## Performance analysis of bit error rate for free space optical communication with tip-tilt compensation based on gamma–gamma distribution

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In this paper, the gamma–gamma probability distribution is used to model turbulent channels. The bit error rate (BER) performance of free space optical (FSO) communication systems employing on-off keying (OOK) or subcarrier binary phase-shift keying (BPSK) modulation format is derived. A tip-tilt adaptive optics system is also incorporated with a FSO system using the above modulation formats. The tip-tilt compensation can alleviate effects of atmospheric turbulence and thereby improve the BER performance. The improvement is different for different turbulence strengths and modulation formats. In addition, the BER performance of communication systems employing subcarrier BPSK modulation is much better than that of compatible systems employing OOK modulation with or without tip-tilt compensation.

Keywords: free space optical communication, atmospheric turbulence, adaptive optics, bit error rate, tip-tilt compensation.

### 1. Introduction

Free space optical (FSO) communication links have some distinct advantages over conventional microwave and optical fiber communication systems by virtue of their high carrier frequencies that permit large capacity, enhanced security, high data rate and so on. However, a number of limitations due to atmospheric turbulence make it difficult to achieve the desired level of performance [1, 2]. Atmospheric turbulence causes phase disturbances along propagation paths that are manifested as intensity fluctuation (scintillation), beam broadening and beam wandering at the receiver. These disturbances are generally considered to be a multiplicative noise source that reduces the capability of receiver to distinguish the information contained in the modulated optical wave [3]. They make the received signal fade and impair the link performance severely.

In order to mitigate signal fading, many techniques were proposed, such as error control coding, spatial diversity and adaptive optics technique. In [4, 2, 5], the error performance of coded free space optical links is studied for log-normal (LN), K and gamma-gamma distributed turbulence channels, respectively. However, optical links with transmission rates of order of gigabits exhibit high temporal correlation. This requires large-size interleavers to achieve the promised coding gains in many scenarios [6]. With spatial diversity, the bit error rate (BER) performance of FSO links over log-normal atmospheric turbulence channels is investigated [6]. But with the number of transmitter/receiver apertures increasing, the overall system will become more complex. According to previous work of NoLL [7], the so-called tip-tilt distortion is the main component of the above disturbances induced upon a wavefront, which is passing through the turbulent atmosphere. Performance deteriorations induced by the turbulent atmosphere can be alleviated by a tip-tilt adaptive optics (AO) system, which is simpler, less expensive and more practically feasible comparing to a conventional AO system. Thus, the tip-tilt compensation approach is incorporated and evaluated in this paper.

Many statistical quantities such as bit error rate [1-6], outage probability [8, 9] and ergodic capacity [9] in free space optical communication are associated with atmospheric turbulence-induced irradiance fluctuations. Thus, the probability density function (PDF) of irradiance fluctuations is absolutely necessary in quantitatively estimating these statistical quantities. Over years, many irradiance PDF models [1] have been proposed to describe irradiance fluctuations due to atmospheric turbulence, such as the log-normal distribution model, *K* distribution model, gamma–gamma distribution model.

The most widely used model for the PDF is log-normal distribution due to its simplicity. However, it is only applicable to the weaker turbulence regime. As the strength of turbulence increases, multiple scattering effects become important and significant deviations from log-normal statistics are exhibited in experimental data. Log-normal PDF underestimates the behavior in the tails as compared with measurement results. For communication systems, accurate detection and fade probabilities primarily depend on the tails of the PDF, hence underestimating this region significantly affects the accuracy of the communication performance. Gamma– gamma distribution is a two-parameter distribution which is based on a doubly stochastic theory of scintillation, and assumes that small-scale irradiance fluctuations are modulated by large-scale irradiance fluctuations of the propagating wave, both governed by independent gamma distributions. It has been shown that gamma–gamma distribution is valid for both weaker and stronger turbulence regimes [1, 10].

Based on the log-normal distribution and on-off keying (OOK) modulation, the BER expressions for FSO links with multiple transmitter and/or receiver apertures with and without channel state information considering both spatially independent and correlated channels are derived [6]. In a *K* turbulence channel, the BER performance with pointing error [11], outage probability and ergodic capacity [9] of FSO links over a *K* turbulence channel are investigated. Some channel statistics are derived in closed

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form. In wireless optical heterodyne communication systems over gamma-gamma turbulence channles, the outage probability and the average BER are also studied [8].

