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A COUPLED MODEL FOR MULTIPHASE FLUID FLOW AND SEDIMENTATION DEFORMATION IN OIL RESERVOIR AND ITS NUMERICAL SIMULATION

Ran Qiquan (冉启全) Gu Xiaoyun (顾小芸) (Institute of Mechanics, Chinese Academy of Sciences, Beijing 100080, China) Li Shilun (李士伦) (Southwest Petroleum Institute, Nanchong 637001, China)

ABSTRACT: A mathematical model for coupled multiphase fluid flow and sedimentation deformation is developed based on fluid-solid interaction mechanism. A finite difference-finite element numerical approach is presented. The results of an example show that the fluid-solid coupled effect has great influence on multiphase fluid flow and reservoir recovery performances, and the coupled model has practical significance for oilfield development.

KEY WORDS: reservoir, multiphase flow, solid deformation, coupled model, numerical simulation

1 INTRODUCTION

In the recovery process of oil and gas reservoir, there is strong dynamic interaction between the multiphase fluid flow and sedimentation deformation. On the one hand, the multiphase fluid flow or reservoir recovery will cause the change of pore fluid pressure and the redistribution of effective stress on rock/soil skeleton, which lead to the sedimentation deformation. On the other hand, the change in pore volume as a result of sedimentation deformation will lead to the alterations of reservoir porosity and permeability, which greatly affect the multiphase fluid flow or reservoir recovery. So the fluid flow through the porous medium of the geomaterial is fluid-solid coupled flow, while the deformation of the porous medium is fluid-solid coupled deformation. Many previous studies on the problem of reservoir flow and recovery are based on the theory of pure seepage flow mechanics, and the coupling effect is ignored, thus great deviation between research and practice occurs.

Recently, with the development of seepage flow mechanics and rock/soil mechanics as well as the needs for solving complex petroleum engineering problem, the study on coupled problem of multiphase fluid flow and sediment deformation in oil and gas reservoir becomes more and more important and has been paid high attention^[1]. The early works of coupled problem are mainly concentrated on experimental study^[2~4]. Fung^[5] developed a coupled model of oil-water two-phase flow and sedimentation deformation, and the model is solved by the control-volume finite-element method. In the present work, based on the basic principle of fluid-solid interaction, a coupled mathematical model of oil-gas-water three-phase flow

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and sediment deformation is established, and it is solved by a finite difference-finite element hybrid approach. Thus the three-phase flow process of oil-gas-water and the deformation process of sedimentation can be simulated, and the scientific basis for the reservoir recovery is provided.

2 MATHEMATICAL MODEL

The coupled model consists of two parts: the multiphase flow model under fluid-solid interaction and the sedimentation deformation model under fluid-solid interaction. The flow model describes the motion law of oil-gas-water multiphase flow through deformable porous medium. The coupled effect is reflected by the changes in porosity and permeability induced by sedimentation deformation. The deformation model describes the stress-strain law of sedimentation under seepage forces. The coupled effect herein is expressed by the additional loads induced by the changes of seepage forces, reservoir porosity and fluid saturation, which may lead to further deformation and stress redistribution.

2.1 Multiphase Flow Model

The coupled effects are not involved in conventional motion equation, so it is necessary to establish the motion equation suitable for fluid flow coupled with deformation. The motion equation of fluid flow through deformable porous medium can be developed based on the concept of velocity.

The velocities of different phases are defined as follows

$$\boldsymbol{v_{ro}} = \boldsymbol{v_o} - \boldsymbol{v_s} \tag{1}$$

$$\boldsymbol{v_{rg}} = \boldsymbol{v_g} - \boldsymbol{v_s} \tag{2}$$

$$\boldsymbol{v}_{\boldsymbol{r}\boldsymbol{w}} = \boldsymbol{v}_{\boldsymbol{w}} - \boldsymbol{v}_{\boldsymbol{s}} \tag{3}$$

where v_r — relative velocity, v — absolute velocity, subscripts o, g, w represent oil, gas, water respectively. v_s , the absolute velocity of solid matrix, can be defined as

$$\boldsymbol{v}_{\boldsymbol{s}} = \frac{\partial \boldsymbol{U}}{\partial t} \tag{4}$$

where U — displacement vector of solid matrix, t — time.

