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MICROMECHANICS ANALYSES OF INTERFACE CRACK*

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ABSTRACT: A new mechanics model based on Peierls concept is presented in this paper, which can clearly characterize the intrinsic features near a tip of an interfacial crack. The stress and displacement fields are calculated under general combined tensile and shear loadings. The near tip stress fields show some oscillatory behaviors but without any singularity and the crack faces open completely without any overlapping when remote tensile loading is comparable with remote shear loading. A fracture criterion for predicting interface toughness has been also proposed, which takes into account for the shielding effects of emitted dislocations. The theoretical toughness curve gives excellent prediction, as compared with the existing experiment data.

KEY WORDS: micromechanics, interface crack, toughness

1 INTRODUCTION

Interfaces are found in many advanced materials including fiber reinforced and layered composites, metal/ceramic systems, polymetric adhesive joints, coating and film/substrate for electronic packaging systems among others. Interface fracture is a common phenomenon and plays an important role for the overall mechanical properties of advanced materials.

The classical works[1-3] on fracture mechanics of interfaces between two dissimilar elastic materials have revealed several important features, such as oscillatory singularity, penetration of crack faces and peculiar dimension of stress intensity factors. In order to overcome these unrealistic features, a new mechanics model for interface cracks has been developed in this paper, which can clearly describe the intrinsic features near the crack tip. A fracture criterion for predicting interface toughness has been also proposed, which takes into account for the shielding effects of emitted dislocations. Theoretical results for an epoxy/glass interface system are presented and compared with existing experimental results.

2 BASIC FORMULAS

We consider a semi-infinite crack lying on an interface between two isotropic elastic materials as shown in Fig.1. The crack lies on the negative x axis with its tip at $x = 0$. The problem of interaction of a dislocation at $z = z_0$ with a semi-infinite interface crack was solved by Suo[4]. If the z_0 is real, the solution can be expressed as follows

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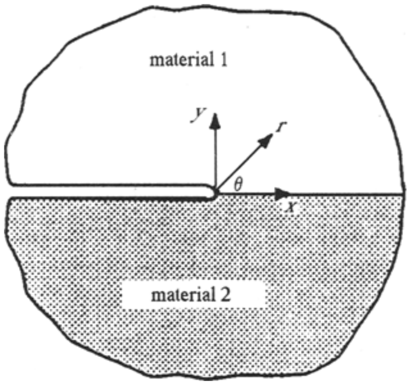

Fig.1 An interfacial crack between two dissimilar elastic materials

$$\sigma_y - \mathrm{i}\tau_{xy} = \Phi(z) + \Omega(\bar{z}) + (z-\bar{z})\overline{\Phi'(z)} \tag{1}$$

$$\Phi(z) = \Omega(z) = \begin{cases} \dfrac{1}{(1-\beta)}H(z) & z \in S_1 \\ \dfrac{1}{(1+\beta)}H(z) & z \in S_2 \end{cases} \tag{2}$$

$$\left.\begin{aligned} H(z) &= \Big(\frac{z}{z_0}\Big)^{-1/2-\mathrm{i}\varepsilon}\frac{B}{z-z_0} \\ B &= \frac{\mu}{\pi\mathrm{i}(\kappa+1)}(b_x+\mathrm{i}b_y) \end{aligned}\right\} \tag{3}$$

$$\frac{\kappa+1}{\mu} = \frac{\kappa_1+1}{2\mu_1} + \frac{\kappa_2+1}{2\mu_2} \tag{4}$$

$$\varepsilon = \frac{1}{2\pi}\ln\frac{1-\beta}{1+\beta} \tag{5}$$

α, β Dundurs parameters are

$$\alpha = \frac{(\kappa_2+1)/\mu_2 - (\kappa_1+1)/\mu_1}{(\kappa_2+1)/\mu_2 + (\kappa_1+1)/\mu_1} \qquad \beta = \frac{(\kappa_2-1)/\mu_2 - (\kappa_1-1)/\mu_1}{(\kappa_2+1)/\mu_2 + (\kappa_1+1)/\mu_1} \tag{6}$$

where $b_e = b_x + ib_y$ is the Burgers vector of an edge dislocation. For a distribution of dislocation ahead of a crack tip, we have

$$H(z) = \frac{\mu}{(\kappa+1)}\frac{1}{\pi\mathrm{i}}\int_0^R \frac{(\tau/2)^{1/2+\mathrm{i}\varepsilon}b_e(\tau)\mathrm{d}\tau}{(z-\tau)} \tag{7}$$

