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任意非正交曲线坐标系在叶轮机械气动计算中的应用*

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摘　　要

本文应用向量分析的方法，推导出非正交曲线坐标系的梯度、散度和旋度的计算公式，从而得到了无粘性流体在叶轮机械中稳定相对运动的三维流动的气动热力学基本方程。然后，讨论了用任意非正交曲线坐标表示的势函数方程，以及两类相对流面（S_1 和 S_2 流面）的流函数方程(吴仲华方程)．最后也给出了速度梯度法（或称流线曲率法）方程的三种形式．

一、任意非正交曲线坐标系

吴仲华同志的论文[2]的发表，引起了国内外的重视．他用张量的方法首次推导得到了任意非正交曲线坐标的叶轮机械三维流动的基本方程，促进了叶轮机械气动力设计方法的发展．我们自 1973 年也开始了此项工作．文献[2]的发表启发促进了我们的工作．

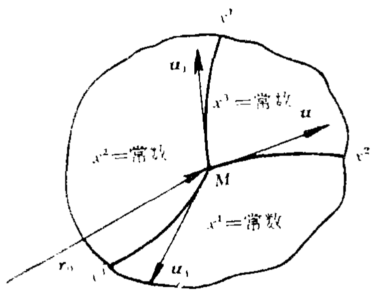

图 1　任意非正交曲线坐标系

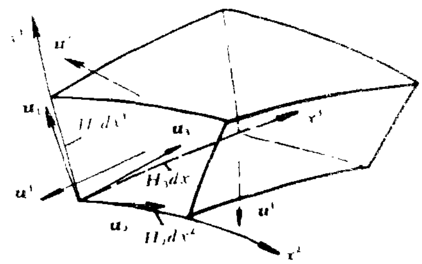

图 2　非正交曲线六面形微元体

在任意非正交曲线坐标中，任意点M的位置可以用三个坐标值 x^1, x^2, x^3 来表示．于M点在三个变数 x^1, x^2, x^3 增加的方向取切线方向的三个单位向量 $\boldsymbol{u}_1$, $\boldsymbol{u}_2$, $\boldsymbol{u}_3$．而 $\boldsymbol{u}^1$, $\boldsymbol{u}^2$, $\boldsymbol{u}^3$ 分别表示垂直于体积为 $d\mathscr{V}$ 的非正交曲线形六面体的三个面的单位法线向量．

向径 $\boldsymbol{r}_0(x^1, x^2, x^3)$ 的偏导数可写为:

$$\partial \boldsymbol{r}_0/\partial x^i = H_i \boldsymbol{u}_i, \qquad (i = 1, 2, 3) \tag{1.1}$$

这里 H_i 为向量 $\partial \boldsymbol{r}_0/\partial x^i$ 的长，称之为**拉梅系数**，它们可以从下式计算中得到:

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$$H_i^2 = (\partial \boldsymbol{r}_0/\partial x^i)^2 = (\partial x/\partial x^i)^2 + (\partial y/\partial x^i)^2 + (\partial z/\partial x^i)^2. \qquad (1.2)$$

某一向量 $\boldsymbol{C}$ 在两类三个不共平面的向量 $\boldsymbol{u}_1$, $\boldsymbol{u}_2$, $\boldsymbol{u}_3$ 和 $\boldsymbol{u}^1$, $\boldsymbol{u}^2$, $\boldsymbol{u}^3$ 方向分解，可写为：

$$\left.\begin{aligned} \boldsymbol{C} &= C^1\boldsymbol{u}_1 + C^2\boldsymbol{u}_2 + C^3\boldsymbol{u}_3, \\ \boldsymbol{C} &= C_1\boldsymbol{u}^2 + C_2\boldsymbol{u}^2 + C_3\boldsymbol{u}^3. \end{aligned}\right. \qquad (1.3)$$

C_i 与 C^i 之间的关系为：

$$C_i = [C^1\cos(\boldsymbol{u}_i, \boldsymbol{u}_1) + C^2\cos(\boldsymbol{u}_i, \boldsymbol{u}_2) + C^3\cos(\boldsymbol{u}_i, \boldsymbol{u}_3)]/\cos(\boldsymbol{u}^i, \boldsymbol{u}_i) \qquad (1.4)$$

众所周知，一指向无向量 Φ 增加最快的方向，且大小等于此方向的导数值的向量，称之为 Φ 的**梯度**，而且在任意方向 $\boldsymbol{m}$ 的导数等于 Φ 的梯度 $(\nabla\Phi)$ 在此方向的投影．根据此定义，不难求出梯度的计算公式：

$$\nabla\Phi = \frac{1}{\cos(\boldsymbol{u}^1, \boldsymbol{u}_1)}\frac{\partial\Phi}{H_1\partial x^1}\boldsymbol{u}^1 + \frac{1}{\cos(\boldsymbol{u}^2, \boldsymbol{u}_2)}\frac{\partial\Phi}{H_2\partial x^2}\boldsymbol{u}^2 + \frac{1}{\cos(\boldsymbol{u}^3, \boldsymbol{u}_3)}\frac{\partial\Phi}{H_3\partial x^3}\boldsymbol{u}^3. \qquad (1.5)$$

向量 $\boldsymbol{C}$ 经过包围某点的无穷小体积的面上对单位体积的通量，称为该向量的**散度**，即：

$$\nabla\cdot\boldsymbol{C} = \lim_{d\mathscr{V}\to 0}\frac{\oint_A \boldsymbol{C}\cdot d\boldsymbol{A}}{d\mathscr{V}}.$$

按此定义写出通过 dA_1, dA_2, dA_3, dA_1', dA_2', dA_3' 六个面的通量之和，并除以六面体的体积 $d\mathscr{V}$，便得：

$$\nabla\cdot\boldsymbol{C} = [\partial(H_2H_3C^1\Pi)/\partial x^1 + \partial(H_3H_1C^2\Pi)/\partial x^2 + \partial(H_1H_2C^3\Pi)/\partial x^3]/H_1H_2H_3\Pi. \qquad (1.6)$$

$$\Pi = \sin(\boldsymbol{u}_2, \boldsymbol{u}_3)\cos(\boldsymbol{u}^1, \boldsymbol{u}_1) = \sin(\boldsymbol{u}_1, \boldsymbol{u}_2)\cos(\boldsymbol{u}^3, \boldsymbol{u}_2) = \sin(\boldsymbol{u}_3, \boldsymbol{u}_1)\cos(\boldsymbol{u}^2, \boldsymbol{u}_2) \qquad (1.7)$$