Although OOK modulation has been the dominant modulation format for intensity modulation of optical communication due to its simplicity and low implementation cost, a threshold is required for the demodulation [6]. So, phase shift keying (PSK)-based subcarrier intensity modulation (SIM) can be employed to avoid the need for the threshold required by OOK modulation and improve the performance of communication systems [12–15]. In comparison with OOK, SIM-based format offers less power efficiency. However, various techniques for reducing the average optical power requirement have been reported to resolve this problem [12]. HUANG et al. [13] briefly analyzed subcarrier PSK intensity modulation in FSO communication for the first time. An experiment on differential binary phase-shift keying (BPSK) was reported. It was shown that subcarrier PSK intensity modulation was superior to OOK modulation in the presence of atmospheric turbulence. LI et al. [14] derived the BER for FSO communication systems employing OOK or subcarrier PSK and compared system performances for different modulation formats. An explanation from frequency domain was developed to answer why subcarrier PSK intensity modulation can perform better. On the basis of these analyses, KUMAR and JAIN [15] investigated the effects of aperture averaging on BER performance in a FSO communication link between a satellite and a ground station. In the higher-turbulence case, although OOK modulation format was not usable due to severe degradation in the BER performance, it can be used in satellite communication by utilizing the aperture averaging method.

In this paper, turbulence channels are described by the gamma-gamma probability distribution and then we derive the BER performance of FSO communication system with OOK or subcarrier BPSK modulation. A decision criterion is developed to obtain an optimal threshold for OOK modulation under various turbulent conditions. A tip-tilt AO system is also incorporated with a FSO communication system operating over atmospheric turbulence. The effects of tip-tilt compensation on the BER performance of OOK and subcarrier BPSK modulation format are evaluated and compared. This work mainly considers the spatial phase correction introduced by tip-tilt compensation. Other factors, such as the temporal response of AO systems, are not included.

### 2. Irradiance probability density function

The gamma–gamma distribution is used in this paper for evaluating the BER of communication systems. Its form is given by [1]:

$$p(I) = \frac{2}{\Gamma(\alpha)\Gamma(\beta)I} (\alpha\beta I)^{\frac{\alpha+\beta}{2}} K_{\alpha-\beta} \left(2\sqrt{\alpha\beta I}\right), \quad I > 0$$
(1)

where I is the normalized irradiance,  $\alpha$  and  $\beta$  are parameters of the PDF,  $\Gamma$  is the gamma function, and  $K_{\alpha-\beta}$  is the modified Bessel function of the second kind of

order  $\alpha - \beta$ . For plane wave and zero inner scale, the parameters  $\alpha$  and  $\beta$  are given by the expressions:

$$\alpha = \frac{1}{\exp(\sigma_{\ln x}^2) - 1}$$
(2)

$$\beta = \frac{1}{\exp(\sigma_{\ln y}^2) - 1}$$
(3)

where  $\sigma_{\ln x}^2$  and  $\sigma_{\ln y}^2$  are large-scale and small-scale log-irradiance variances, respectively. For the plane wave propagation,  $\sigma_{\ln x}^2$  and  $\sigma_{\ln y}^2$  are defined by [1]:

$$\sigma_{\ln x}^2 = 8\pi^2 k^2 \int_0^L \int_0^\infty \kappa \Phi_n(\kappa) G_x(\kappa) \left[ 1 - \cos\left(\frac{\kappa^2 z}{k}\right) \right] d\kappa dz$$
(4)

$$\sigma_{\ln y}^{2} = 8\pi^{2}k^{2}\int_{0}^{L}\int_{0}^{\infty}\kappa \Phi_{n}(\kappa)G_{y}(\kappa)\left[1-\cos\left(\frac{\kappa^{2}z}{k}\right)\right]d\kappa dz$$
(5)

where  $\Phi_n(\kappa) = 0.033 C_n^2(z) \kappa^{-11/3}$  is the Kolmogorov spectrum,  $C_n^2(z)$  is the refractive index structure parameter,  $k = 2\pi/\lambda$  is the wave-number,  $\lambda$  is the wavelength, *D* is the receiving aperture diameter and *L* is the propagation distance,  $G_x(\kappa)$  and  $G_y(\kappa)$  are large-scale and small-scale filters, respectively, which have the forms:

$$G_{x}(\kappa) = \exp\left(-\frac{\kappa^{2}}{\kappa_{x}^{2}}\right)$$
(6)

$$G_{y}(\kappa) = \exp\left(-\frac{\kappa^{11/3}}{(\kappa^{2} + \kappa_{y}^{2})^{11/6}}\right)$$
(7)

where  $\kappa_x^2 = \frac{2.61}{1+1.11\sigma_1^2} \frac{k}{L}$ ,  $\kappa_y^2 = 3(1+0.69\sigma_1^2) \frac{k}{L}$ ,  $\sigma_1^2$  is the Rytov variance given by:

$$\sigma_1^2 = 1.23 C_n^2 k^{7/6} L^{11/6}$$
(8)

where  $C_n^2$  is the refractive-index structure constant. For simplicity, the discussion below assumes that the refractive-index structure parameter is a constant.

According to Andrews's analysis [1], the variance of log-irradiance takes the form

$$\sigma_{\ln I}^2 = \sigma_{\ln x}^2 + \sigma_{\ln y}^2 \tag{9}$$

# **3.** Atmospheric turbulence channel and adaptive optics filter function

In an optical communication system employing intensity modulation through turbulent atmosphere, the received optical intensity P(t) can be written as

$$P(t) = I(t) P_0(t) + n(t)$$
(10)

where  $P_0(t)$  is the received optical intensity without turbulence, and  $n(t) \sim N(0, \sigma_n^2)$  is the additive white Gaussian noise (AWGN). I(t) is a stationary random process representing the intensity gain induced by atmospheric turbulence, its probability density function (PDF) is given by Eq. (1).

Because atmospheric turbulence is considered as a spatial noise that degrades the optical wave signal, a transverse spectral filter, *i.e.*, the AO filter, can be used to filter out this spatial noise. The AO filter function is derived from the Fourier transforms of Zernike polynomials. Large-scale and small-scale log-irradiance variances with tip-tilt components correction are written as [3]

$$\sigma_{\ln x, t-\text{removed}}^{2} = \frac{8\pi^{2}k^{2}}{2\pi} \int_{0}^{L} \int_{0}^{\infty} \int_{0}^{2\pi} \kappa \Phi_{n}(\kappa) G_{x}(\kappa) \left[1 - \cos\left(\frac{\kappa^{2}z}{k}\right)\right] \left[1 - F(\kappa, D, \varphi)\right] d\kappa dz d\varphi$$
(11)

$$\sigma_{\ln y, t-\text{removed}}^{2} = \frac{8\pi^{2}k^{2}}{2\pi} \int_{0}^{L} \int_{0}^{\infty} \int_{0}^{2\pi} \kappa \Phi_{n}(\kappa) G_{y}(\kappa) \left[1 - \cos\left(\frac{\kappa^{2}z}{k}\right)\right] \left[1 - F(\kappa, D, \varphi)\right] d\kappa dz d\varphi$$
(12)

In the above equations,  $[1 - F(\kappa, D, \varphi)]$  is the tip-tilt AO filter function that operates on the transverse spatial spectrum ( $\kappa$  dependence) and represents the removal of tip-tilt aberration modes by phase conjugation. *D* is the receiving aperture diameter and  $F(\kappa, D, \varphi)$  is given by

$$\begin{cases} F_{x}(\kappa, D, \varphi) \\ F_{y}(\kappa, D, \varphi) \\ F(\kappa, D, \varphi) \end{cases} = \left[ \frac{4 J_{2}(\kappa D/2)}{\kappa D/2} \right]^{2} \begin{cases} \cos^{2} \varphi \\ \sin^{2} \varphi \\ 1 \end{cases}$$
(13)

where  $J_{n+1}$  is the Bessel function.

By removing the tip-tilt aberration modes (equivalent to a tip-tilt AO system), the parameters  $\alpha$  (or  $\beta$ ) can be obtained by replacing  $\sigma_{\ln x}^2$  (or  $\sigma_{\ln y}^2$ ) in Eq. (2) or (3) with  $\sigma_{\ln x, t\text{-removed}}^2$  (or  $\sigma_{\ln y, t\text{-removed}}^2$ ). We can also obtain the variance of log-irradiance with tip-tilt compensation according to Eq. (9).