The traction on the cohesive zone ahead of the crack tip is

$$\begin{aligned} \sigma_y - \mathrm{i}\tau_{xy} = \sigma_y^{(0)} - \mathrm{i}\tau_{xy}^{(0)} + \frac{2\mu}{(\kappa+1)(1-\beta^2)}\Big[\beta(b_x+\mathrm{i}b_y) + \\ \frac{1}{\pi\mathrm{i}}\int_0^{R_1}\frac{\tau^{1/2+\mathrm{i}\varepsilon}b_x(\tau)\mathrm{d}\tau}{x^{1/2+\mathrm{i}\varepsilon}(x-\tau)} + \frac{1}{\pi}\int_0^{R_2}\frac{\tau^{1/2+\mathrm{i}\varepsilon}b_y(\tau)\mathrm{d}\tau}{x^{1/2+\mathrm{i}\varepsilon}(x-\tau)}\Big] \end{aligned} \tag{8}$$

where $\sigma_y^{(0)}, \tau_{xy}^{(0)}$ are the singular stress field, R_1 denotes the length of the cohesive zone for sliding displacement and R_2 the length of the decohesive zone for opening displacement. Beyond R_1, there is no discontinuity for sliding displacement, meanwhile beyond R_2, discontinuity of the opening displacement vanishes.

Let Δ_x and Δ_y denote the relative sliding and opening displacements of two atomic planes adjacent to the interface plane, respectively. According to Ref.[5], we have

$$\left.\begin{aligned} \tau_x &= \tau_{\max}A(\Delta_y)\sin\Big(2\pi\frac{\Delta_x}{b}\Big) \\ \sigma_y &= \sigma_{\max}B(\Delta_x)\frac{\Delta_y}{\Delta_m}\mathrm{e}^{1-(\Delta_y/\Delta_m)} \end{aligned}\right\} \tag{9}$$

where

$$A(\Delta_y) = \Big(1+\frac{\Delta_y}{\Delta_m}\Big)\mathrm{e}^{-\Delta_y/\Delta_m} \qquad B(\Delta_x) = 1 - q\sin^2\Big(\pi\frac{\Delta_x}{b}\Big) \tag{10}$$

The functions $b_x(x)$ and $b_y(x)$ can be expressed as the following sine series[6]

$$\left.\begin{aligned} \frac{2\mu}{(\kappa+1)}t_1^{1/2}b_x(x) &= \sum_{m=1}^{\infty}\alpha_m \sin m\theta \qquad t_1=\frac{1}{2}(1+\cos\theta) \quad 0\le\theta\le\pi \\ \frac{2\mu}{(\kappa+1)}t_2^{1/2}b_y(x) &= \sum_{m=1}^{\infty}\beta_m \sin m\varphi \qquad t_2=\frac{1}{2}(1+\cos\varphi) \quad 0\le\varphi\le\pi \end{aligned}\right\} \tag{11}$$

where $t_1 = x/R_1$ and $t_2 = x/R_2$.

Substitute Eqs.(11) into (8), one obtains

$$\left.\begin{aligned} \sigma_y - \mathrm{i}\tau_{xy} = \sigma_y^{(0)} - \mathrm{i}\tau_{xy}^{(0)} &+ \frac{1}{(1-\beta^2)}\Big\{\beta\Big[\frac{1}{\sqrt{t_1}}\sum_{m=1}^{\infty}\alpha_m \sin m\theta + \\ &\mathrm{i}\frac{1}{\sqrt{t_2}}\sum_{m=1}^{\infty}\beta_m \sin m\varphi\Big] - \mathrm{i}t_1^{-(1/2+\mathrm{i}\varepsilon)}\sum_{m=1}^{\infty}\alpha_m\Big[A_m(\theta)+\mathrm{i}B_m(\theta)\Big] + \\ &t_2^{-(1/2+\mathrm{i}\varepsilon)}\sum_{m=1}^{\infty}\beta_m\Big[A_m(\varphi)+\mathrm{i}B_m(\varphi)\Big]\Big\} \end{aligned}\right\} \tag{12}$$

where

$$\left.\begin{aligned} A_m(\theta) &= \sum_{n=1}^{\infty}(P_{|m-n|} - P_{m+n})\cos n\theta \\ B_m(\theta) &= \sum_{n=1}^{\infty}(Q_{|m-n|} - Q_{m+n})\cos n\theta \end{aligned}\right\} \tag{13}$$

$$P_k = \frac{1}{\pi}\int_0^{\pi}\cos(\varepsilon\ln t_1)\cos k\theta \mathrm{d}\theta \tag{14a}$$

$$Q_k = \frac{1}{\pi}\int_0^{\pi}\sin(\varepsilon\ln t_1)\cos k\theta \mathrm{d}\theta \tag{14b}$$