某一向量 $\boldsymbol{C}$ 的旋度 $\nabla\times\boldsymbol{C}$ 在任意方向 $\boldsymbol{m}$ 的投影，可以由下面的定义给出：

$$(\nabla\times\boldsymbol{C})_{\boldsymbol{m}} = \lim_{dA\to 0}\frac{\oint_C \boldsymbol{C}\cdot d\boldsymbol{r}_0}{dA}$$

上式中的 dA 为积分 $\oint_C$ 封闭周线所包含的面积．由此可见，$(\nabla\times C)_{\boldsymbol{m}}$ 与所选择的坐标系无关．例如，向量 $\boldsymbol{C}$ 的旋度在面积 dA_1 法向 $\boldsymbol{u}^1$ 的投影为：

$$(\nabla\times\boldsymbol{C})_{\boldsymbol{u}^1} = \{\partial[H_3C_3\cos(\boldsymbol{u}^3,\boldsymbol{u}_3)]/\partial x^2 - \partial[H_2C_2\cos(\boldsymbol{u}^2,\boldsymbol{u}_2)]/\partial x^3\}/H_2H_3\sin(\boldsymbol{u}_2, \boldsymbol{u}_3)$$

最后得**旋度**三个分式的表示式：

$$\left.\begin{aligned} \xi^1 &= \{\partial[H_2C_2\cos(\boldsymbol{u}^2, \boldsymbol{u}_2)]/\partial x^3 - \partial[H_3C_3\cos(\boldsymbol{u}^3, \boldsymbol{u}_3)]/\partial x^2\}/H_2H_3\sin(\boldsymbol{u}_2, \boldsymbol{u}_3)\cos(\boldsymbol{u}^1, \boldsymbol{u}_1); \\ \xi^2 &= \{\partial[H_3C_3\cos(\boldsymbol{u}^3, \boldsymbol{u}_3)]/\partial x^1 - \partial[H_1C_1\cos(\boldsymbol{u}^1, \boldsymbol{u}_1)]/\partial x^3\}/H_3H_1\sin(\boldsymbol{u}_3, \boldsymbol{u}_1)\cos(\boldsymbol{u}^2, \boldsymbol{u}_2); \\ \xi^3 &= \{\partial[H_1C_1\cos(\boldsymbol{u}^1, \boldsymbol{u}_1)]/\partial x^2 - \partial[H_2C_2\cos(\boldsymbol{u}^2, \boldsymbol{u}_2)]/\partial x^1\}/H_1H_2\sin(\boldsymbol{u}_1, \boldsymbol{u}_2)\cos(\boldsymbol{u}^3, \boldsymbol{u}_3). \end{aligned}\right\} \qquad (1.8)$$

$$\nabla\times\boldsymbol{C} = \xi^1\boldsymbol{u}_1 + \xi^2\boldsymbol{u}_2 + \xi^3\boldsymbol{u}_3. \qquad (1.9)$$

应用向量分析的方法，不难求得用任意非正交曲线坐标表示的叶轮机械中相对稳定流动的气动热力学方程组．由式 (1.6) 和 $\nabla\cdot(\rho\boldsymbol{W}) = 0$ 得到**连续方程**为：

$$\frac{\partial(H_2H_3\rho W^1\Pi)}{\partial x^1} + \frac{\partial(H_3H_1\rho W^2\Pi)}{\partial x^2} + \frac{\partial(H_1H_2\rho W^3\Pi)}{\partial x^3} = 0. \qquad (1.10)$$

由式 (1.8) (1.9) 代入运动方程的两种形式，即

$$\nabla(|\boldsymbol{W}|^2/2) - \boldsymbol{W} \times (\nabla \times \boldsymbol{W}) + 2\boldsymbol{\omega} \times \boldsymbol{W} - \omega^2 \boldsymbol{r} = -\nabla p/\rho;$$

$$\boldsymbol{W} \times (\nabla \times \boldsymbol{V}) = Jg(\nabla I - T\nabla s),$$

经过向量运算，不难求得用压力梯度和滞止转焓 I 与熵 s 的梯度来表示的**运动方程** $\boldsymbol{u}^1$，$\boldsymbol{u}^2$，$\boldsymbol{u}^3$ 间的三个分式：

$$\frac{1}{2}\frac{\partial(W)^2}{H_1\partial x^1} + \frac{W^3}{H_1H_3}\left\{\frac{\partial[H_1W_1\cos(\boldsymbol{u}^1,\boldsymbol{u}_1)]}{\partial x^3} - \frac{\partial[H_3W_3\cos(\boldsymbol{u}^3,\boldsymbol{u}_3)]}{\partial x^1}\right\}$$
$$-\frac{W^2}{H_1H_2}\left\{\frac{\partial[H_2W_2\cos(\boldsymbol{u}^2,\boldsymbol{u}_2)]}{\partial x^1} - \frac{\partial[H_1W_1\cos(\boldsymbol{u}^1,\boldsymbol{u}_1)]}{\partial x^2}\right\}$$
$$+2\omega[W^2\sin(\boldsymbol{u}_1,\boldsymbol{u}_2)\cos(\boldsymbol{u}^3,\boldsymbol{e}_z) - W^3\sin(\boldsymbol{u}_3,\boldsymbol{u}_1)\cos(\boldsymbol{u}^2,\boldsymbol{e}_z)]$$
$$-\omega^2 r\cos(\boldsymbol{u}_1,\boldsymbol{e}_r) = -\frac{1}{\rho H_1}\frac{\partial p}{\partial x^1}; \tag{1.11a}$$

$$\frac{1}{2}\frac{\partial(W)^2}{H_2\partial x^2} + \frac{W^1}{H_1H_2}\left\{\frac{\partial[H_2W_2\cos(\boldsymbol{u}^2,\boldsymbol{u}_2)]}{\partial x^1} - \frac{\partial[H_1W_1\cos(\boldsymbol{u}^1,\boldsymbol{u}_1)]}{\partial x^2}\right\}$$
$$-\frac{W^3}{H_2H_3}\left\{\frac{\partial[H_3W_3\cos(\boldsymbol{u}^3,\boldsymbol{u}_3)]}{\partial x^2} - \frac{\partial[H_2W_2\cos(\boldsymbol{u}^2,\boldsymbol{u}_2)]}{\partial x^3}\right\}$$
$$+2\omega[W^3\sin(\boldsymbol{u}_2,\boldsymbol{u}_3)\cos(\boldsymbol{u}^1,\boldsymbol{e}_z) - W^1\sin(\boldsymbol{u}_1,\boldsymbol{u}_2)\cos(\boldsymbol{u}^3,\boldsymbol{e}_z)]$$
$$-\omega^2 r\cos(\boldsymbol{u}_2,\boldsymbol{e}_r) = -\frac{1}{\rho H_2}\frac{\partial p}{\partial x^2}; \tag{1.11 b}$$