The Darcy velocities for the respective phases are

$$\boldsymbol{V}_{o} = \phi S_{o} \boldsymbol{v}_{ro} = -\frac{KK_{ro}}{\mu_{o}} (\nabla P_{o} - \rho_{o} g \nabla D)$$
⁽⁵⁾

$$\boldsymbol{V}_{\boldsymbol{g}} = \phi S_{\boldsymbol{g}} \boldsymbol{v}_{r\boldsymbol{g}} = -\frac{KK_{r\boldsymbol{g}}}{\mu_{\boldsymbol{g}}} (\boldsymbol{\nabla} P_{\boldsymbol{g}} - \rho_{\boldsymbol{g}} \boldsymbol{g} \boldsymbol{\nabla} D)$$
(6)

$$\boldsymbol{V}_{w} = \phi S_{w} \boldsymbol{v}_{rw} = -\frac{KK_{rw}}{\mu_{w}} (\nabla P_{w} - \rho_{w} g \nabla D)$$
(7)

where V — Darcy velocity, ϕ — porosity, S — saturation, K — permeability, K_r — relative permeability, μ — viscosity, P — pressure, ρ — density, g — gravity acceleration, D — relative height with respect to a fixed reference level.

The partial differential equations of multiphase flow through deformable porous medium under fluid-solid interaction can be established based on the motion equation of fluid flow and the mass conservation principle with consideration of source terms and dissolution of gas in oil and water. The following three equations are for oil, gas, and water respectively

$$\nabla \cdot \left[\frac{KK_{ro}}{B_o\mu_o}(\nabla P_o - \rho_o g \nabla D) - \nabla \cdot \left[\frac{\phi S_o}{B_o} \boldsymbol{v}_s\right] + Q_o = \frac{\partial}{\partial t} \left[\frac{\phi S_o}{B_o}\right]$$
(8)

$$\nabla \cdot \left[\frac{KK_{rg}}{B_{g}\mu_{g}}(\nabla P_{g} - \rho_{g}g\nabla D) + \nabla \cdot \left[\frac{R_{so}KK_{ro}}{B_{o}\mu_{o}}(\nabla P_{o} - \rho_{o}g\nabla D)\right] + \nabla \cdot \left[\frac{R_{sw}KK_{rw}}{B_{w}\mu_{w}}(\nabla P_{w} - \rho_{w}g\nabla D) - \nabla \cdot \left[\frac{\phi S_{g}}{B_{g}}\boldsymbol{v}_{s}\right] - \nabla \cdot \left[\frac{\phi R_{so}S_{o}}{B_{o}}\boldsymbol{v}_{s}\right] - \nabla \cdot \left[\frac{\phi R_{sw}S_{w}}{B_{w}}\boldsymbol{v}_{s}\right] + Q_{g} = \frac{\partial}{\partial t}\left[\frac{\phi S_{g}}{B_{g}} + \frac{\phi R_{so}S_{o}}{B_{o}} + \frac{\phi R_{sw}S_{w}}{B_{w}}\right]$$
(9)

$$\nabla \cdot \left[\frac{KK_{rw}}{B_w\mu_w}(\nabla P_w - \rho_w g \nabla D) - \nabla \cdot \left[\frac{\phi S_w}{B_w} \boldsymbol{v}_s\right] + Q_w = \frac{\partial}{\partial t} \left[\frac{\phi S_w}{B_w}\right]$$
(10)

where B — volumetric coefficient, R_{so} , R_{sw} are dissolved gas/oil ratio and dissolved gas/water ratio. $Q = q/\rho_{sc}$ is the expression of volume flow rate of production or injection, where ρ_{sc} — density under the standard condition, q — mass flow rate of production or injection.

Each equation contains a "solid" term related to the solid velocity. The changes of porosity in accumulation term and permeability in convection term are related to solid deformation. Thus the coupling effect of multiphase flow and sedimentation deformation is embodied in the equations.