The opening and sliding displacements take the form

$$\delta_x + \mathrm{i}\delta_y = \int_x^R b(\tau)\mathrm{d}\tau \tag{15a}$$

$$\frac{\mu}{(\kappa+1)}\delta_x = \frac{R_1}{4}\cdot\sum_{m=1}^{\infty}\alpha_m V_m(\theta) \qquad \frac{\mu}{(\kappa+1)}\delta_y = \frac{R_2}{4}\cdot\sum_{m=1}^{\infty}\beta_m V_m(\varphi) \tag{15b}$$

$$V_m(\theta) = \frac{\sin\left(m-\frac{1}{2}\right)\theta}{m-\frac{1}{2}} - \frac{\sin\left(m+\frac{1}{2}\right)\theta}{m+\frac{1}{2}} \tag{16}$$

3 CALCULATION METHOD AND RESULTS

The calculation method is similar to that of analyses for dislocation nucleation and emission from crack tip for homogeneous material by Wang[6]. The cohesive zone $0 \le x \le R_1$ is discretized into M elements

$$x_i = \frac{1}{2}R_1\Big[1+\cos\Big(\frac{(i-1)\pi}{M}\Big)\Big] \tag{17}$$

Similarly, the decohesive zone $0 \le x \le R_2$ is divided into N elements

$$x_j^* = \frac{1}{2}R_2\Big[1 + \cos\Big(\frac{(j-1)\pi}{N}\Big)\Big] \tag{18}$$

The cohesive relation (9) should be satisfied on these nodal points. Thus the governing equations are transformed into a set of nonlinear algebraic equations.

$$\left.\begin{aligned}
&\sum_{k=1}^{M} a_{ik}\alpha_k + \sum_{k=1}^{N} b_{ik}\beta_k - A(\Delta_{yi})\sin\big(2\pi\frac{\Delta_{xi}}{b}\big) = 0 \qquad && i = 1, 2, \cdots, M \\
&\sum_{k=1}^{N} a_{jk}^*\beta_k + \sum_{k=1}^{N} b_{jk}^*\beta_k - B(\Delta_{xi}^*)\frac{\Delta_{yj}^*}{L}\exp\big(1 - \frac{\Delta_{yj}^*}{L}\big) = 0 \qquad && j = 1, 2, \cdots, N
\end{aligned}\right\} \tag{19}$$

Equations.(19) are solved by the Newton-Raphson method. The iterating convergence is obtained after five to ten iterations. Most calculation in this paper was carried out with five digits of accuracy for stress fields in the cohesive zone.

The fracture toughness testing for an epoxy/glass interface was carried out by Liechti and Chai[7]. The Young's modulus of epoxy is $E_1 = 2.07$ GPa, and the Poisson's ratio is $\nu_1 = 0.37$, while the Young's modulus of glass is $E_2 = 68.9$ GPa, and the Poisson's ratio is $\nu_2 = 0.20$. The Dundurs parameters are $\alpha = -0.935$ and $\beta = -0.188$.

The shear modulus μ_s and Poisson's ratio ν_s of the interface layer are taken to be equal to the equivalent shear modulus μ and equivalent Poisson ratio ν of the bimaterial, respectively in the present calculation. The parameters related to the microstructure and the mechanical properties of the interfacial layer are chosen as

$$\frac{h}{b} = 1, \frac{\Delta_m}{b} = 0.4, \nu = 0.3, \frac{\tau_0}{\mu} = 0.01, \frac{\sigma_0}{\mu} = 0.0208, \frac{\tau_{\max}}{\mu} = 0.159, \frac{\sigma_{\max}}{\mu} = 0.390$$

and $M = 180, N = 180$.

The parameter b is chosen as the reference length scale to characterize the nondimensional stress intensity factor as follows

$$q = q_1 + \mathrm{i}q_2 = \frac{Kb^{\mathrm{i}\omega}}{\sqrt{2b\pi}\mu} = |q|\mathrm{e}^{\mathrm{i}\omega} \tag{20}$$

Fig.2 and Fig.3 show the calculation results for the case of $\frac{R_1}{b} = 0.05$ and $\frac{R_2}{b} = 1.0$. The corresponding nondimensional stress intensity factors and the phase angle are $q_1 = 0.0202$ and $q_2 = 0.00198$ and $\omega = 5.6°$, respectively. The classical elastic results are denoted by solid lines with small white circles and the present results are denoted by solid lines with small black circles. Both results agree well with each other for $(x/b) > 1$. When x approaches to zero, the elastic stress fields show an oscillatory singularity, while the present stress results approach to finite values. It can be seen from Fig.3 that the crack faces completely open without any penetration for the present results, while the crack faces contact near the crack tip for the elastic solution.