$$\frac{1}{2}\frac{\partial(W)^2}{H_3\partial x^3} + \frac{W^2}{H_2H_3}\left\{\frac{\partial[H_3W_3\cos(\boldsymbol{u}^3,\boldsymbol{u}_3)]}{\partial x^2} - \frac{\partial[H_2W_2\cos(\boldsymbol{u}^2,\boldsymbol{u}_2)]}{\partial x^3}\right\}$$
$$-\frac{W^1}{H_3H_1}\left\{\frac{\partial[H_1W_1\cos(\boldsymbol{u}^1,\boldsymbol{u}_1)]}{\partial x^3} - \frac{\partial[H_3W_3\cos(\boldsymbol{u}^3,\boldsymbol{u}_3)]}{\partial x^1}\right\}$$
$$+2\omega[W^1\sin(\boldsymbol{u}_3,\boldsymbol{u}_1)\cos(\boldsymbol{u}^2,\boldsymbol{e}_z) - W^2\sin(\boldsymbol{u}_2,\boldsymbol{u}_3)\cos(\boldsymbol{u}^1,\boldsymbol{e}_z)]$$
$$-\omega^2 r\cos(\boldsymbol{u}_3,\boldsymbol{e}_r) = -\frac{1}{\rho H_3}\frac{\partial p}{\partial x^3}. \tag{1.11 c}$$

$$-W^3\left\{\frac{\partial[H_1V_1\cos(\boldsymbol{u}^1,\boldsymbol{u}_1)]}{\partial x^3} - \frac{\partial[H_3V_3\cos(\boldsymbol{u}^3,\boldsymbol{u}_3)]}{\partial x^1}\right\}\Big/H_3$$
$$+W^2\left\{\frac{\partial[H_2V_2\cos(\boldsymbol{u}^2,\boldsymbol{u}_2)]}{\partial x^1} - \frac{\partial[H_1V_1\cos(\boldsymbol{u}^1,\boldsymbol{u}_1)]}{\partial x^2}\right\}\Big/H_2$$
$$= Jg\left(\frac{\partial I}{\partial x^1} - \frac{T\partial s}{\partial x^1}\right); \tag{1.12a}$$

$$-W^1\left\{\frac{\partial[H_2V_2\cos(\boldsymbol{u}^2,\boldsymbol{u}_2)]}{\partial x^1} - \frac{\partial[H_1V_1\cos(\boldsymbol{u}^1,\boldsymbol{u}_1)]}{\partial x^2}\right\}\Big/H_1$$
$$+W^3\left\{\frac{\partial[H_3V_3\cos(\boldsymbol{u}^3,\boldsymbol{u}_3)]}{\partial x^2} - \frac{\partial[H_2V_2\cos(\boldsymbol{u}^2,\boldsymbol{u}_2)]}{\partial x^3}\right\}\Big/H_3$$
$$= Jg\left(\frac{\partial I}{\partial x^2} - T\frac{\partial s}{\partial x^2}\right); \tag{1.12b}$$

$$-W^2\left\{\frac{\partial[H_3V_3\cos(\boldsymbol{u}^3,\boldsymbol{u}_3)]}{\partial x^2} - \frac{\partial[H_2V_2\cos(\boldsymbol{u}^2,\boldsymbol{u}_2)]}{\partial x^3}\right\}\Big/H_2$$
$$+W^1\left\{\frac{\partial[H_1V_1\cos(\boldsymbol{u}^1,\boldsymbol{u}_1)]}{\partial x^3} - \frac{\partial[H_3V_3\cos(\boldsymbol{u}^3,\boldsymbol{u}_3)]}{\partial x^1}\right\}\Big/H_1$$

$$= Jg\left(\frac{\partial I}{\partial x^3} - \frac{T\partial s}{\partial x^3}\right). \tag{1.12c}$$

二、叶轮机械三维位势流动的势函数方程

设在叶轮机械内部的流动是等熵过程，且进口的总焓和熵都均匀，从而($\boldsymbol{V}/\!/\nabla\times\boldsymbol{V}$除外) $\nabla\times\boldsymbol{V}=0$，则在绝对流场中存在一个势函数 Φ，它的梯度与非正交速度分量的关系为：

$$V_i = \left(\frac{\partial\Phi}{H_i dx^i}\right)\Big/\cos(\boldsymbol{u}^i, \boldsymbol{u}_i) \quad (i = 1, 2, 3); \tag{2.1}$$

$$V^i = \frac{1}{\cos(\boldsymbol{u}^i, \boldsymbol{u}_i)}\left[\frac{\cos(\boldsymbol{u}^i, \boldsymbol{u}^1)}{\cos(\boldsymbol{u}^1, \boldsymbol{u}_1)}\frac{\partial\Phi}{H_1\partial x^1} + \frac{\cos(\boldsymbol{u}^i, \boldsymbol{u}^2)}{\cos(\boldsymbol{u}^2, \boldsymbol{u}_2)}\frac{\partial\Phi}{H_2\partial x^2} + \frac{\cos(\boldsymbol{u}^i, \boldsymbol{u}^3)}{\cos(\boldsymbol{u}^3, \boldsymbol{u}_3)}\frac{\partial\Phi}{H_3\partial x^3}\right] \quad (i = 1, 2, 3). \tag{2.2}$$

将上式代入式(1.10)，经整理化简后，得**势函数方程**：

$$\begin{aligned} &B_{11}(\partial^2\Phi/\partial x^{12}) + B_{22}(\partial^2\Phi/\partial x^{22}) + B_{33}(\partial^2\Phi/\partial x^{32}) + 2B_{12}(\partial^2\Phi/\partial x^1\partial x^2) \\ &\quad + 2B_{23}(\partial^2\Phi/\partial x^2\partial x^3) + 2B_{31}(\partial^2\Phi/\partial x^3\partial x^1) + (\partial\Phi/\partial x^1)(\partial B_{11}/\partial x^1 \\ &\quad + \partial B_{21}/\partial x^2 + \partial B_{31}/\partial x^3) + (\partial\Phi/\partial x^2)(\partial B_{12}/\partial x^1 + \partial B_{22}/\partial x^2 + \partial B_{32}/\partial x^3) \\ &\quad + (\partial\Phi/\partial x^3)(\partial B_{31}/\partial x^1 + \partial B_{23}/\partial x^2 + \partial B_{33}/\partial x^3) = \omega(\partial A_1/\partial x^1 \\ &\quad + \partial A_2/\partial x^2 + \partial A_3/\partial x^3) - H_1H_2H_3\Pi(W^1\partial\ln\rho/H_1\partial x^1 \\ &\quad + W^2\partial\ln\rho/H_2\partial x^2 + W^3\partial\ln\rho/H_3\partial x^3). \end{aligned} \tag{2.3}$$