# 4. Theoretical analysis of BER for free space optical communication systems over turbulent channels

#### 4.1. OOK modulation format and optimum decision strategy

In this modulation format, the received electrical signal [14] can be written as

$$r(t) = I(t) + \sum_{i = -\infty}^{\infty} I(t) a_i g(t - iT_s) + n(t)$$
(14)

where  $a_i$  is the level of the *i*-th symbol and  $a_i \in \{-1, 1\}$ , the transmission probabilities of bit -1 and 1 are  $P_0$  and  $P_1$ , respectively; g(t) is the rectangle pulse shape function and  $T_s$  is the symbol time.

When there is no turbulence and only AWGN is present, the BER can be written as [14]

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2\sigma_n^2}}\right)$$
(15)

where  $E_b = a_i^2 = 1$  is the normalized bit energy,  $\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} \exp(-t^2/2) dt$ . In decibels, the signal-to-noise ratio (SNR) can be defined as

SNR (dB) = 
$$10 \log \left(\frac{E_b}{\sigma_n^2}\right)$$
 (16)

As the transmitted signal corresponding to bit "0" is zero, the received signal for this bit will have only AWGN. However, both turbulence noise and AWGN are present for bit "1". So, the signal currents for bit "0" and "1" can be written as

$$r(t) = \begin{cases} n(t), & a_i = -1 \\ 2I(t) + n(t), & a_i = 1 \end{cases}$$
(17)

Let the variance of n(t) and I(t) be  $\sigma_n^2$  and  $\sigma_I^2$ , respectively. The PDF of the converted electrical signal when bit "0" or "1" is sent is given by (x > 0):

$$p(r|0) = \frac{1}{\sqrt{2\pi} \sigma_n} \exp\left(-\frac{r^2}{2\sigma_n^2}\right)$$
(18)

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$$p(r|1) = \frac{1}{\sqrt{2\pi} \sigma_n} \frac{2(\alpha\beta)^{(\alpha+\beta)/2}}{\Gamma(\alpha)\Gamma(\beta)} \int_0^\infty x^{\frac{\alpha+\beta}{2}-1} K_{\alpha-\beta}(2\sqrt{\alpha\beta}x) \exp\left[-\frac{(r-2x)^2}{2\sigma_n^2}\right] dx$$
(19)

Let the threshold T > 0 and the BER in atmospheric environment be given by [6]

$$P_{e} = P_{0}P(r > T|0) + P_{1}P(r < T|1) = P_{0}\int_{T}^{\infty} p(r|0)dr + P_{1}\int_{-\infty}^{T} p(r|1)dr =$$

$$= \frac{1}{2}P_{0}\operatorname{erfc}\left(\frac{T}{\sqrt{2}\sigma_{n}}\right) + \frac{P_{1}(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)}\int_{0}^{\infty} x^{\frac{\alpha+\beta}{2}-1}K_{\alpha-\beta}(2\sqrt{\alpha\beta x})\operatorname{erfc}\left[\frac{2x-T}{\sqrt{2}\sigma_{n}}\right]dx$$
(20)

According to the optimal maximum a posteriori (MAP) symbol-by-symbol detection with equiprobable OOK data [6, 16], the likelihood function is given by:

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$$A(r) = \frac{p(r|1)}{p(r|0)} = \frac{2(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)} \int_{0}^{\infty} x^{\frac{\alpha+\beta}{2}-1} K_{\alpha-\beta}(2\sqrt{\alpha\beta x}) \exp\left[-\frac{4x(x-r)}{2\sigma_{n}^{2}}\right] dx$$
(21)

It is noteworthy that Eqs. (20) and (21) are also presented in a more compact form [6] based on the log-normal distribution model. In Eq. (21), the threshold *T* is given by the value of *r* that satisfies the equation  $\Lambda(r) = 1$ , which can be numerically calculated according to different Rytov variances  $\sigma_1^2$  and noise variances  $\sigma_n^2$ . Figure 1



Fig. 1. Optimal threshold of OOK versus the standard deviation  $\sigma_1$  for different noise variances.

shows the optimal threshold T versus  $\sigma_1$ . It clearly illustrates the dynamism in the OOK threshold. Therefore, the receiver must be able to select the threshold point adaptively for the optimal performance.

