Commun. Theor. Phys. (Beijing, China) **44** (2005) pp. 303–306
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Vol. 44, No. 2, August 15, 2005

# Synchronization of Modified Chua's Circuit with $x|x|$ Function*

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(Received December 14, 2004; Revised January 24, 2005)

***Abstract*** *This paper considers the chaos synchronization of the modified Chua's circuit with $x|x|$ function. We firstly show that a couple of the modified Chua systems with different parameters and initial conditions can be synchronized using active control when the values of parameters both in drive system and response system are known aforehand. Furthermore, based on Lyapunov stability theory we propose an adaptive active control approach to make the states of two identical Chua systems with unknown constant parameters asymptotically synchronized. Moreover the designed controller is independent of those unknown parameters. Numerical simulations are given to validate the proposed synchronization approach.*

**PACS numbers:** 05.45.Gg
**Key words:** chaos synchronization, modified Chua's circuit, active control, adaptive active control

## 1 Introduction

Since the idea of chaos synchronization for two identical chaotic systems from different initial conditions was introduced by Pecora and Carroll in 1990,[1] synchronization of chaotic systems has attracted much attention and numerous methods have been proposed due to its potential applications in secure communication, laser and biological systems, and so on. Many possible applications have been discussed by computer simulations and realized in laboratory conditions.[2−4] In order to achieve synchronization, it needs to design a controller based on signals from the drive system and the response system.

It has been proved that many nonlinear functions (maps) can generate chaos, such as the smooth quadratic functions (e.g. Lorenz system and Henon map), the piecewise linear functions (e.g. Chua's circuit). Recently Tang *et al.*[5] demonstrated that in some systems function $x|x|$ also can generate chaos, and the circuit implementation of the modified Chua's circuit with $x|x|$ function was reported.[6] In this paper, we study the dynamical characteristics of the modified Chua's circuit for different parameters and show that a couple of the modified Chua's circuits with different system parameters and initial conditions can be synchronized using active control.[7] However, it needs to precisely know the information of the drive system parameters and the response system parameters to construct the controller. In practice, most system parameters cannot be exactly known in advance. So, based on Lyapunov stability theory we propose an adaptive active control method to synchronize two modified Chua systems in the presence of multiple unknown system parameters. And the proposed method guarantees that the controller is independent of those uncertain parameters. Numerical simulation results demonstrate the effectiveness of the proposed approach.

The organization of this paper is as follows. In Sec. 2, the modified Chua's circuit model is introduced and its dynamical characteristics are given. In Sec. 3, we study the application of active control to the modified Chua systems with all known parameters. In Sec. 4, an adaptive active control method is proposed to synchronize the modified Chua systems with unknown parameters, and the numerical simulations are given. In Sec. 5, we end with some conclusions.

## 2 Modified Chua's Circuit

Chua's circuit[8] has been widely used as the experimental vehicle for research on nonlinear science. By replacing its piecewise linear V-I characteristic of the nonlinear component with function $x|x|$,[6] we hereby consider its following variant (say the modified Chua's circuit),

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \alpha[y - g(x)]\,, \quad \frac{\mathrm{d}y}{\mathrm{d}t} = x - y + z\,, \quad \frac{\mathrm{d}z}{\mathrm{d}t} = -\beta y\,, \quad (1)$$

where $x$, $y$, and $z$ are dimensionless variables, $\alpha$ and $\beta$ are positive constant parameters, and $g(x) = ax + bx|x|$, in which $a$ and $b$ are chosen as $-1/6$ and $1/16$ respectively.

The phase portraits of the modified circuit for $\beta = 14.0$ with different $\alpha$ are depicted in Fig. 1.

Seen from Fig. 1, when $\alpha$ is small, two equilibria $P^+$ and $P^-$ are stable (Fig. 1(a)). With the increase of $\alpha$, the stability margin of the equilibrium begins to deteriorate at the Hopf point and a small periodic orbit can be observed. As $\alpha$ increases further, the system generates a cascaded period-doubling bifurcation for asymmetric periodic orbits (a period-2 limit cycle with $\alpha = 8.835$ is shown in Fig. 1(b)). At the end of the period-doubling cascade,

*The project partially supported by National Natural Science Foundation of China under Grant Nos. 60204008 and 60374060
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two asymmetric chaotic attractors can be observed. For the sake of clarity, only the right-hand one is shown in Fig. 1(c). These two asymmetric attractors are moving closer and closer to each other, and eventually "glued" together to form a double scroll attractor (the result with $\alpha = 9.267$ is illustrated in Fig. 1(d)).

