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DYNAMIC ANALYSIS OF ARREST OF BUCKLE PROPAGATION ON A BEAM ON A NONLINEAR ELASTIC FOUNDATION BY FEM*

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ABSTRACT Based on the dynamic governing equation of propagating buckle on a beam on a nonlinear elastic foundation. this paper deals with an important problem of buckle arrest by combining the FEM with a time integration technique. A new conclusion completely different from that by the quasi-static analysis about the buckle arrestor design is drawn. This shows that the inertia of the beam cannot be ignored in the analysis under consideration, especially when the buckle propagation is suddenly stopped by the arrestors.

KEY WORDS buckle propagation, arrest of buckle, beam on a nonlinear foundation, finite element method

I. INTRODUCTION

As pointed out in [1] the buckle and buckle propagation phenomena often take place when a long pipe undergoes an extra external load. and usually a buckle is initiated from a local dent on the pipe. Once initiated the buckle can spread at a load substantially below the initial buckling load, which means a large-scale destruction of the pipeline and a heavy financial loss. To stop a propagating buckle to minimize the length of pipeline damaged, arrestors (usually thick and stiff rings) are placed apart at a certain distance along the pipeline. But how to make a choice of their material, geometry and separation asks for theoretical analyses. Only some experimental studies can be cited[2]. For a beam on a nonlinear elastic foundation, however, by modelling a discontinuous distribution of foundation stiffness, Youn[3] first discussed a quasi-static buckle arrest numerically using the FEM, and obtained some relationships between the width and stiffness of the arrestor for buckle arrestor design in 1991. But in engineering practice, the velocity of propagation may increase to 120 m/sec rather than a slow propagation[2] so that the inertia of the pipeline cannot be ignored, especially when the buckle is suddenly stopped by the arrestors.

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In this paper, the quasi-static state assumption used in the past is replaced with a nonlinear finite element method combined with a time integration technique to analyze the arrest of a propagating buckle on a beam on a nonlinear foundation. A new group of parameters for arrestor design which differs greatly from [3] is obtained. For convenience, Youn's idea of arrestor distribution is adopted as well in the present work.

Ⅱ. DESCRIPTION OF THE ARRESTOR

Although the buckle arrestors could be designed in various types and forms to meet different needs in engineering, the basic idea still consists in placing stiff rings along a pipeline to improve the pipe's local ability to resist a further deformation so that a propagating buckle can be stopped. This idea enables us to modify the foundation stiffness, k_0(see(6)), into a discontinuous function with respect to $\xi(\xi=x/(EI/k_0)^{1/4}$, in which EI is the flexural rigidity of the beam) denoted by $k_0(\xi)$, so that some arrestors with a certain width and strength can be assumed to be put on the beam under consideration as done by Youn[3], i.e.,

$$k_0(\xi)=\begin{cases}k_0, & |\xi|<\bar{\xi} \text{ or } |\xi|>\bar{\xi}+\bar{L}\\ \bar{k}k_0, & \bar{\xi}\leqslant|\xi|\leqslant\bar{\xi}+\bar{L}\end{cases} \tag{1}$$

where $\bar{\xi}$ is one half of the distance between two adjacent arrestors, $\bar{L}$ is the width of the arrestor and $\bar{k}$ a multiple coefficient for increasing the foundation stiffness beyond its original value, which are shown in Fig. 1. $\bar{k}=1$ means that no arrestor is placed.

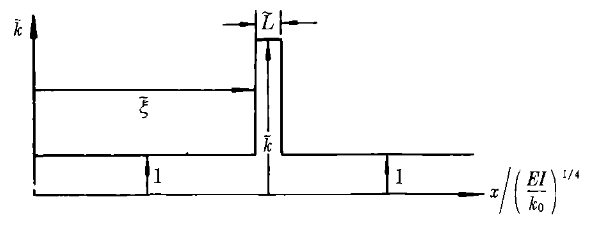

Fig. 1 Geometric description of arrestor.