To solve the above equations, two sets of constraint equations are needed, i.e. capillary pressure equations and saturation equation

$$P_g = P_o + P_{cgo} \tag{11}$$

$$P_w = P_o - P_{cwo} \tag{12}$$

where P_{cgo} , P_{cwo} are the capillary pressure of oil-gas and oil-water.

$$S_o + S_g + S_w = 1 \tag{13}$$

When porosity ϕ , permeability K, and solid velocity v_s are known, six variables $(P_o, P_g, P_w, S_o, S_g, \text{ and } S_w)$ are to be solved. Thus the mathematical model of multiphase flow coupled with deformation can be solved under given boundary conditions and initial conditions.

2.2 Sedimentation Deformation Model

The solid velocity, porosity, and permeability involved in the equations of multiphase flow depend on the deformation of porous medium. Thus it is necessary to develop a deformation model under fluid-solid interaction.

The deformation and strength of reservoir sediments containing oil, gas, and water depend on effective stress. Assume stresses are positive in tension and negative in compression, then the effective stress principle can be expressed as

$$\boldsymbol{\sigma} = \boldsymbol{\sigma}^T + \delta_{ij} P \tag{14}$$

where δ_{ij} — Kronecker delta, σ — effective stress, σ^T — total stress. P is equivalent pore fluid pressure, which equals the average of oil, gas, and water pressure.

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The equilibrium differential equations based on effective stress principle may be expressed as

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} - \frac{\partial P}{\partial x} = 0$$
(15)

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} - \frac{\partial P}{\partial y} = 0$$
(16)

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} - \frac{\partial P}{\partial z} + \left[(1 - \phi)\rho_s + \phi S_o \rho_o + \phi S_w \rho_w + \phi S_g \rho_g \right] g = 0$$
(17)

where ρ_s — density of solid matrix.

The body forces involved in the above equations comprise two parts: the self-weight (gravity) of porous medium with multiphase fluid, $[(1-\phi)\rho_s + \phi S_o\rho_o + \phi S_w\rho_w + \phi S_g\rho_g]g$, and the seepage force as distributed body force, $\partial P/\partial x$, $\partial P/\partial y$, and $\partial P/\partial z$. The seepage force and the self-weight vary dynamically with the change of pressure, saturation, and porosity in the recovery process. Thus the coupling effect of multiphase flow and sedimentation deformation is embodied in the equilibrium equations.

The equilibrium equations stated above, the geometric equation describing the straindisplacement relationship of sedimentation deformation, and the constitutive equations describing the stress-strain relationship of the sediment, constitute the equation group of reservoir sedimentation deformation under fluid-solid interaction.

When porosity ϕ , fluid pressure $(P_o, P_g, \text{ and } P_w)$, and saturation $(S_o, S_g, \text{ and } S_w)$ are known, fifteen variables involving 6 stress components, 6 strain components and 3 displacement components are to be solved. Since there are fifteen equations, the mathematical model can be solved under given boundary conditions and initial conditions.

3 NUMERICAL SOLUTION OF THE COUPLED MODEL

The coupled model is solved by a finite difference-finite element hybrid approach. The flow model is solved by finite difference method, and the results provide the pressure and saturation of oil, gas, and water. The deformation model is solved by finite element method, and the displacement, strain, and stress can be given.

3.1 Numerical Solution of the Flow Model by Finite Difference Method

The finite difference approach to solve the flow model is that the pressures are implicitly calculated, the saturations are explicitly calculated. The solution sequence is as follows:

(1) The oil Eq.(8), gas Eq.(9), and water Eq.(10) are respectively multiplied by some coefficients to eliminate the variables of saturations $(S_o, S_g, \text{ and } S_w)$, and then are incorporated into a new equation which only contains the variables of pressures $(P_o, P_g, \text{ and } P_w)$.

(2) Gas pressure P_g and water pressure P_w are substituted by oil pressure P_o and capillary pressure $(P_{cgo} \text{ and } P_{cwo})$ based on capillary pressure Eqs.(11) and (12). Then the equation has only one variable of oil pressure P_o .