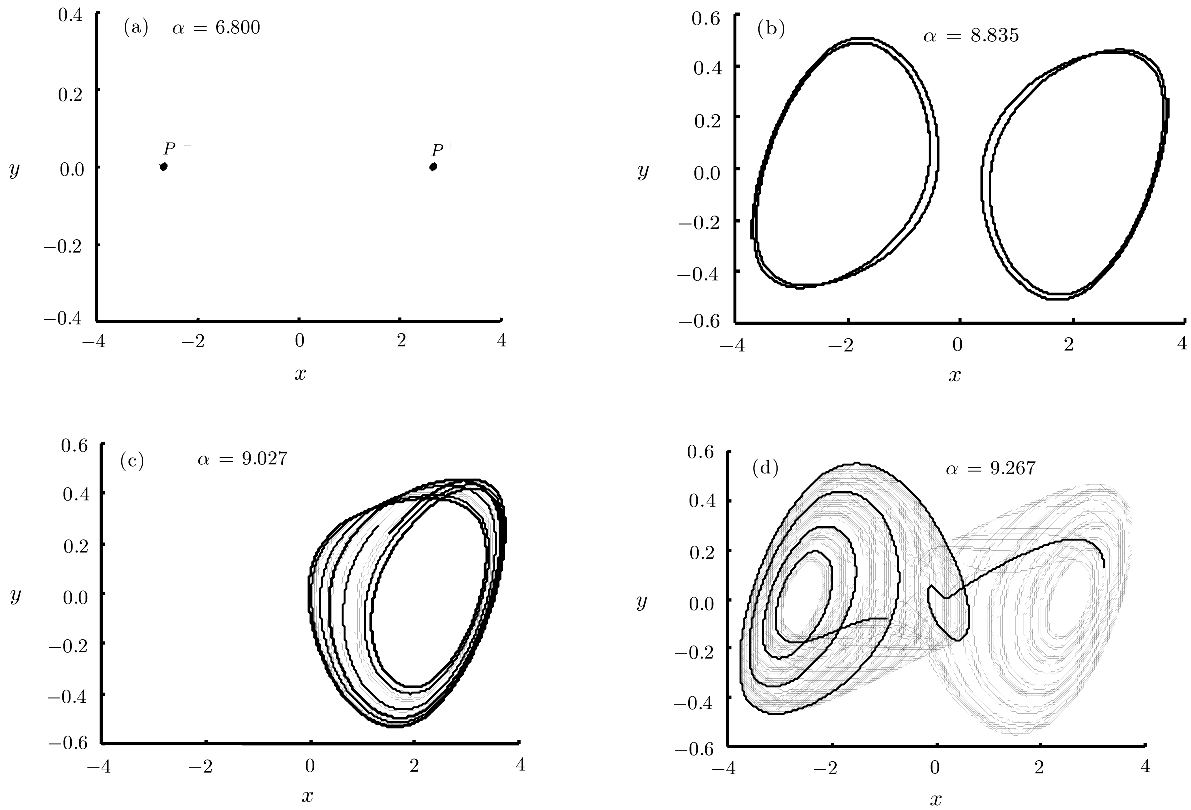


**Fig. 1** Phase portraits: The left periodic orbits of (a) and (b) are generated with initial conditions $x(0) = 1.01$, $y(0) = z(0) = 0.01$, while those on the right are associated with the initial conditions $x(0) = -1.01$, $y(0) = z(0) = -0.01$.