系数 $A_1, A_2, A_3, B_{11}, B_{12}\cdots\cdots$ 只是几何参数的函数，其表示式为：

$$A_i = \frac{rH_1H_2H_3\Pi}{H_i}\frac{\cos(\boldsymbol{u}^i, \boldsymbol{e}_\varphi)}{\cos(\boldsymbol{u}^i, \boldsymbol{u}_i)}; \qquad B_{ij} = B_{ji} = \frac{H_1H_2H_3}{H_iH_j}\frac{\Pi\cos(\boldsymbol{u}^i, \boldsymbol{u}^j)}{\cos(\boldsymbol{u}^i, \boldsymbol{u}_i)\cos(\boldsymbol{u}^j, \boldsymbol{u}_j)}. \tag{2.4}$$

势函数的方程属于第二类边值问题的求解，因此它的边界条件可按壁面的法向单位向量与速度向量的标量相乘为零的条件得出，即 $\boldsymbol{n}\cdot\boldsymbol{W}=0$. 对于远离叶片上下游处，气流速度的方向往往平行于内、外壳，例如对于轴流式叶轮机械出口，就有：

$$W_r = \partial\Phi/\partial r = 0 \quad (z = \pm\infty).$$

对于几种特殊情况，势函数方程可以得到某些简化. 例如：x^2 坐标选为角坐标 φ，x^1 与 x^3 为子午面任意非正交的曲线坐标，且 $\boldsymbol{e}_\varphi$ 与 $\boldsymbol{u}_1$, $\boldsymbol{u}_3$ 组成的平面正交. 则有：

$$\begin{aligned} &\frac{H_3H_2}{H_1\sin\theta_{13}}\frac{\partial^2\Phi}{\partial x^{12}} + \frac{H_3H_1}{H_2\sin\theta_{13}}\frac{\partial^2\Phi}{\partial x^{22}} + \frac{H_1H_2}{H_3\sin\theta_{13}}\frac{\partial^2\Phi}{\partial x^{32}} - 2\frac{H_2\cos\theta_{13}}{\sin\theta_{13}}\frac{\partial^2\Phi}{\partial x^3\partial x^1} \\ &\quad + \left[\partial\left(\frac{H_3H_2}{H_1\sin\theta_{13}}\right)\Big/\partial x^1 - \partial\left(\frac{H_2\cos\theta_{13}}{\sin\theta_{13}}\right)\Big/\partial x^3\right]\frac{\partial\Phi}{\partial x^1} \\ &\quad + \left[\partial\left(\frac{H_3H_1\sin\theta_{13}}{H_2}\right)\Big/\partial x^2\right]\frac{\partial\Phi}{\partial x^2} + \left[\partial\left(\frac{H_1H_2}{H_3\sin\theta_{13}}\right)\Big/\partial x^3 \right. \\ &\quad \left. - \partial\left(\frac{H_2\cos\theta_{13}}{\sin\theta_{13}}\right)\Big/\partial x^1\right]\frac{\partial\Phi}{\partial x^3} = -\omega\frac{\partial(rH_2H_3\sin\theta_{13})}{H_1\partial x^1} \\ &\quad - H_1H_2H_3\sin\theta_{13}\left(\frac{W^1}{H_1}\frac{\partial\ln\rho}{\partial x^1} + \frac{W_\varphi}{H_2}\frac{\partial\ln\rho}{\partial x^2} + \frac{W^3}{H_3}\frac{\partial\ln\rho}{\partial x^3}\right). \end{aligned} \tag{2.5}$$

其近壁处的边界条件为:

$$\boldsymbol{n}\cdot\boldsymbol{W}=n^1(\partial\Phi/H_1\partial x^1)+n_\varphi(\partial\Phi/H_2\partial x^2-\omega r)+n^3(\partial\Phi/H_3\partial x^3)=0 \tag{2.6}$$

式中的 n^1, n_φ, n^3 分别为壁面法向单位向量在 $\boldsymbol{u}_1$, $\boldsymbol{u}_2$, $\boldsymbol{u}_3$ 三个方向上的分量.

三、两类相对流面的任意非正交曲线坐标的流函数方程

早在 1952 年吴仲华教授就提出了一个通用的计算模型，利用两类相对流面 (S_1, S_2) 把三维流动简化为两个二维流动问题，由这两类相对流面的互相迭代，可以得到三维流动的解. 求解这两类相对流面可以用通流矩阵法或速度梯度法(流线曲率法). 前一个方法就是应用流函数 Ψ 作为未知数来求解流场中的诸参数. 在 S_1 **流面**的计算中，常应用第二种形式的运动方程. 此时，x^2 与 x^3 为 S_1 流面上的任意曲线坐标，x^1 垂直于流面; $\boldsymbol{u}_1$ 向分速和 $\boldsymbol{V}$ 的旋度分量为零.

$$\left\{\frac{\partial[H_2W_2\cos(\boldsymbol{u}^2,\boldsymbol{u}_2)]}{\partial x^3}-\frac{\partial[H_3W_3\cos(\boldsymbol{u}^3,\boldsymbol{u}_3)]}{\partial x^2}\right\}\Big/H_2H_3\sin(\boldsymbol{u}_2,\boldsymbol{u}_3)\cos(\boldsymbol{u}^1,\boldsymbol{u}_1)$$
$$+2\omega\cos(\boldsymbol{u}^1,\boldsymbol{e}_z)/\cos(\boldsymbol{u}^1,\boldsymbol{u}_1)=0. \tag{3.1}$$

对于均匀分布的滞止转焓和熵的情况. 对于迴转面的 S_1 流面，满足连续方程

$$\frac{\partial(H_3\tau\rho W^2\sin\theta_{23})}{\partial x^2}+\frac{\partial(H_2\tau\rho W^3\sin\theta_{23})}{\partial x^3}=0 \tag{3.2}$$

的流函数 Ψ 有如下等式:

$$\frac{\partial\Psi}{\partial x^2}=H_2\tau\rho gW^3\sin\theta_{23},\quad \frac{\partial\Psi}{\partial x^3}=-H_3\tau\rho gW^2\sin\theta_{23}. \tag{3.3}$$