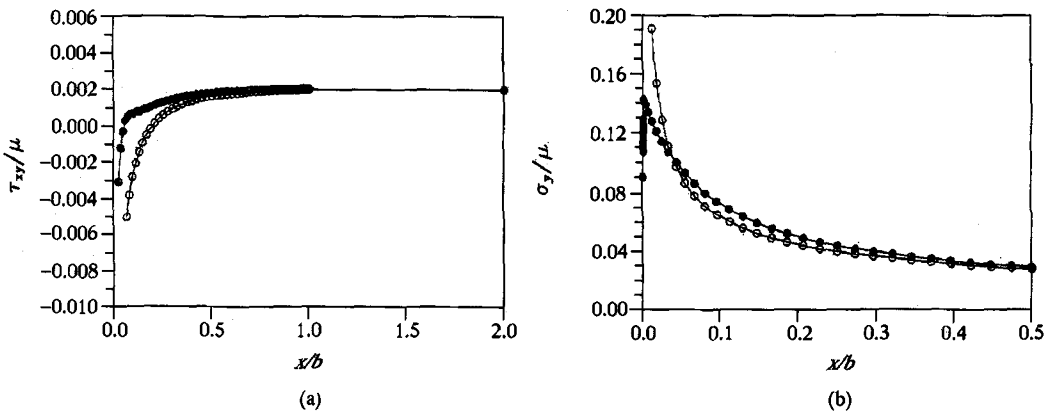

Fig.2 Shear stress and normal stress distribution ahead of the crack tip for the case of $R_1/b = 0.05$ and $R_2/b = 1.0, q_1 = 0.0202$ and $q_2 = 0.00198$ and $\omega = 5.6°$

• present results o elastic results

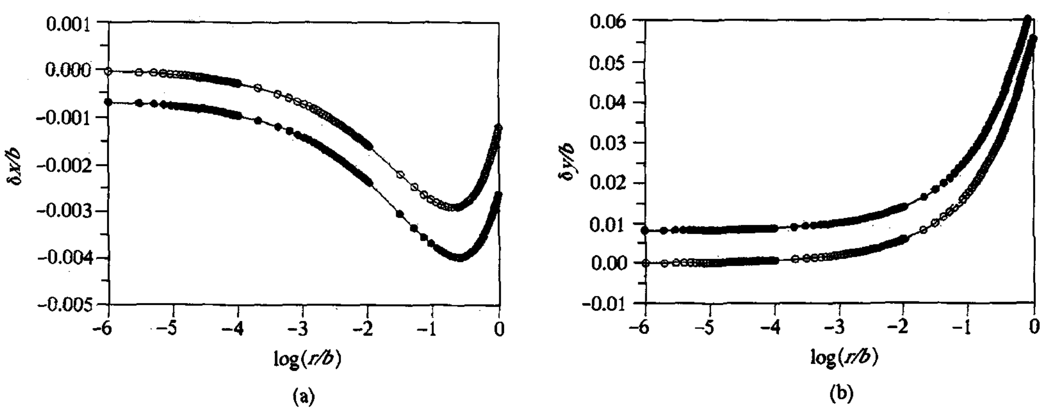

Fig.3 Shear and opening displacement profiles. The corresponding parameters are as same as that of Fig.2

• present results o elastic results

4 THEORETICAL PREDICTION ON INTERFACE TOUGHNESS CURVES

Numerous experimental results[7,8] show that the interface fracture toughness strongly depends on the mode mixty.

This section presents a micromechanics analysis for the interface fracture toughness. The idea adopted here is essentially the same as that proposed by Rice[5]. Suppose both the material I and material II are brittle or semi-brittle materials. The fracture process zone is in a nanon scale and the plastic deformation is concentrated on a narrow strip near the interface. Assume that an emitted dislocation is located at $x = x_c$ ahead of the tip of interface crack with Burgers vector $b_e = b_x$.