## 3 Synchronization of Modified Chua's Circuit Using Active Control

Obviously, parameter $\alpha$ plays an important role in determining the characteristics of the modified Chua's circuit. So here we only consider two modified Chua's circuits governed by Eq. (1) with different initial conditions and different but known $\alpha$, where the drive system and the response system are denoted with subscript 1 and subscript 2 respectively,

$$\frac{\mathrm{d}x_1}{\mathrm{d}t} = \alpha[y_1 - g(x_1)]\,,$$
$$\frac{\mathrm{d}y_1}{\mathrm{d}t} = x_1 - y_1 + z_1\,, \quad \frac{\mathrm{d}z_1}{\mathrm{d}t} = -\beta y_1\,, \tag{2}$$

$$\frac{\mathrm{d}x_2}{\mathrm{d}t} = \alpha'[y_2 - g(x_2)] + u_1\,,$$
$$\frac{\mathrm{d}y_2}{\mathrm{d}t} = x_2 - y_2 + z_2 + u_2\,,$$
$$\frac{\mathrm{d}z_2}{\mathrm{d}t} = -\beta y_2 + u_3\,. \tag{3}$$

The goal of the control is to find a controller $u = [u_1, u_2, u_3]^{\mathrm{T}}$ to force the response system to follow the drive system. Subtracting Eq. (2) from Eq. (3) yields the error dynamical system between the drive system and the response system as follows:

$$\dot{e}_1 = \alpha'(y_2 - g(x_2)) - \alpha(y_1 - g(x_1)) + u_1\,,$$
$$\dot{e}_2 = e_1 - e_2 + e_3 + u_2\,, \qquad \dot{e}_3 = -\beta\, e_2 + u_3\,, \tag{4}$$

where $e_1 = x_2 - x_1$, $e_2 = y_2 - y_1$, and $e_3 = z_2 - z_1$.

The synchronization for modified Chua's circuits is to achieve the asymptotic stability of the zero solution of the error system. In Ref. [6] the practical realization of $g(x)$ in the modified Chua's circuit was presented, so if the system parameters can be exactly measured, then the active control function can be defined in the following equation:

$$u_1 = \alpha'(g(x_2) - y_2) - \alpha(\, g(x_1) - y_1) - e_1\,,$$
$$u_2 = -e_1 - e_3\,, \qquad u_3 = \beta e_2 - e_3\,. \tag{5}$$

With suitable choice of control signals, the error system Eq. (4) has the eigenvalues $-1$, $-1$, and $-1$, which will lead to the error states $e(t) = [e_1, e_2, e_3]^{\mathrm{T}}$ converge to zero as time tends to infinity and hence the synchronization of two modified Chua's circuits is achieved.

Let the values of $\alpha$ and $\alpha'$ be 8.835 and 9.027 respectively, $\beta = 14.0$, the initial state of the drive system be $\{x(0) = 1.01, y(0) = z(0) = 0.01\}$, and the initial state of the response system be $\{x(0) = -1.01, y(0) = z(0) = -0.01\}$. Seen from Fig. 1, the drive system has a periodic solution on the right of the phase portrait (as Fig. 1(b)), while the response system generates a chaotic attractor on the left (as Fig. 1(c)).

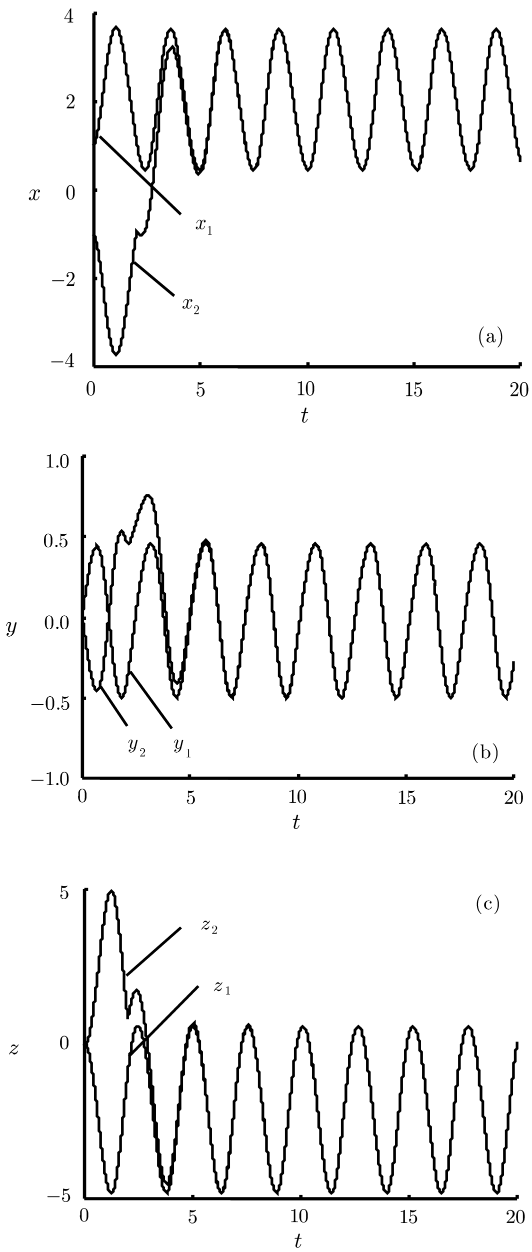


**Fig. 2** Solution of the coupled Chua systems with adaptive active control. (a) Signals $x_1$ and $x_2$; (b) Signals $y_1$ and $y_2$; (c) Signals $z_1$ and $z_2$.

