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A GENERAL METHOD FOR SOLVING THE PROBLEM OF BOTH OPEN AND CLOSED MULTIPLE CRACKS

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For brittle materials microcracks often control overall deformation and failure mechanics. The study of solids containing multiple cracks has been the subject of many investigations. Due to the difficulty in the exact solution of the problem involving multiple cracks, a number of simplified methods have been proposed, see a critical review by Kachanov [1].

Recently, we presented a very accurate and efficient method for solving multiple arbitrary cracks problems in plane elastic solids [2]. The method takes into direct account the interactions between different cracks without any simplification assumption.

In the method, each crack is described by a distribution of complex dislocations (with both Type I and Type II dislocations). The complex dislocation density function $\mathbf{b}=b_{\mathrm{I}}+ib_{\mathrm{II}}$ is expressed as a Chebyshev polynomial series:

$$b(s)=\frac{i(k+1)}{\mu}\sum_{0}^{\infty}\alpha_m(1-\tau^2)^{-\frac{1}{2}}T_m(s/a) \tag{1}$$

where a is half crack length, α_m are unknowns to be determined.

Then the complex potentials for the crack in the local system can be expressed as:

$$\begin{aligned}\Phi(z)=\Omega(z)&=\frac{\mu}{\pi i(k+1)}\int_{-a}^{a}\frac{b(s)ds}{z-s}\\ &=\sum_{m=0}^{\infty}\alpha_m\left(\frac{z}{a}-\sqrt{\frac{z^2}{a^2}-1}\right)^m/\sqrt{\frac{z^2}{a^2}-1}\end{aligned} \tag{2}$$

The stresses at any point due to the crack can be expanded as series also.

For a solid containing multiple N cracks, the problem can be represented as a superposition of N individual crack problems, each involving only one isolated crack but loaded by unknown tractions, induced by all cracks and remote loadings. On the basis of superposition technique and the traction free conditions on crack surfaces, a set of governing equations for the multiple crack problem are developed. Using boundary collocation method, the governing equations are reduced to a set of algebraic equations. When the algebraic equations are solved, the complex potentials and the stress components produced by each crack are known. According to the superposition principle, the stress fields produced by the multiple cracks are obtained with the aid of the transformation formulas.

On crack surfaces, traction free conditions are used, these mean that all cracks are open ones.

Through some modification, the method can be used to solve both open and closed multiple cracks problems. For a solid with multiple cracks under remote loadings, perhaps some crack surfaces are open, some others are closed. First all crack surfaces weres considered to be open, and the multiple cracks problem was solved by using the traction free conditions on all crack surfaces as before. When it was solved, we calculated the stress intensity factors and crack surface displacements of all cracks. If the Type I stress intensity factors and crack normal surface displacements of all cracks are positive, this means all cracks are indeed open with this condition, and the problem is solved. On the other hand, if the Type I stress intensity factors and crack normal surface displacements of some or all cracks are negative, this irrational result means that the assumption that all cracks are open is not correct. Then some cracks are considered as closed sliding cracks, they are represented by distribution of only Type II dislocations, and shear traction free conditions on these sliding (without friction) cracks are used. Then we resolved the multiple cracks problem again checking if crack surface conditions are reasonable or not, that is crack surfaces should not interpenetrate and the normal tractions on closed sliding crack surfaces should be negative. If reasonable, then the problem is solved. Otherwise, the assumption of open and closed crack conditions must be changed until a reasonable result is reached.

An example is given to show the process. A solid with two cracks under remote loadings is shown in Fig. 1. The half lengths of the two equal cracks are *as*. The space of the two crack centers is $2a$. The remote loading $\sigma_0=\tau_0$. First, assuming the surfaces of the two cracks are open, we solved the two cracks problem. The stress intensity factors of the cracks are given in Table 1. One can find that the Type I stress intensity factors of Crack 2 are negative, and the calculated normal surface displacement of the crack is also negative.

Then, assuming Crack 2 is a closed one, we solved it again. The stress intensity factors are also given in Table 1. The crack surface displacements and stresses on crack lines of the two cracks are shown in Figs. 2 and 3, respectively. On the open Crack 1 surface, the normal and shear tractions are zero (not shown in the figure), the normal traction at the crack out tips and the normal surface displacement are positive. On the closed Crack 2 surface, the normal displacement and shear traction are zero (not shown in the figure), the normal traction is negative, the normal surface displacement is zero. The displacements and tractions on the two crack surfaces, show that the assumption of the two crack surface conditions is correct now, that is Crack 1 is open and Crack 2 is the closed one. The two cracks problem is solved then.

In some complex multiple cracks problems, some cracks may even be partially closed under some loading conditions. This kind of cracks problem is under investigation now.

REFERENCES

[1] M. Kachanov, *Applied Mechanics Reviews* 45 (1992) 304-335.
[2] Han Xueli and Wang Tzuchiang, in *Progress in Advanced Materials and Mechanics*, Wang Tzuchiang and Tsu-Wei Chou (eds.), Chinese Journal of Mechanics Press, Beijing (1996) 688-693.