Generally, the behavior of an arrestor can be distinguished by the efficiency of the arrestor which was given a definition by Kyriakides and Babcock[2] as follows:

$$\eta_0=\frac{P_0/P_p-1}{P_c/P_p-1} \tag{2}$$

where P_0 is the pressure at which a buckle will go through the arrestor (crossover pressure). As a rule, the value of the efficiency is not allowed to be higher than 1 which means that the size of the arrestor should not be too big. The value of η_0 for an excellent arrestor is near 1.

It is seen from[5] that the width of the arrestor is restrained by a numerical integration formula from improving computational accuracy. To take the value of $\tilde{L}$ in terms of the following expression[3] is a good way:

$$\tilde{L} = l\sum_{i=1}^{i}\omega_i \tag{3}$$

where l is the length of the beam element, ω_i the weighted coefficient in numerical integration and i the segment number chosen according to engineering need. It follows from (3) that there may be ten types of arrestors with different widths for offering the use to determine the value of $\tilde{L}$ if the number of the Gauss points is ten. Youn[3] took the width of the arrestor as the element length ($l=\tilde{L}$). This way often leads to the result that the total amount of beam elements is so great that the computation cost increases greatly.

Ⅲ. GOVERNING EQUATION AND ITS FE SOLUTION

3.1 Governing Equation

If the mass of beam per unit length is $\bar{m}$, one can write the governing equation for the beam with infinite length on the nonlinear foundation as (Fig. 2)

$$\bar{m}\frac{\partial^2 w}{\partial t^2} + EI\frac{\partial^4 w}{\partial x^4} + k(w,t)w = p(t) \tag{4}$$

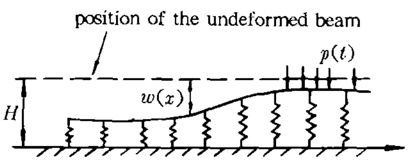

Fig. 2 A linear beam on a nonlinear elastic foundation.

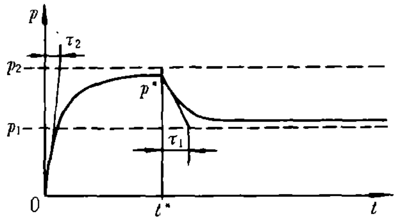

Fig. 3 A standard loading history for dynamic analysis.

The boundary conditions are

$$x \to \pm\infty, \quad \frac{\partial w}{\partial x} = \frac{\partial^2 w}{\partial x^2} = 0 \tag{5a}$$

$$[M]^e = \frac{\bar{m}l}{420}\begin{bmatrix} 156 & 22l & 54 & -13l \\ & 4l^2 & 13l & -3l^2 \\ & & 156 & -22l \\ & \text{Sym} & & 4l^2 \end{bmatrix} \tag{8a}$$

$$[K]^e = \frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ & 4l^2 & -6l & 2l^2 \\ & & 12 & -6l \\ & \text{Sym} & & 4l^2 \end{bmatrix} \tag{8b}$$

$$\begin{aligned}(K_F)^e_{ij} &= \int_0^l k(w,\bar{x})N_i(\bar{x})N_j(\bar{x})\mathrm{d}\bar{x} \\ &= c_0\beta^0_{ij} + c_1\beta^1_{ijk}w_k + c_2\beta^2_{ijmn}w_m w_n\end{aligned} \tag{8c}$$

$$\begin{cases} \beta^0_{ij} = \int_0^l a(\bar{x})N_i(\bar{x})N_j(\bar{x})\mathrm{d}\bar{x} \\ \beta^1_{ijm} = \int_0^l a(\bar{x})N_i(\bar{x})N_j(\bar{x})N_m(\bar{x})\mathrm{d}\bar{x} \\ \beta^2_{ijmn} = \int_0^l a(\bar{x})N_i(\bar{x})N_j(\bar{x})N_m(\bar{x})N_n(\bar{x})\mathrm{d}\bar{x} \\ a(\bar{x}) = 1 - \eta\exp\left[-\lambda\sqrt{\frac{k_0}{EI}}(x_F + \bar{x})^2\right] \\ c_0 = \tilde{k}k_0, \quad c_1 = -\frac{4.5}{H}\tilde{k}k_0, \quad c_2 = \frac{5.25}{H^2}\tilde{k}k_0 \\ \tilde{k} = \begin{cases} 1, & \text{in position without the arrestors} \\ \tilde{k}, & \text{in position with the arrestors} \end{cases} \end{cases} \tag{8d}$$

in which l is the length of the beam element, $N_i(\bar{x})$ Hermite's interpolating function, x_F the distance from the origin to the left node of each beam element. The Einstein summation convention is here adopted for $i,j,k,m,n = 1,2,3,4$.