将以上诸式代入式 (3.1)，整理得与文献[2]相同的**流函数方程**(吴仲华方程):

$$\frac{1}{H_2^2}\frac{\partial^2\Psi}{\partial(x^2)^2}+\frac{D}{H_2H_3}\frac{\partial^2\Psi}{\partial x^2\partial x^3}+\frac{1}{H_3^2}\frac{\partial^2\Psi}{\partial(x^3)^2}+E\frac{\partial\Psi}{H_2\partial x^2}+F\frac{\partial\Psi}{H_3\partial x^3}=G. \tag{3.4}$$

式中: $D=-2\cos\theta_{23}$,

$$E=\frac{\partial}{H_2\partial x^2}\left(\ln\frac{H_3}{H_2\sin\theta_{23}}\right)-\frac{\partial\ln\tau}{H_2\partial x^2}+\cos\theta_{23}\frac{\partial\ln\tau}{H_3\partial x^3}+\frac{1}{\sin\theta_{23}}\frac{\partial\theta_{23}}{H_3\partial x^3},$$

$$F=\frac{\partial}{H_3\partial x^3}\left(\ln\frac{H_2}{H_3\sin\theta_{23}}\right)-\frac{\partial\ln\tau}{H_3\partial x^3}+\cos\theta_{23}\frac{\partial\ln\tau}{H_2\partial x^2}+\frac{1}{\sin\theta_{23}}\frac{\partial\theta_{23}}{H_2\partial x^2},$$

$$G=2\omega\tau\rho g\sin\sigma\sin^2\theta_{23}+\frac{\partial\Psi}{H_2\partial x^2}\left(\frac{\partial\ln\rho}{H_2\partial x^2}-\cos\theta_{23}\frac{\partial\ln\rho}{H_3\partial x^3}\right)$$
$$+\frac{\partial\Psi}{H_3\partial x^3}\left(\frac{\partial\ln\rho}{H_3\partial x^3}-\cos\theta_{23}\frac{\partial\ln\rho}{H_2\partial x^2}\right)+Jg\left(\frac{\partial I}{H_2\partial x^2}-T\frac{\partial s}{H_2\partial x^2}\right)$$
$$\cdot\tau^2\rho^2g^2\sin^2\theta_{23}\Big/\left(\frac{\partial\Psi}{H_2\partial x^2}\right). \tag{3.5}$$

S_2 **流面**的流函数方程. 坐标系可以如此选取: $x^2=\varphi$, x^1 与 x^3 为子午面上任意非正交的曲线坐标. 同时，设垂直于 S_2 流面的单位法线向量为 $\boldsymbol{n}$. 沿流面上对 x^1 和 x^3 的偏导数为:

$$\frac{\boldsymbol{\partial}(\ )}{H_1\boldsymbol{\partial}x^1}=\frac{\partial(\ )}{H_1\partial x^1}+\frac{n_1\cos(\boldsymbol{u}^1,\boldsymbol{u}_1)}{n_2}\frac{\partial(\ )}{r\partial\varphi};\quad \frac{\boldsymbol{\partial}(\ )}{H_3\boldsymbol{\partial}x^3}=\frac{\partial(\ )}{H_3\partial x^3}+\frac{n_3\cos(\boldsymbol{u}^3,\boldsymbol{u}_3)}{n_2}\frac{\partial(\ )}{r\partial\varphi}. \tag{3.6}$$

考虑到由于周向压力梯度所造成的加于流面上的力

$$\boldsymbol{F} = -\left(\frac{1}{n_\varphi}\frac{1}{\rho}\frac{\partial p}{r\partial\varphi}\right)\boldsymbol{n} = F_1\boldsymbol{u}^1 + F_2\boldsymbol{u}^2 + F_3\boldsymbol{u}^3.$$

以及 $\boldsymbol{F}$ 力与流速 $\boldsymbol{W}$ 的垂直关系和单位法线向量 $\boldsymbol{n}$ 与流速 $\boldsymbol{W}$ 的关系，整理并化简后，便得到 S_2 流面的运动方程的三个分式：

$$\begin{aligned}&W^1\frac{\partial W^1}{H_1\partial x^1} + W^2\frac{\partial W^3}{H_3\partial x^3} + \left(W^1\frac{\partial W^2}{H_1\partial x^1} + W^3\frac{\partial W^3}{H_3\partial x^3}\right)\cos\theta_{31}\\&\quad-\left(2\omega W_\varphi + \omega^2 r + \frac{W_\varphi^2}{r}\right)\cos\theta_1\\&\quad+W^3(W^1 + W^3\cos\theta_{31})\frac{1}{H_1}\frac{\partial H_1}{H_3\partial x^3}\\&\quad-W^3(W^3 + W^1\cos\theta_{31})\frac{1}{H_3}\frac{\partial H_3}{H_1\partial x^1} = -\frac{1}{\rho}\frac{\partial p}{H_1\partial x^1} + F_1^*;\end{aligned}\qquad(3.7\text{a})$$

$$W^1\frac{\partial W_\varphi}{H_1\partial x^1} + W^3\frac{\partial W_\varphi}{H_3\partial x^3} + \left(\frac{W_\varphi}{r} + 2\omega\right)(W^1\cos\theta_1 + W^3\sin\theta_3) = F_2^*\qquad(3.7\text{b})$$

或

$$\frac{W^1}{r}\frac{\partial(V_\varphi r)}{H_1\partial x^1} + \frac{W^3}{r}\frac{\partial(V_\varphi r)}{H_3\partial x^3} = F_2^*;\qquad(3.7\text{b}')$$

$$\begin{aligned}&W^1\frac{\partial W^3}{H_1\partial x^1} + W^3\frac{\partial W^3}{H_3\partial x^3} + \left(W^1\frac{\partial W^1}{H_1\partial x^1} + W^3\frac{\partial W^1}{H_3\partial x^3}\right)\cos\theta_{31}\\&\quad-\left(2\omega W_\varphi + \omega^2 r + \frac{W_\varphi^2}{r}\right)\sin\theta_3 + W^1(W^1\cos\theta_{31} + W^3)\frac{1}{H_3}\frac{\partial H_3}{H_1\partial x^1}\\&\quad-W^1(W^1 + W^3\cos\theta_{31})\frac{1}{H_1}\frac{\partial H_1}{H_3\partial x^3} - (W^1)^2\sin\theta_{31}\frac{\partial\theta_{31}}{H_3\partial x^1}\\&= -\frac{1}{\rho}\frac{\partial p}{H_3\partial x^3} + F_3^*.\end{aligned}\qquad(3.7\text{c})$$