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Table 1. The stress intensity factors assuming different crack surfaces conditions

	Crack 1 open		Crack 2 open	
	Tip A	Tip B	Tip C	Tip D
$k_I/\sigma_0\sqrt{a}$	0.9751386	0.9897022	-0.2531224	-0.4970980
$k_{II}/\sigma_0\sqrt{a}$	1.014654	0.9906443	0.5296456	0.5574737
	Crack 1 open		Crack 2 close	
	Tip A	Tip B	Tip C	Tip D
$k_I/\sigma_0\sqrt{a}$	1.011600	1.068195		
$k_{II}/\sigma_0\sqrt{a}$	0.9897251	0.9716754	0.5419421	0.5587892

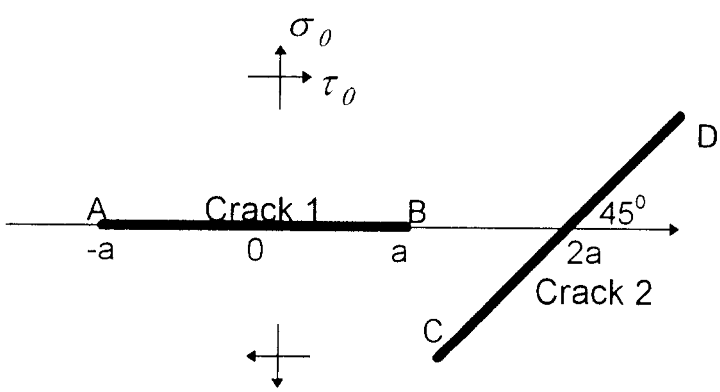

Figure 1. Two equal cracks under remote loadings.

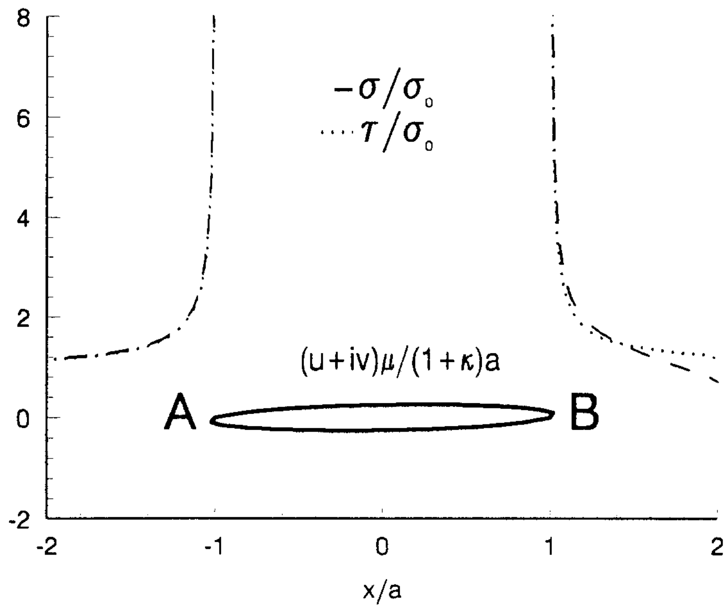

Figure 2. Crack 1 surface displacement and stresses in the crack plane.

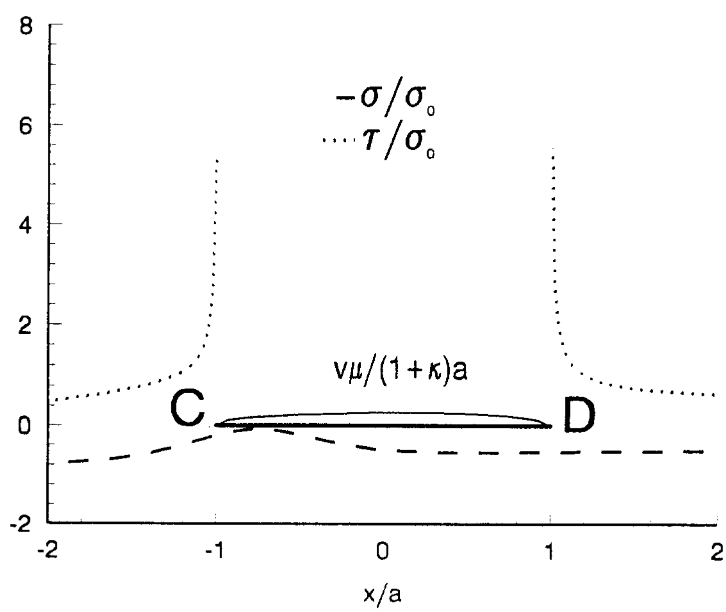

Figure 3. Crack 2 shear surface displacement and stresses in the crack plane.