(3) The difference discretization of the equation is conducted, and the necessary algebraic manipulation is performed to form the difference equation as follows

$$a \cdot P_{oijk-1}^{n+1} + c \cdot P_{oij-1k}^{n+1} + e \cdot P_{oi-1jk}^{n+1} + g \cdot P_{oijk}^{n+1} + f \cdot P_{oi+1jk}^{n+1} + d \cdot P_{oij+1k}^{n+1} + b \cdot P_{oijk+1}^{n+1} = h$$
(18)

where the superscript n is time level, the subscripts i, j, and k are respectively the numbers of difference grid in x, y, and z directions, a, b, c, d, e, f, g are the coefficient terms and h the constant term. Eq.(18) which is nonlinear can be solved implicitly to obtain oil pressure P_o^{n+1} by LSOR method, when linearization is performed with the coefficient terms and the constant term being the values at the last time level. Whence gas pressure P_g^{n+1} and water pressure P_w^{n+1} may be calculated as follows

$$P_{q}^{n+1} = P_{o}^{n+1} + P_{cgo}^{n} \qquad P_{w}^{n+1} = P_{o}^{n+1} - P_{cwo}^{n} \qquad (19)$$

(4) When the difference discretization of oil Eq.(8) and water Eq.(10) are performed, oil pressure P_o^{n+1} and P_w^{n+1} are respectively substituted into the difference equations of oil and gas. Oil saturation S_o^{n+1} and water saturation S_w^{n+1} can be explicitly obtained from the difference equations of oil and gas, while gas saturation S_g^{n+1} can be obtained from saturation Eq.(13).

3.2 Numerical Solution of the Deformation Model by Finite Element Method

The effective stress formulation of the equilibrium equations can be obtained based on the principle of effective stress and virtual work. The effective formulation may be expressed as the equilibrium of named loads

$$[K]^{e}\{\delta\}^{e} = \{F\}^{e}$$
(20)

where $\{\delta\}^e$ is the element nodal displacements, $[K]^e$ is the element stiffness matrix, which may be written as

$$[K]^{e} = \iiint_{V} [B]^{\mathrm{T}}[D][B] \mathrm{d}V = \iiint_{V} [B]^{\mathrm{T}}[D][B] \mathrm{d}x \mathrm{d}y \mathrm{d}z$$
(21)

where [D] is elastic constitutive matrix, [B] is strain matrix, and $[B]^{T}$ is the turning matrix of [B].

 $\{F\}^e$ is the element equivalent nodal forces, which may be expressed as

$$\{F\}^{e} = \iiint_{V} [N]^{\mathrm{T}} \{p_{v}\} \mathrm{d}V + \iint_{A} [N]^{\mathrm{T}} \{p_{s}\} \mathrm{d}A + \iiint_{V} [B]^{\mathrm{T}} P\{m\} \mathrm{d}V - \iiint_{V} [B]^{\mathrm{T}} \{\sigma^{0}\} \mathrm{d}V$$
(22)

where [N] is shape function matrix, $[N]^{T}$ is the turning matrix of $[N], \{p_v\}$ is body fore per unit volume, $\{p_s\}$ is surface force per unit area, $\{\sigma^0\}$ is initial stress, while $\{m\} = [1 \ 1 \ 1 \ 0 \ 0 \ 0]^{T}$.

The first term of the right hand of formulation (22) is the equivalent nodal forces of internal body forces (self-weight), the second term is the equivalent nodal forces of surface forces, the third term is the equivalent nodal forces of pore pressures, and the fourth term is the equivalent nodal forces of initial stress.

The element equilibrium equation can be formed when the element stiffness matrix and the element equivalent nodal forces are obtained. All the element equilibrium equations assemble the total equilibrium equation

$$[K]{\delta} = {F}$$

$$(23)$$

where $[K], \{\delta\}$, and $\{F\}$ are the total stiffness matrix, nodal displacements, and equivalent nodal forces.

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The nodal displacements, strains, and stresses can be obtained by solving the total equilibrium Eq.(23), combining the geometric equations and the constitutive equations.

3.3 Coupling Schemes of Solution between the Models

The coupled models can be solved in a staggered mode, where the solution of deformation model lags behind the solution of flow model by one time step. The solution results of flow model provide the increments of pressure, porosity, and saturation as loads to the finite element model. The resultant nodal displacement and volumetric strain of solid model are then fed back to the finite difference model. The implementation of the coupling solution between the two models is stated as follows:

(1) The initial stress field of reservoir under undisturbed condition is calculated.