#### 4.2. Subcarrier PSK modulation format

For an optical communication system employing subcarrier PSK intensity modulation, the data sequence is first modulated with PSK, which can be implemented with existing microchips at very low cost. Secondly, the PSK signal is upconverted to an intermediate frequency (IF). This process can be implemented in the electrical circuit of the transmitter by any of the currently used RF modulation formats. Then, the modulated electrical signal is utilized to control the irradiance of optical beam in the transmitter. In the receiver, the optical signal is firstly converted to an electrical signal. Then, the receiver demodulates the electrical signal by using RF devices like selective filters and stable oscillators.

For subcarrier BPSK intensity modulation, the output electrical signal is [14]

$$r(t) = I(t) + mI(t) \left[ s_i(t) \cos(2\pi f_c t) - s_q(t) \sin(2\pi f_c t) \right] + n_i(t) \cos(2\pi f_c t) - n_q(t) \sin(2\pi f_c t)$$
(22)

where  $s_i(t)$  and  $s_q(t)$  are the in-phase and quadrature signals, respectively, *m* is the modulation index with  $m \in (0, 1]$ , and  $f_c$  is the intermediate carrier frequency;  $n_i(t)$  and  $n_q(t)$  are the narrow band white Gaussian processes with the variance  $\sigma_n^2$ .

For subcarrier BPSK modulation format, the threshold is chosen at 0, *i.e.*, T = 0. The conditional PDF p(r|x) of the received signal can be written as

$$p(r|x) = \begin{cases} \frac{2}{\sqrt{2\pi} \sigma_n \Gamma(\alpha) \Gamma(\beta)} \left(\frac{\alpha \beta}{m}\right)^{\frac{\alpha+\beta}{2}} \int_0^\infty t^{\frac{\alpha+\beta}{2}-1} K_{\alpha-\beta} \left(2\sqrt{\frac{\alpha\beta t}{m}}\right) \exp\left[-\frac{(r-t)^2}{2\sigma_n^2}\right] dt, \quad x = +1 \\ \frac{2}{\sqrt{2\pi} \sigma_n \Gamma(\alpha) \Gamma(\beta)} \left(\frac{\alpha \beta}{m}\right)^{\frac{\alpha+\beta}{2}} \int_0^\infty t^{\frac{\alpha+\beta}{2}-1} K_{\alpha-\beta} \left(2\sqrt{\frac{\alpha\beta t}{m}}\right) \exp\left[-\frac{(r+t)^2}{2\sigma_n^2}\right] dt, \quad x = -1 \end{cases}$$
(23)

and its BER in the atmospheric environment is given by

$$P_{e} = \frac{(\alpha\beta)^{\frac{\alpha+\beta}{2}}}{\Gamma(\alpha)\Gamma(\beta)} \int_{0}^{\infty} x^{\frac{\alpha+\beta}{2}-1} K_{\alpha-\beta}(2\sqrt{\alpha\beta x}) \operatorname{erfc}\left[\frac{mx}{\sqrt{2}\sigma_{n}}\right] dx$$
(24)

Without turbulence and in the presence of AWGN only, the BER will be [14]

$$P_e = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{m^2}{2\sigma_n^2}}\right)$$
(25)

Without loss of generality, here m = 1 is assumed.

### 5. Results

#### 5.1. Comparison between fixed and optimal threshold for OOK modulation

For OOK modulation format, LI *et al.* [14] and KUMAR and JAIN [15] have considered the BER in an optical communication system by using a fixed threshold, which is chosen as half of the mean of received signal corresponding to bit 1. However, the fixed threshold will not optimize the performance of the communication systems over changing atmospheric conditions, while the optimal threshold can optimize the performance of the communication systems. A comparison between fixed and optimal threshold for the OOK modulation format is shown in Fig. 2. Here, the fixed threshold is also chosen as half of the mean of received signal corresponding to bit 1. We know that the mean of received intensity is 1 from Eq. (1), so the fixed threshold is T = 0.5. The optimal threshold is obtained by Eq. (21). Figure 2 shows that the optimal threshold can significantly optimize the performances of communication systems by significantly reducing the BER of systems. When  $\sigma_1^2$  is 0.3, the BER of OOK is more than  $10^{-2}$  and does not almost change with increasing SNR.



Fig. 2. The BER is plotted as a function of the SNR by using a fixed or optimal threshold.

The discussion below assumes that the OOK modulation format employs the optimal threshold.