用同样方法求得用滞止转焓、熵的梯度表示的运动方程：

$$\begin{aligned}&-\frac{W_\varphi}{r}\frac{\partial(V_\varphi r)}{H_1\partial x^1} + W^3\left(\frac{\partial W^1}{H_3\partial x^3} - \frac{\partial W^3}{H_1\partial x^1}\right) + W^3\cos\theta_{31}\left(\frac{\partial W^3}{H_3\partial x^3} - \frac{\partial W^1}{H_1\partial x^1}\right)\\&\quad+W^3(W^1 + W^3\cos\theta_{31})\frac{1}{H_1}\frac{\partial H_1}{H_3\partial x^3} - W^3(W^1\cos\theta_{31} + W^3)\frac{1}{H_3}\frac{\partial H_3}{H_1\partial x^1}\\&\quad+W^3\sin\theta_{31}\left(W^1\frac{\partial\theta_{31}}{H_1\partial x^1} - W^3\frac{\partial\theta_{31}}{H_3\partial x^3}\right) = Jg\left(-\frac{\partial I}{H_1\partial x^1} + T\frac{\partial s}{H_1\partial x^1}\right)\\&\quad+F_1^*;\end{aligned}\qquad(3.8\text{a})$$

$$\frac{W^1}{r}\frac{\partial(V_\varphi r)}{H_1\partial x^1} + \frac{W^3}{r}\frac{\partial(V_\varphi r)}{H_3\partial x^3} = F_2^*;\qquad(3.8\text{b})$$

$$\begin{aligned}&-\frac{W_\varphi}{r}\frac{\partial(V_\varphi r)}{H_3\partial x^3} + W^1\left(\frac{\partial W^3}{H_1\partial x^1} - \frac{\partial W^1}{H_3\partial x^3}\right) + W^1\cos\theta_{31}\left(\frac{\partial W^1}{H_1\partial x^1} - \frac{\partial W^3}{H_3\partial x^3}\right)\\&\quad+W^1(W^1\cos\theta_{31} + W^3)\frac{1}{H_3}\frac{\partial H_3}{H_1\partial x^1} - W^1(W^1 + W^3\cos\theta_{31})\frac{1}{H_1}\frac{\partial H_1}{H_3\partial x^3}\end{aligned}$$

$$+ W^1 \sin\theta_{31}\left(W^3 \frac{\partial\theta_{31}}{H_3\partial x^3} - W^1 \frac{\partial\theta_{31}}{H_1\partial x^1}\right) = Jg\left(-\frac{\partial I}{H_3\partial x^3} + T\frac{\partial s}{H_3\partial x^3}\right) + F_3^*. \quad (3.8c)$$

$$\left.\begin{aligned} F_1^* &= -F_1\sin\theta_{31} = -\frac{1}{\rho}\frac{\partial p}{r\partial\varphi}\frac{n_1}{n_2}\sin\theta_{31}; \\ F_2^* &= -F_2 = -\frac{1}{\rho}\frac{\partial p}{r\partial\varphi} = F_\varphi; \\ F_3^* &= -F_3\sin\theta_{31} = -\frac{1}{\rho}\frac{\partial p}{r\partial\varphi}\frac{n_3}{n_2}\sin\theta_{31} \end{aligned}\right\} \quad (3.9)$$

$$F_1^* = F_2^*\frac{n_1}{n_2}\sin\theta_{31}; \qquad F_3^* = F_2^*\frac{n_3}{n_2}\sin\theta_{31}. \quad (3.10)$$

$$F_1^*W^1 + F_2^*W_\varphi + F_3^*W^3 = 0. \quad (3.11)$$

设满足 S_2 流面的连续方程

$$\frac{\partial(H_3\tau\rho W^1\sin\theta_{31})}{\partial x^1} + \frac{\partial(H_1\tau\rho W^3\sin\theta_{31})}{\partial x^3} = 0 \quad (3.12)$$

的流函数的偏导数为:

$$\frac{\partial\Psi}{\partial x^1} = H_1\tau\rho g W^3\sin\theta_{31}; \qquad \frac{\partial\Psi}{\partial x^3} = -H_3\tau\rho g W^1\sin\theta_{31}. \quad (3.13)$$

将上式代入运动方程第一分式(3.7a)便得 S_2 流面的流函数方程[2]:

$$\frac{1}{H_1^2}\frac{\partial^2\Psi}{\partial(x^1)^2} + \frac{D}{H_1H_3}\frac{\partial^2\Psi}{\partial x^1\partial x^3} + \frac{1}{H_3^2}\frac{\partial^2\Psi}{\partial(x^3)^2} + E\frac{\partial\Psi}{H_1\partial x^1} + F\frac{\partial\Psi}{H_3\partial x^3} = G. \quad (3.14)$$

式中诸系数分别为:

$$\left.\begin{aligned} D &= -2\cos\theta_{31}, \\ E &= \frac{\partial}{H_1\partial x^1}\left(\ln\frac{H_3}{H_1\sin\theta_{31}}\right) - \frac{\partial\ln\tau}{H_1\partial x^1} + \cos\theta_{31}\frac{\partial\ln\tau}{H_3\partial x^3} + \frac{1}{\sin\theta_{31}}\frac{\partial\theta_{31}}{H_3\partial x^3}, \\ F &= \frac{\partial}{H_3\partial x^3}\left(\ln\frac{H_1}{H_3\sin\theta_{31}}\right) - \frac{\partial\ln\tau}{H_3\partial x^3} + \cos\theta_{31}\frac{\partial\ln\tau}{H_1\partial x_1} + \frac{1}{\sin\theta_{31}}\frac{\partial\theta_{31}}{H_1\partial x^1}, \\ G &= \frac{\partial\Psi}{H_1\partial x^1}\left(\frac{\partial\ln\rho}{H_1\partial x^1} - \frac{\partial\ln\rho}{H_3\partial x^3}\cos\theta_{31}\right) + \frac{\partial\Psi}{H_3\partial x^3}\left(\frac{\partial\ln\rho}{H_3\partial x^3} - \frac{\partial\ln\rho}{H_1\partial x^1}\cos\theta_{31}\right) \\ &\quad + \tau\sin\theta_{31}\rho g H, \\ H &= \left[-\frac{W_\varphi}{r}\frac{\partial(V_\varphi r)}{H_1\partial x^1} + Jg\left(\frac{\partial I}{H_1\partial x^1} - T\frac{\partial s}{H_1\partial x^1}\right) - F_1^*\right]\Big/W^3 \end{aligned}\right\} \quad (3.15)$$

正、反问题的解法见文献[2].