(2) The flow model is solved for one time step, which provides the pressure and saturations of oil, gas, and water, and gives the recovery performances of reservoir.

(3) At the end of this time step, the change in pressure and saturation is used to compute the increment of distributed loads on the system. Then the solid model can be solved to calculate the increment of displacement, strain, and stress induced by the increment of loads. Whence the solid velocity and volumetric strain within this time step can also be obtained.

(4) The new reservoir parameters, such as porosity and permeability, can be calculated as the function of volumetric strain.

Returning to step (2) of the procedure, the solution is performed for the next time step until the end.

The numerical simulator of the coupled models is developed based on the procedure described above.

4 NUMERICAL EXAMPLE AND RESULT ANALYSIS

In the numerical example, how the coupling effect of multiphase flow and sedimentation deformation affects the reservoir flow and recovery performances is examined, and the significance of the coupled model for guiding oilfield development is shown.

The example is a depletion type of exploitation problem for reservoir. An oil production well is located at the center of the reservoir. The flowing bottom hole pressure specified is 5.0 MPa. The reservoir and fluid parameters are taken from [6]: depth and thickness of the reservoir are 2537.5 m and 15.3 m respectively; porosity, 0.25; permeability, $100 \times 10^{-3} \mu m^2$; initial reservoir pressure, 30.3 MPa; saturation pressure, 27.7 MPa; initial oil saturation, 0.88; initial water saturation, 0.12; and initial gas saturation, 0. In the recovery process, two stages exist. When reservoir pressure is greater than the saturation pressure, there is no gas. When reservoir pressure is decreased to the saturation pressure, free gas starts to emerge from oil, and the two-phase flow turns to be three-phase flow.

The uncoupled simulation and coupled simulation are first conducted for oil-water twophase flow problem (saturation pressure is 5.0 MPa). Figures 1 and 2 show that the porosity and permeability decrease gradually in the recovery process due to the deformation of porous medium. Figure 3 shows the distinction of cumulative oil production between the uncoupled model and coupled model. From these figures, the significance of coupling effects is quite apparent.







Fig.2 Profile of permeability with simulation time



Fig.3 Profile of oil production with simulation time

The sensitivity study on the material constitutive behavior of the coupled model is also carried out. Coupled model 1 is a linear elastic constitutive model with elastic modulus of 1 300.0 MPa and poison's ratio of 0.15. Coupled model 2 is an elastic-perfectly-plastic model. Zero cohesion, friction angle of 30°, hardening parameter of 0.0 MPa, and the Mohr-Coulomb yield criterion are used in the model. Figures 1, 2, and 3 show the differences in porosity, permeability, and cumulative oil production between coupled model 1 and coupled Vol.13, No.3 Ran Qiquan et al.: Multiphase Fluid Flow and Sedimentation Deformation 271

model 2, which illustrate the importance of the selected material constitutive model in the coupled models.

Then the simulation is done for the oil-gas-water three-phase flow problem (saturation pressure is 27.2 MPa), where the linear elastic constitutive model is used. Figure 4 shows the changes of the saturations. Figures 5, 6, and 7 show the differences of simulated results between three-phase flow problem and two-phase flow problem. It implies that the existence and the flow of free gas are important factors in the recovery process.



Fig.4 Profile of saturation with simulation time



Fig.5 Profile of oil production with simulation time



Fig.6 Profile of gas production with simulation time



Fig.7 Profile of water production with simulation time

5 CONCLUSIONS

- A coupled model which can represent the physics of oil-gas-water multiphase flow and sedimentation deformation in reservoirs is developed. The example shows the great significance of coupled model for the recovery of oil reservoirs.
- (2) The fluid-solid coupled effect has great influence on the physical parameters of the sediments, which positively affect the multiphase fluid flow and reservoir production performances, thus the coupling effects can't be ignored in reservoir simulation.
- (3) Since the material constitutive model controls the geomechanical response, it is very important to select the constitutive model correctly.
- (4) The existence and the flow of free gas are important factors to be considered in the simulation of multiphase flow in reservoir recovery process.

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