The simulation results of the active control for the above two Chua systems are shown in Fig. 2. We observe that the drive system and the response system synchronize after $t = 5$.

## 4 Chaos Synchronization Using Adaptive Active Control

For the above active control, one should know the information about the system parameters of drive system and the response system to obtain control signals that have a significant effect on the control performance. However, in practice system parameters often cannot be exactly known in advance. Assume that the drive and the response systems (modified Chua's circuits) with different initial conditions have the same but unknown parameters. Our goal is to design a controller $u = [u_1, u_2, u_3]^{\mathrm{T}}$ that is independent of all parameters and an update law for all parameters to make the response system and the drive system synchronize asymptotically.

Choosing a Lyapunov function for Eq. (4) as follows:

$$V(e, \tilde{\alpha}, \tilde{\beta}) = \frac{1}{2} e^{\mathrm{T}} e + \frac{1}{2}\tilde{\alpha}^2 + \frac{1}{2}\tilde{\beta}^2 ,$$

$$\text{Subject to} \quad \frac{\mathrm{d}V(e, \tilde{\alpha}, \tilde{\beta})}{\mathrm{d}t} \le -W(e) , \tag{6}$$

where $\tilde{\alpha} = \alpha - \hat{\alpha}$, $\tilde{\beta} = \beta - \hat{\beta}$, and $\hat{\alpha}$, $\hat{\beta}$ are estimated values of unknown parameters $\alpha$, $\beta$, respectively, and $W(e) = e_1^2 + e_2^2 + e_3^2$.

We design a controller $u$ and update laws for estimated parameters as follows, which is named adaptive active control in this paper,

$$\begin{aligned} &u_1 = \hat{\alpha}(g(x_2) - g(x_1) - e_2) - e_1 , \\ &u_2 = -e_1 - e_3 , \qquad u_3 = \hat{\beta} e_2 - e_3 , \\ &\dot{\hat{\alpha}} = -(g(x_2) - g(x_1) - e_2)e_1 , \quad \dot{\hat{\beta}} = -e_2 e_3 . \end{aligned} \tag{7}$$

Thus, the time derivation of $V(e, \tilde{\alpha}, \tilde{\beta})$ along the solutions of Eq. (4) is

$$\begin{aligned} \frac{\mathrm{d}V(e, \tilde{\alpha}, \tilde{\beta})}{\mathrm{d}t} &= e^{\mathrm{T}}\dot{e} + \tilde{\alpha}\dot{\tilde{\alpha}} + \beta\dot{\tilde{\beta}} \\ &= e_1(\alpha e_2 - \alpha g(x_2) + \alpha g(x_1) + u_1) \\ &\quad + e_2(e_1 - e_2 + e_3 + u_2) + e_3(-\beta e_2 + u_3) \\ &\quad + \tilde{\alpha}(-\dot{\hat{\alpha}}) + \tilde{\beta}(-\dot{\hat{\beta}}) \\ &= -e_1^2 - e_2^2 - e_3^2 \le -W(e) . \end{aligned} \tag{8}$$

Hence, the synchronization of two modified Chua's circuits can be achieved. Moreover, it is obvious that the controller in Eq. (7) is independent of unknown parameters.

It has been shown in Sec. 2 that the modified Chua's circuit presents chaos when $\alpha$=9.027 and $\beta$=14.0. In the following simulation, suppose 9.027 and 14.0 are the true values of "unknown" parameters $\alpha$ and $\beta$ in both drive system and response system, and the initial estimated values are $\hat{\alpha}(0) = 8.0$ and $\hat{\beta}(0) = 10.0$. The initial states of the drive system and the response system are

$$\{x(0) = 1.01, y(0) = z(0) = 0.01\} ,$$
$$\{x(0) = -1.01, y(0) = -1.01, z(0) = -0.01\} ,$$

respectively.

