Sample-Specific Behavior in Failure Models of Disordered Media

Meng-Fen XIA
Department of Physics, Peking University, Beijing 100871, China and
Center of Nonlinear Science, Peking University, Beijing 100871, China and
LNM, Institute of Mechanics, Academia Sinica, Beijing 100080, China
Zhong-Qing SONG and Jian-Bo XU
Department of Physics, Peking University, Beijing 100871, China
Kai-Hua ZHAO
Department of Physics, Peking University, Beijing 100871, China and
Center of Nonlinear Science, Peking University, Beijing 100871, China
Yi-Long BAI
LNM, Institute of Mechanics, Academia Sinica, Beijing 100080, China
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Abstract
The concept “sample-specific” is suggested to describe the behavior of disordered media close to macroscopic failure. It is pointed out that the transition from universal scaling to sample-specific behavior may be a common phenomenon in failure models of disordered media. The dynamical evolution plays an important role in the transition.

I. Introduction

Mechanical and electrical failure in disordered media is a problem of scientific and technological importance and has been intensively studied for years. However, a series of fundamental questions are still open due to its incredible richness in complexity. In recent years, emphasis has been focused on the universal behavior in failure based on several simple models.\textsuperscript{[1-9]} These models showed that:

1) The macroscopic properties of these models obey universal scaling laws in weakly damaged regime.\textsuperscript{[1-4]} For example, the data of force $F$ and displacement $\lambda$ collapse with quite weak statistical fluctuations into a scaling law

$$F = L^\alpha \psi(\lambda L^{-\beta})$$

where $L$ is the size of the system, and exponents $\alpha \simeq \beta \simeq 1$ for two-dimensional network.

2) The macroscopic failure exhibits catastrophic feature resulting from cascade of damage from small to large scales. This is related to the evolution far from equilibrium, so it may be called evolution induced catastrophe (EIC).\textsuperscript{[5]}

In addition to the above-mentioned universal behavior, there is still a noticeable common feature in the failure models of disordered media. It was found that\textsuperscript{[1-4]} in growth models, the statistical fluctuations become very strong in catastrophic regime, and the scaling law no longer works. The behavior closely related to macroscopic failure demonstrates significant differences from sample to sample. In the EIC model, it was pointed out that this behavior apparently results from its sensitive dependence on mesoscopic pattern. In this paper, we would like to suggest a unified term, sample-specific behavior, to describe such a kind of common phenomena in the failure processes.

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II. Failure Model

For simplicity, we adopt a random fuse network, however, the obtained results seem to be suitable for a variety of the failure models.

Consider a two-dimensional network with $N \times N$ sites shown in Fig. 1, accordingly, the number of bonds is given by $\Omega = gL^2$, where $L = N - 1$ and $g = (2N - 1)/(N - 1) \approx 2$. As an electric network, the voltage $V$ is applied vertically and so does the total current $I$. The counterparts in a mechanical network are $V \rightarrow \lambda$ and $I \rightarrow F$.

The resistance of each bond is assumed to be a unit, and a bond would break if the current $i$ through it exceeds its threshold $i_c$. The quenched disorder of the system is taken into account by randomly assigning the threshold according to a given distribution $f(i_c/i_o)$, where $i_o$ is the maximum threshold. We consider Weibull distribution

$$f(x) = cmx^{m-1}e^{-x^m}, \quad 0 \leq x \leq 1$$

and power law distribution

$$f(x) = (1 + \nu)x^n, \quad 0 \leq x \leq 1,$$

where $m$ and $n$ are two parameters, and $c$ is a normalized constant.

We can construct a sample space consisting of samples with the same threshold distribution function. These samples look alike macroscopically, and the differences between them are in the details of their mesoscopic structures only.

For a specified sample, the calculating procedure is as follows: After giving an applied voltage, we determine the current $i$ passing each bond, and remove the bond with $i > i_c$ and having maximum ratio $i/i_o$. Then, we recalculate the current $i$ and pick up another bond to be removed, until an equilibrium state is reached. To start next round, we increase the applied voltage gradually and repeat the above procedure until the network finally becomes macroscopically disconnected.

III. Scaling Law

Now, we would like to derive a scaling law based on the mean-field approximation in weakly damaged regime. In this regime, the breakdown of symmetry and homogeneity of the system is still insignificant, so, averagings in controlling field as well as in the sample space are reasonable. According to the mean-field approximation, the voltage acting on each bond is $V/L$, and the current $i = V/L$ because of unit resistance. For applied voltage $V$, the number of unbroken bonds is $\Omega \int_{i/i_o}^1 f(x)dx$. So, the total current $I$ can be expressed as

$$I = iL \int_{i/i_o}^1 f(x)dx = LV/V/L,$$

where the scaling function $\psi$ is given by

$$\psi(x) = gx \int_{x/i_o}^1 f(x')dx',$$

which is dependent on the threshold distribution function $f$. In comparison with Eq. (1), we can see that the scaling exponents $\alpha = \beta = 1$ in the two-dimensional system. Similarly, for a three-dimensional network, we have obtained $\alpha = 1$ and $\beta = 1$.