四、适用于速度梯度法的 S_1 和 S_2 流面的基本方程组

吴仲华教授早在1950年就已经提出了逐站计算的速度梯度法(或称流线曲率法)[7],把全场的二维问题简化为每个计算站的一维问题. 本文所推导的 S_1 流面的方程是适用于迴转面的情况.

S_1 **流面**的速度梯度方程通常写成三种形式,为节省篇幅,**速度梯度方程**写成下面的形

表 1　S_1 与 S_2 流面速度梯度方程的系数 P, Q 的计算公式

方程形式 \ 系数		P	Q
S_1 流面	I	$\frac{1}{1+A_1\cos\theta_{23}}\Big[-\frac{\partial\ln W^3}{H_3\partial x^3}(A_1+\cos\theta_{23})+\Big(\cos\theta_{23}\frac{\partial A_1}{r\partial\varphi}-\frac{\partial A_1}{H_3\partial x^3}\Big)-A_1\Big(\frac{\partial\ln r}{H_3\partial x^3}-\cos\theta_{23}\frac{\partial\ln H_3}{r\partial\varphi}+\sin\theta_{23}\frac{\partial\theta_{23}}{r\partial\varphi}\Big)+\Big(\frac{\partial\ln H_3}{r\partial\varphi}-\cos\theta_{23}\frac{\partial\ln r}{H_3\partial x^3}+\sin\theta_{23}\frac{\partial\theta_{23}}{H_3\partial x^3}\Big)\Big]$	$-\frac{2\omega r\sin\sigma\sin\theta_{23}}{1+A_1\cos\theta_{23}}$
	II	$\frac{1}{1-(M^3)^2(A_1^2+2A_1\cos\theta_{23}+1)}\cdot\frac{A_1+\cos\theta_{23}}{A_1^2+2A_1\cos\theta_{23}+1}\Big\{A_1\Big(\frac{\partial\ln H_3}{r\partial\varphi}+\mathrm{ctg}\theta_{23}\frac{\partial\theta_{23}}{r\partial\varphi}+\frac{\partial\ln\tau}{r\partial\varphi}\Big)+\Big(\frac{\partial\ln r}{H_3\partial x^3}+\mathrm{ctg}\theta_{23}\frac{\partial\theta_{23}}{H_3\partial x^3}+\frac{\partial\ln\tau}{H_3\partial x^3}\Big)+\frac{\partial A_1}{r\partial\varphi}-(M^3)^2\Big[(A_1+\cos\theta_{23})\frac{DA_1}{H_3dx^3}-A_1\sin\theta_{23}\frac{D\theta_{23}}{H_3dx^3}-\frac{\omega^2r}{(W_3)^2}\frac{Dr}{H_3dx^3}\Big]\Big\}+\frac{1}{A^2+2A_1\cos\theta_{23}+1}\Big[\Big(\cos\theta_{23}\frac{\partial A_1}{r\partial\varphi}-\frac{\partial A_1}{H_3\partial x^3}\Big)-A_1\Big(\frac{\partial\ln r}{H_3\partial x^3}-\cos\theta_{23}\frac{\partial\ln H_3}{r\partial\varphi}+\sin\theta_{23}\frac{\partial\theta_{23}}{r\partial\varphi}\Big)+\Big(\frac{\partial\ln H_3}{r\partial\varphi}-\cos\theta_{23}\frac{\partial\ln r}{H_3\partial x^3}+\sin\theta_{23}\frac{\partial\theta_{23}}{H_3\partial x^3}\Big)\Big]$	$-\frac{2\omega r\sin\sigma\sin\theta_{23}}{A_1^2+2A_1\cos\theta_{23}+1}$
	III	$\Big\{A_1^2\Big(\frac{\partial\ln H_3}{r\partial\varphi}+\mathrm{ctg}\theta_{23}\frac{\partial\theta_{23}}{r\partial\varphi}+\frac{\partial\ln\tau}{r\partial\varphi}\Big)+A_1\Big(2\cos\theta_{23}\frac{\partial\ln H_3}{r\partial\varphi}+\frac{\cos2\theta_{23}}{\sin\theta_{23}}\cdot\frac{\partial\theta_{23}}{r\partial\varphi}+\mathrm{ctg}\theta_{23}\frac{\partial\theta_{23}}{H_3\partial x^3}+\cos\theta_{23}\frac{\partial\ln\tau}{r\partial\varphi}+\frac{\partial\ln\tau}{H_3\partial x^3}\Big)+\Big(\frac{\partial\ln H_3}{r\partial\varphi}+\frac{1}{\sin\theta_{23}}\frac{\partial\theta_{23}}{H_3\partial x^3}+\cos\theta_{23}\frac{\partial\ln\tau}{H_3\partial x^3}\Big)+\frac{\partial A_1}{r\partial\varphi}(A_1+2\cos\theta_{23})-\frac{\partial A_1}{H_3\partial x^3}+(A_1+\cos\theta_{23})\Big(\frac{\partial\ln\rho}{H_3\partial x^3}+\cos\theta_{23}\frac{\partial\ln\rho}{H_3\partial x^3}\Big)\Big\}\Big/(A_1^2+2A_1\cos\theta_{23}+1)$	$-\frac{2\omega r\sin\sigma\sin\theta_{23}}{A_1^2+2A_1\cos\theta_{23}+1}$
S_2 流面	I	$\frac{1}{1+A\cos\theta_{31}}\Big[-\frac{\partial\ln W^3}{H_3\partial x^3}(A+\cos\theta_{31})+\Big(\cos\theta_{31}\frac{\partial A}{H_1\partial x^1}-\frac{\partial A}{H_3\partial x^3}\Big)-A\Big(\frac{\partial\ln H_1}{H_3\partial x^3}-\cos\theta_{31}\frac{\partial\ln H_3}{H_1\partial x^1}+\sin\theta_{31}\frac{\partial\theta_{31}}{H_1\partial x^1}\Big)+\Big(\frac{\partial\ln H_3}{H_1\partial x^1}-\cos\theta_{31}\frac{\partial\ln H_1}{H_3\partial x^3}+\sin\theta_{31}\frac{\partial\theta_{31}}{H_3\partial x^3}\Big)\Big]$	$\frac{1}{W^3(1+A\cos\theta_{31})}\Big[\frac{W_\varphi}{r}\frac{\partial(V_\varphi r)}{H_1\partial x^1}+Jg\Big(-\frac{\partial I}{H_1\partial x^1}+T\cdot\frac{\partial s}{H_1\partial x^1}\Big)+F_1^*\Big]$
	II	$\Big\{A^2\Big(\frac{\partial\ln H_3}{H_1\partial x^1}+\mathrm{ctg}\theta_{31}\frac{\partial\theta_{31}}{H_1\partial x^1}+\frac{\partial\ln\tau}{H_1\partial x^1}\Big)+A\Big(2\cos\theta_{31}\frac{\partial\ln H_3}{H_1\partial x^1}+\frac{\cos2\theta_{31}}{\sin\theta_{31}}\frac{\partial\theta_{31}}{H_1\partial x^1}+\mathrm{ctg}\theta_{31}\frac{\partial\theta_{31}}{H_3\partial x^3}+\cos\theta_{31}\frac{\partial\ln\tau}{H_1\partial x^1}+\frac{\partial\ln\tau}{H_3\partial x^3}\Big)+\Big(\frac{\partial\ln H_3}{H_1\partial x^1}+\frac{1}{\sin\theta_{31}}\frac{\partial\theta_{31}}{H_3\partial x^3}+\cos\theta_{31}\frac{\partial\ln\tau}{H_3\partial x^3}\Big)+\frac{\partial A}{H_1\partial x^1}(A+2\cos\theta_{31})-\frac{\partial A}{H_3\partial x^3}+(A+\cos\theta_{31})\cdot\Big(\frac{\partial\ln\rho}{H_3\partial x^3}+A\frac{\partial\ln\rho}{H_1\partial x^1}\Big)\Big\}\Big/(A^2+2A\cos\theta_{31}+1)$	$\Big[\frac{W_\varphi}{r}\frac{\partial(V_\varphi r)}{H_1\partial x^1}+Jg\cdot\Big(-\frac{\partial I}{H_1\partial x^1}+T\frac{\partial s}{H_1\partial x^1}\Big)+F_1^*\Big]\div W_3(A^2+2A\cos\theta_{31}+1)$