The simulation results of the adaptive active control are shown in Fig. 3. The results display that the synchronization of the two modified Chua's circuits can be achieved successfully after $t = 5$.

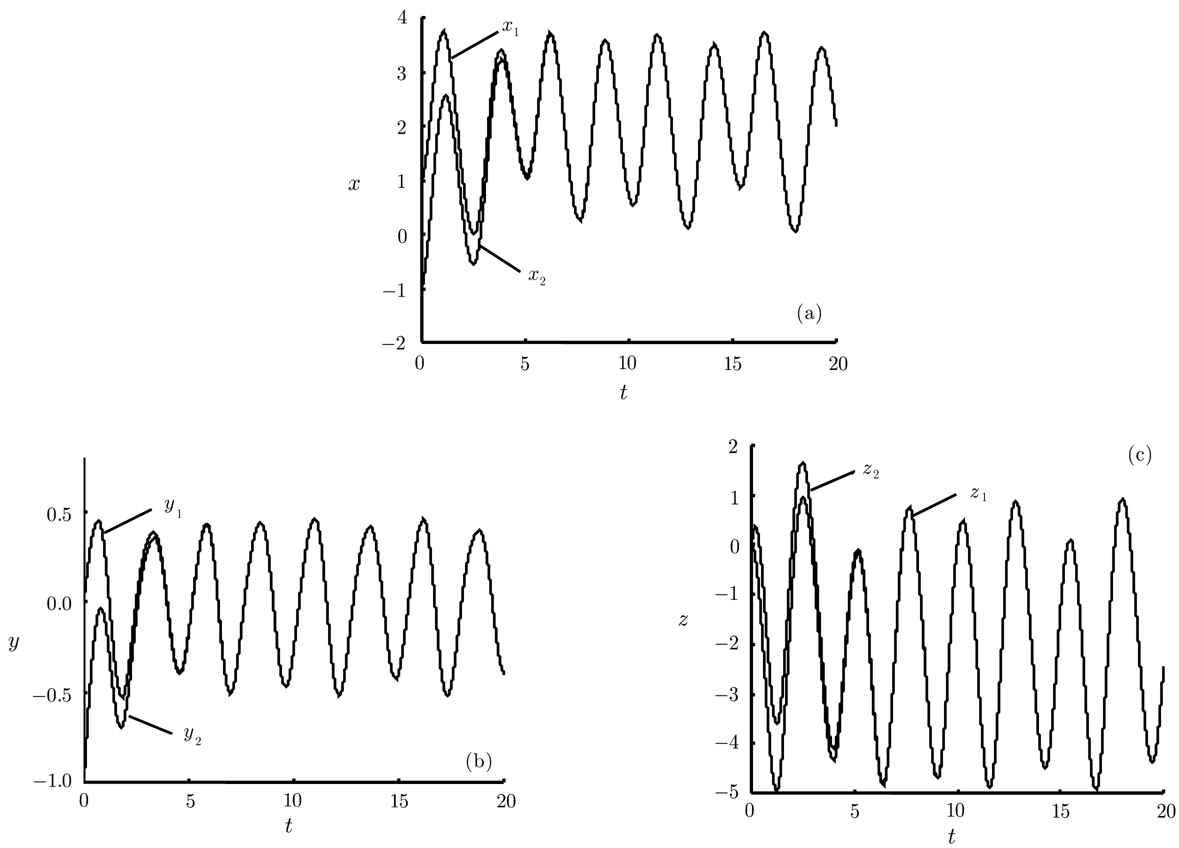


**Fig. 3** Solution of the coupled Chua systems with adaptive active control. (a) Signals $x_1$ and $x_2$; (b) Signals $y_1$ and $y_2$; (c) Signals $z_1$ and $z_2$.

## 5 Conclusion

In conclusion, an active control method is applied to the chaos synchronization of the modified Chua's circuit, and based on Lyapunov stability theory an adaptive active control approach is proposed to achieve chaos synchronization in the presence of multiple unknown system parameters. The proposed adaptive active control method guarantees that the controller is independent of those uncertain parameters so that it is very practicable and easy to be implemented. The simulation results have validated the effectiveness of the proposed approach.

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