Some numerical results for the case of power law and Weibull distribution are shown in Figs 2a–2c, respectively. The curves marked by $\psi$ in each figure represent scaling law (4). Figure 2 demonstrates that numerical data do collapse into the scaling law for the samples of different sizes in initial regime, and there are only very weak fluctuations.
Fig. 1. Sketch map of the network.

Fig. 2a. \( (I/L)-(V/L) \) characteristics: power law distribution, \( \nu = 0.65, L = 20, 40 \) and 80.

Fig. 2b. \( (I/L)-(V/L) \) characteristics: power law distribution, \( \nu = 0.45, L = 20, 40 \) and 80.

Fig. 2c. \( (I/L)-(V/L) \) characteristics: Weibull distribution, \( m = 4, L = 20, 40 \) and 80.

IV. Sample-Specific Behavior

However, the fluctuations are getting stronger and stronger as the damage proceeds. Especially, the fluctuations become a predominate feature in catastrophic regime. This means that the samples even with slight initial mesoscopic differences may exhibit significant different failure behaviors from sample to sample. Obviously, in this situation, averaging over sample space becomes meaningless. Therefore, when approaching to macroscopic failure, the term "strong fluctuations" may not make an appropriate description of the behavior. So, we would like to suggest a new concept, the sample-specific behavior, to describe such a kind of sensitive dependence of failure on mesoscopic details of samples.

The transition of macroscopic behavior from scaling to sample-specific is a common phenomenon in a variety of the failure models of disordered media. Obviously, it is the dynamical evolution which leads to the strong amplification of differences between the samples as they are approaching to eventual failures. Meanwhile, the derivation of controlling field from the mean field is also enhanced.

The distribution of controlling field (or current) in network for the case of power law distribution is shown in Fig. 3. Figure 3a shows the very early stage, the current is homogeneous, i.e., the mean-field assumption works well. Figure 3b demonstrates the scaling regime with
approximately homogeneous current, so the mean-field approximation is also suitable. Figure 3c shows the current distribution at the vicinity of maximum of $I-V$ characteristics, there is a significant derivation of current from the homogeneous distribution. Figure 3d is close to eventual failure, there are only a few current-carrying bonds, i.e., the current distribution becomes localized. Therefore, we can see that, as the damage proceeds, the deviation of controlling field from the mean field is getting stronger and stronger, and when close to macroscopic failure the mean-field approximation is no longer meaningful.

![Graphs](image)

**Fig. 3.** Current distribution in network for the case of power law distribution with $\nu = 0.85$ and $L = 80$. The abscissa represents the current passing a bond (in arbitrary unit), and the ordinate represents the number of bonds: a) at very early stage; b) in the scaling law regime; c) in the vicinity of maximum of $I-V$ characteristics; d) close to eventual failure.

The distribution of threshold of the broken bonds is shown in Fig. 4 for the power law case, and the curve marked by $f$ represents the distribution function $f$. Figure 4a is in the scaling law regime, the most of broken bonds are the weak bonds. Figure 4b is in the stage of
macroscopic failure, we can see that a series of bonds with high thresholds have been broken due to the deviation from the mean field.

![Graph](image)

**Fig. 4.** Distribution of threshold of broken bonds, the curve marked by $f$ represents the distribution function $f(x)$: a) in the scaling law regime; b) after macroscopic failure.

The mechanism of transition from scaling to sample-specific may be attributed to the deviation of controlling field from the mean field and the enhancement of differences between samples due to the dynamical evolution.

The implication of "sample-specific" can be delineated as follows:

1) The behaviors of macroscopic failure may be significantly different from sample to sample. That is to say, the failure may be quite sensitive to mesoscopic details of the system. Macroscopically, instead of deterministic, a statistical formulation of failure should be introduced.
2) The different failure behaviors of samples could not be simply attributed to initial differences between the samples. Although, as a dynamical system, a final state is determined by its initial state uniquely, the final differences between the samples can be strongly enhanced by the dynamical evolution. Both the initial differences and the dynamical evolution are inherent in the sample-specific behavior of failure.

3) As the damage proceeds from initial to catastrophic regime, the transition from universal scaling to sample-specific seems to be a common phenomenon in the failure processes of disordered media. The mechanism of the transition should be attributed to the serious deviation of controlling field from the mean field and the radical development of differences in sample space. In this sense, a serious deviation from scaling law may be regarded as a precursor to the macroscopic failure.

The sample-specific behavior discussed in the present paper may be beneficial to understanding universal features in failure processes.

References