3.3 Time Integration and Iterative Procedure

In order to integrate Eq. (7) with time, the N-wmark constant acceleration method is employed as

$$w^{n+1} = w^n + \frac{1}{2}\Delta t(v^{n+1} + v^n) \tag{9a}$$

$$v^{n+1} = v^n + \frac{1}{2}\Delta t(a^{n+1} + a^n) \tag{9b}$$

where v and a are velocity and acceleration of the beam, respectively. Combining Eqs. (9a) and (9b) leads to

$$w^{n+1} = \tilde{w}^n + \frac{1}{4}(\Delta t)^2 a^{n+1} \tag{10a}$$

$$\tilde{w}^n = w^n + \Delta t v^n + \frac{1}{4}(\Delta t)^2 a^n \tag{10b}$$

Such a relationship brings Eq. (7) into a new form:

$$([M_D] + [K_S])\{w\}_{n+1} = P_{n+1}\{P\} + [M_D]\{\tilde{w}\}_n \tag{11}$$

where $[M_D] = \frac{4}{(\Delta t)^2}[M]$.

For each time increment, those two terms on the right-hand side of Eq. (11) are known, so that the unknown $\{w\}_{n+1}$ can be determined by the Newton-Raphson iterative process.

If the mth iterative vector is marked as $\{w\}_m$, then the $(m+1)$th correction can be easily given by

$$\{\Delta w\}_m = -[K_T^D]^{-1}\{\Psi\}_m \tag{12a}$$

$$\{\Psi\}_m = [K_S^D]^{-1}\{w\}_m - P_{n+1}\{P\} - [M_D]\{\tilde{w}\}_n \tag{12b}$$

where $[K_S^D]$ and $[K_T^D]$ are the secant and tangent stiffness matrixes for the dynamic system of the beam on the nonlinear elastic foundation respectively, and

$$[K_S^D] = [M_D] + [K_S] \tag{13a}$$

$$[K_T^D] = [M_D] + [K_T] \tag{13b}$$

According to the same principle as before, the detailed expression for the matrix $[K_T]_{i,j}$ for a beam element can be written as

$$(K_T)_{ij} = (K_S)_{ij} + c_1\beta^1_{ijk}w_k + 2c_2\beta^2_{ijmn}w_m w_n \tag{13c}$$

Following the above-listed steps one can calculate the response of the beam on the nonlinear elastic foundation under the lateral load given by Table 1. Obviously, the buckle propagation will be obstructed in the presence of arrectors. It is necessary to progressively increase the magnitude of p_1 (Fig. 3) up to P_0 (crossover pressure), at which the buckle will go through the arrestor for the sake of observing the full process of the arrest of dynamically propagating buckle.

Ⅳ. NUMERICAL RESULTS AND CONCLUSIONS

For a comparison with the results of the efficiency of the arrestor obtained by Youn in which a quasi-static state was assumed, all the parameters needed in the calculation are the same as [3]. These values are relisted as follows: $\lambda = 1/6$, $\eta = 0.2$, $\tilde{L} = 0.6484$, $\tilde{k} = 4.40$, and two arrestors are placed in $\xi = \pm 30$ which possesses a symmetric position with respect to the origin. In the case where the inertia of the beam is neglected, Youn[3] gained the following results: $P_s/(k_0H) = 0.06647$, $P_p/(k_0H) = 0.0108$, and $P_0/(k_0H) = 0.06647$, which are the dimensionless values of the initiated load, propagation load, and crossover load, respectively. This indicates that $\eta_0 = 100\%$ because P_s is just equal to P_p. However, taking the inertia of the beam into account will lead to different results from the above one.