#### 5.2. Effects of turbulence strength on BER for different modulation formats

In order to investigate the effects of turbulence strength on the BER for different modulation formats, the performance of two modulation formats (OOK and subcarrier BPSK) for different  $\sigma_1^2$  (0.02, 0.1, and 0.32) is compared. The computational results are presented in Fig. 3. There are degradations in the BER performance of both OOK and subcarrier BPSK due to atmospheric turbulence. However, the degradation in the BER performance of OOK modulation format is much more than that of subcarrier BPSK modulation format. In order to obtain the same BER, the difference



Fig. 3. The BER performance of FSO communication systems employing OOK and subcarrier BPSK modulation format with  $\sigma_1^2 = 0.02$ ,  $\sigma_1^2 = 0.1$  and  $\sigma_1^2 = 0.32$ .

in SNR between OOK and subcarrier BPSK reduces with decreasing  $\sigma_1^2$ . At BER = 10<sup>-9</sup>, the difference in SNR is about 2 dB with  $\sigma_1^2 = 0.02$  while it is about 4 dB with  $\sigma_1^2 = 0.1$ . Under all the above turbulence conditions, especially in the higher-turbulence case, the BER performance of subcarrier BPSK is always better than that of OOK irrespective of the turbulence strength.

# 5.3. Effects of tip-tilt compensation on BER for different modulation formats and turbulence strengths

From Equations (9), (11) and (12), it is apparent that tip-tilt compensation can reduce the log-irradiance variance of an optical carrier collected by the receiver and thus improve the performance of a FSO communication system.

A performance comparison of two modulation formats (OOK and subcarrier BPSK) with tip-tilt compensation for different turbulence strengths is shown in Fig. 4.



Fig. 4. Variation of BER versus SNR with tip-tilt compensation at different  $\sigma_1^2$ : OOK (a), subcarrier BPSK (b).

N = 0 denotes the BER without tip-tilt compensation. N = 1 denotes the BER with tip-tilt compensation. By utilizing tip-tilt compensation, a major improvement in the BER performance occurs for both modulation formats. The tip-tilt compensation can be utilized to reduce the power requirements of the transmitter or to reduce the sensitivity requirement of the receiver while maintaining a lower BER.

#### 5.4. Effects of SNR on BER with/without tip-tilt compensation

After evaluation of tip-tilt compensation at different turbulence strengths, the relationship between BER and tip-tilt compensation is further investigated at fixed SNRs. Figure 5 shows the BER as a function of Rytov variances for two fixed values of SNR = 16 dB and SNR = 20 dB. It shows that SNR has significant effect on the BER. For example, the BER for OOK modulation is reduced by eight orders of magnitude,



Fig. 5. The BER employing OOK and BPSK modulation versus Rytov variance  $\sigma_1^2$  with/without tip-tilt compensation.

and the BER for subcarrier BPSK is reduced by nine orders of magnitude when SNR increases from 16 dB to 20 dB at  $\sigma_1^2 = 0.01$ . But in comparison with SNR, the tip-tilt compensation provides a minor improvement in the BER. In addition, the BER is found to be significantly limited by the turbulence strength. At SNR = 20 dB, the BER of the OOK modulation format increases from  $6.3 \times 10^{-16}$  to  $2.1 \times 10^{-7}$  when Rytov variance increases from 0.01 to 0.1.

### 6. Conclusions

The BER performance of FSO communication systems employing OOK or subcarrier BPSK modulation format through turbulence channels is investigated in this paper. Based on the gamma–gamma distribution, the BER performances for two modulation formats are derived. By introducing a tip-tilt filter function, the effects of tip-tilt compensation on the BER performance of FSO communication systems employing OOK and subcarrier BPSK modulation formats over atmospheric turbulence channels are investigated.

For OOK modulation format, the BER performance employing an optimal threshold is superior to that employing a fixed threshold. The incorporation of tip-tilt compensation considerably reduces the BER of a FSO communication system. Furthermore, the BER improvement of subcarrier BPSK modulation format is much more than that of OOK by a tip-tilt compensation. Increasing SNR can notably improve the BER performance. The incorporation of a tip-tilt AO system in a FSO communication system through the turbulent atmosphere is relatively simpler, less expensive, and more practically feasible. It is also shown that a significant reduction of the BER can be obtained by incorporating tip-tilt compensation with a FSO communication system.

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