer is supposed to: (a) 1.0 nm; (b) 10.0 nm; (c) 40.0 nm.

Membrane thickness (μm)	Distance to the center of wetting area (mm)			
	0.0	0.08	0.16	0.2
(a) R = 0.2 mm and δ = 1.0 nm				
1.0	−0.003	−0.003	−0.003	1.165
10.0	−0.025	−0.025	−0.025	1.557
40.0	−0.100	−0.101	−0.090	1.741
∞	−0.500	−0.437	−0.177	1.796
(b) R = 0.2 mm and δ = 10.0 nm				
1.0	−0.003	−0.003	−0.003	0.799
10.0	−0.025	−0.025	−0.025	1.190
40.0	−0.100	−0.101	−0.090	1.374
∞	−0.500	−0.437	−0.177	1.430
(c) R = 0.2 mm and δ = 40.0 nm				
1.0	−0.003	−0.003	−0.003	0.578
10.0	−0.025	−0.025	−0.025	0.970
40.0	−0.100	−0.101	−0.090	1.154
∞	−0.500	−0.437	−0.177	1.209

Table 3
Relative errors between maximum of dimensionless vertical displacement of finite-thickness membrane and that by Lester or Rusanov when the breadth of capillary layer is supposed to be: (a) 1.0 nm; (b) 10.0 nm; (c) 40.0 nm.

Membrane thickness (μm)	$\max(\tilde{U}_z)$ (δ = 1.0 nm)	Relative error (%)
(*a*)		
1.0	1.165	−35.13
10.0	1.557	−13.31
40.0	1.741	−3.06
	$\max(\tilde{U}_z)$ (δ = 10.0 nm)	
(*b*)		
1.0	0.799	−44.13
10.0	1.190	−16.78
40.0	1.374	−3.92
	$\max(\tilde{U}_z)$ (δ = 40.0 nm)	
(*c*)		
1.0	0.578	−52.19
10.0	0.970	−19.77
40.0	1.154	−4.55

and shown in Table 3. From Table 3, we can find that the absolute value of relative errors for a 1.0 μm-thick membrane will be larger than 35%. For a membrane made of hard matter such as silicon (E = 170 GPa, ν = 0.27, $\theta = \sim 0°$, γ_{lv} = 72 mN/m), the maximum vertical displacement induced by a water droplet is in order of 1.0 pico-meter although the membrane is only 1.0 μm thick, thus the surface deformation will be neglected in most cases no matter how thick it is. However, if the membrane is made of soft matter, for example, PDMS, whose Young's modulus is very low and usually below 1 MPa, the contact angle is about 120°, then the maximum vertical displacement might be much larger and in order of several hundred nanometers, so Lester's or Rusanov's solutions may not correctly describe the surface deformation of a PDMS membrane whose thickness is below tens micrometers or even smaller. And in micro/nanofluidics such kind of soft membranes are commonly used. Therefore, the effect of membrane thickness on surface deformation induced by a liquid droplet should be considered. Moreover, even if the vertical displacement induced by a liquid droplet is rather small in the monolayer (though much higher than pico-meter), it is still important for the molecular reorientation associated with the displacement because this results in change in drop-surface intermolecular forces, which was studied by Tadmor [18].

In fact, as in [20], the actual contact angle might differ from Young's contact angle, which will produce a horizontal force and then induce an excess surface deformation. However, the excess deformation including the terms of $(\cos\theta_1 - \cos\theta)$ and $(1–2\nu)$ might be negligible because the difference between the actual contact angles θ_1 and the Young one θ is just only several degrees and the Poisson's ratio of the membrane material is nearly 0.5.

4. Summary

Elastic deformation of finite-thickness membrane induced by a liquid droplet has been studied by using an integral transform method. And the theoretical solution of vertical displacements at the surface was derived and could be reduced to Lester's or Rusanov's solutions without considering the difference of contact angle. Moreover, some simulations of elastic deformation of finite-thickness membrane induced by a water droplet were made. The numerical results showed that there exists a saturated membrane thickness. When a membrane is thicker than such a thickness, the membrane can be treated as a half-infinite body. When a membrane is made of a hard matter such as silicon, the surface deformation induced by a liquid droplet can be neglected for most cases. However, if the membrane is made of a soft material such as PDMS and its thickness is below tens micrometers or even smaller, the maximum vertical displacement induced by a droplet will be of hundreds nanometers and the membrane thickness has great effect on the deformation when the membrane thickness is smaller than several micrometers. This may not be neglected in the design and application of micro/nanodevices such as μ-TAS and lab-on-a-chip.

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Appendix A. Supplementary data

Supplementary data associated with this article can be found, in the online version, at doi:10.1016/j.jcis.2009.08.001.

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