In fact, our result is $\eta^{Dy}=12\%$ by using the present FE formulae (11—13) and (2). This means that the efficiency of the arrestor obtained by dynamic analysis is much lower than by quasi - static analysis. In addition, the full process of the arrest of dynamically propagating buckle is also obtained as shown in Figs. (4—5). It is seen from Fig. 4 that when $p_1/(k_0H) < P_0/(k_0H)$, the propagating buckle is obstructed in $\bar{\xi}=30$ by the arrestor which can not go through the arrestor. But Fig. 5 shows clearly that by progressively increasing p_1 up to P_0 ($=0.04389k_0H$), the buckle will continue propagating after going through the arrestor.

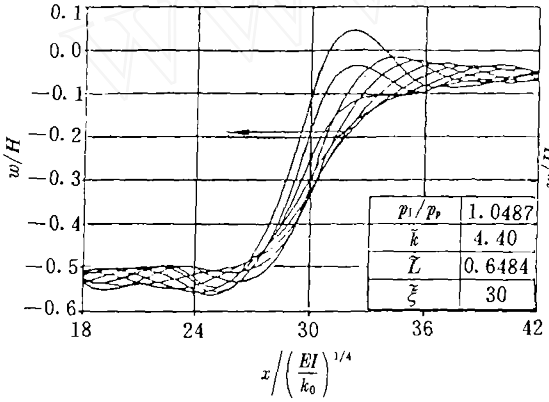

Fig. 1 The propagating buckle is obstructed in $\bar{\xi}=30$ by the arrestor.

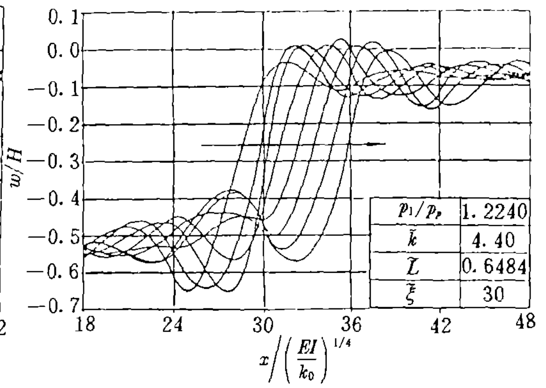

Fig. 5 The buckle continues propagating after going through the arrestor when $p_1 > P_0$.

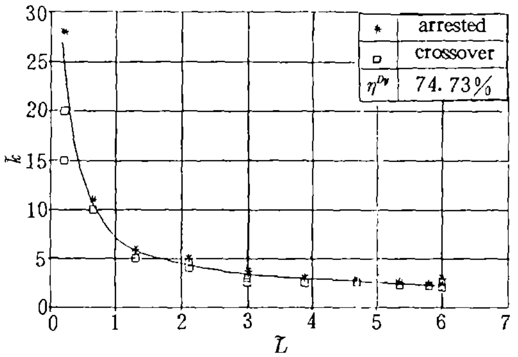

Fig. 6 A fit-matched relationship between $\tilde{L}$ and $\tilde{k}$ for getting a successful arrestor design.

In contrast with the quasi-static case, a fit-matched relationship between $\tilde{L}$ and $\tilde{k}$ different from Youn's[3] for making a successful arrestor design is given in Fig. 6, in which the efficiency of the arrestor based on dynamic analysis, η^{by}, is 74.73%.

Figure 6 tells us that a stiffer and wider arrestor has a higher capability to stop a propagating buckle. However, when the width $\tilde{L}$ is limited to a value lower than 0.1, no increase in $\tilde{k}$ can support a remarkable improvement in the arrestor's ability; on the other hand, if $\tilde{k}$ is limited to a value lower than 2.1 (for the quasi-static case lower than 1.6), then no increase in $\tilde{L}$ can give a remarkable improvement either.

The differences mentioned above demonstrate that the quasi-static design of an arrestor is sometimes unsafe owing to the neglect of the inertia of the beam.

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