续 表 1

方程形式 \ 系数		P	Q
S_2 流面	III	$\frac{1}{1-(M^3)^2(A^2+2A\cos\theta_{31}+1)}\cdot\frac{A+\cos\theta_{31}}{A^2+2A\cos\theta_{31}+1}\Big\{A\frac{\partial\ln H_3}{H_1\partial x^1}$ $+\frac{\partial\ln H_1}{H_3\partial x^3}+\mathrm{ctg}\theta_{31}\frac{\mathbf{D}\theta_{31}}{H_3\mathbf{d}x^3}+\frac{\mathbf{D}\ln\tau}{H_3\mathbf{d}x^3}+\frac{\partial A}{H_1\partial x^1}$ $+(M^3)^2\Big[A\sin\theta_{31}\frac{\mathbf{D}\theta_{31}}{H_3\mathbf{d}x^3}+\frac{(V_\varphi r)^2}{r^3(W^3)^2}\frac{\mathbf{D}r}{H_3\mathbf{d}x^3}-(A+\cos\theta_{31})$ $\cdot\frac{\mathbf{D}A}{H_3\mathbf{d}x^3}-\frac{(V_\varphi r-\omega r^2)}{r^2(W^3)^2}\frac{\mathbf{D}(V_\varphi r)}{H_3\mathbf{d}x^3}\Big]-\frac{J}{R}\frac{\mathbf{D}s}{H_3\mathbf{d}x^3}\Big\}$ $+\frac{1}{A^2+2A\cos\theta_{31}+1}\Big[\Big(\cos\theta_{31}\frac{\partial A}{H_1\partial x^1}-\frac{\partial A}{H_3\partial x^3}\Big)$ $-A\Big(\frac{\partial\ln H_1}{H_3\partial x^3}-\cos\theta_{31}\frac{\partial\ln H_3}{H_1\partial x^1}+\sin\theta_{31}\frac{\partial\theta_{31}}{H_1\partial x^1}\Big)$ $+\Big(\frac{\partial\ln H_3}{H_1\partial x^1}-\cos\theta_{31}\frac{\partial\ln H_1}{H_3\partial x^3}+\sin\theta_{31}\frac{\partial\theta_{31}}{H_3\partial x^3}\Big)\Big]$	$\Big[\frac{W_\varphi}{r}\frac{\partial(V_\varphi r)}{H_1\partial x^1}+Jg$ $\cdot\Big(-\frac{\partial I}{H_1\partial x^1}+T\frac{\partial s}{H_1\partial x^1}\Big)$ $+F_1^*\Big]\div W_3(A^2$ $+2A\cos\theta_{31}+1)$

式:

$$\partial W^3/r\partial\varphi+PW^3+Q=0 \tag{4.1}$$

式中的 P 与 Q 的计算公式见表 1. 表中的 $A_1=W^2/W^3$ 表示两分速的比值; $M^3=W^3/a$ 表示 W^3 分速的马赫数.

连续方程的积分形式. 通过 φ_0 至 φ 厚度为 τ 的 S_1 流片的流量为:

$$G=\int_{\varphi_0}^{\varphi}\rho g\tau W^3\sin\theta_{23}rd\varphi. \tag{4.2}$$

S_2 **流面**的**速度梯度方程**为:

$$\partial W^3/H_1\partial x^1+PW^3+Q=0 \tag{4.3}$$

式中的 P 与 Q 见表 1; $A=W^1/W^3$.

连续方程的积分形式. 通过 φ 向厚度为 τ 的流片(从 h 到 t)之总流量为:

$$G=\int_{x_h^1}^{x_t^1}\rho gW^3\tau\sin\theta_{31}H_1\mathbf{d}x^1. \tag{4.4}$$

关于应用非正交曲线坐标系表示的粘性气体在叶轮机械中流动的基本方程组将在另文中发表.

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APPLICATION OF NON-ORTHOGONAL CURVILINEAR COORDINATES TO CALCULATE THE FLOW IN TURBOMACHINES

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In the first part of the paper, the formulas of calculating gradients of a scalar, divergents and vorticities of a vector in non-orthogonal curvilinear coordinates are presented by vector analysis. With aid of these relations we then obtained basic aero-thermodynamic equations governing relative steady flow of a nonviscous fluid in a turbomachine. General non-orthogonal coordinates are suggested for solving Φ-equation of three-dimensional turbomachine flow. The potential equation for three-dimensional flow calculation is obtained.

Ψ-equations (Wu's equations), expressed in term of general non-orthogonal coordinates, of two kinds of stream surfaces are discussed in the third part.

In the last part of this paper, three forms of velocity gradient formulas are presented.