The stress intensity factor K_d shielded by the emitted dislocation is

$$K_d = \lim_{x \to 0} (\sigma_y + i\tau_{xy})\sqrt{2\pi x} x^{-i\varepsilon} = -\sqrt{2\pi} \frac{2}{(1-\beta^2)} \frac{\mu i b_x}{\pi(\kappa+1)} \frac{x_c^{-i\varepsilon}}{\sqrt{x_c}} \tag{21}$$

As we know when $\varepsilon \neq 0$ the interface crack is of inherently mixed mode. But based on the new mechanics model proposed in section II, one can get an essential pure mode I interface crack under certain conditions. A typical example with parameters $R_1/b = 0.01$ and $R_2/b = 480$, is shown in Fig.4. The calculated nondimentional stress intensity factors $q_1 = 0.460$ and $q_2 = -0.0537$. The shear stress in the cohesive zone is extremely small, as compared with the normal stress at the crack tip $\sigma_y^{\rm tip}$. Hence the interface crack can be considered as "pure mode I" crack. For such kind of interface crack, the phase angle $\omega = -6.65°$.

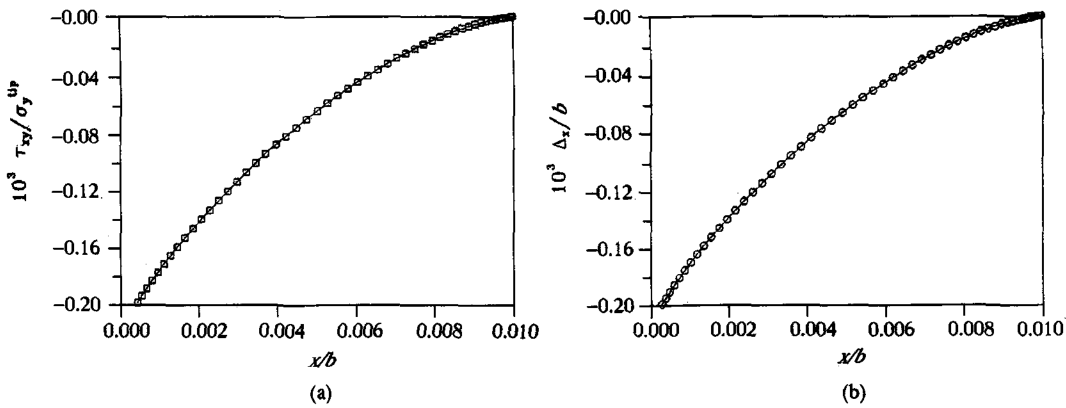

Fig.4 A typical example of a "pure mode I" interfacial crack with parameters $R_1/b = 0.01$ and $R_2/b = 480$

Now we can introduce the characterizing length L, for which the mode mixty or the phase angle ψ for the generalized stress intensity factor $k = KL^{i\varepsilon}$ is zero. We find $L/b = \exp(-\omega/\varepsilon) = 6.93$. Obviously such choice of the characterizing length is consistent with the deformation state in the fracture process zone.

The local stress intensity factors are given by

$$\left.\begin{aligned} k_{\rm I}^{\rm tip} &= k_{\rm I} + k_{{\rm I}d} \\ k_{\rm II}^{\rm tip} &= k_{\rm II} + k_{{\rm II}d} \end{aligned}\right\} \tag{22}$$

where $k_{\rm I}$ and $k_{\rm II}$ are the stress intensity factors due to external combined tensile and shearing loading and $k_{{\rm I}d}$ and $k_{{\rm II}d}$ are the stress intensity factors contributed by the emitted dislocations. From (21), one can obtain

$$\left.\begin{aligned} k_{{\rm I}d} &= -\frac{\mu b}{(1-\nu)(1-\beta^2)} \sum_{i=1}^{N} \frac{1}{\sqrt{2\pi r_i}} \sin\left(\varepsilon \ln\frac{r_i}{L}\right) \\ k_{{\rm II}d} &= -\frac{\mu b}{(1-\nu)(1-\beta^2)} \sum_{i=1}^{N} \frac{1}{\sqrt{2\pi r_i}} \cos\left(\varepsilon \ln\frac{r_i}{L}\right) \end{aligned}\right\} \tag{23}$$

where $r_i (i = 1, 2, \cdots, N)$ is the distance from crack tip of the i-th emitted dislocation.

Suppose that all emitted dislocations are far away from the crack tip and pile up in a small region. Hence we have $r_i \doteq r_c$, $i = 1, 2, \cdots, N$. Thus we obtain

$$k_{\rm I}^{\rm tip} = k_{\rm I} - NC_1 k_{\rm IIe} \qquad k_{\rm II}^{\rm tip} = k_{\rm II} - NC_2 k_{\rm IIe} \tag{24}$$

where

$$\left.\begin{aligned} &C_1 = C_0 \sin\left(\varepsilon \ln\frac{r_c}{L}\right) \qquad C_2 = C_0 \cos\left(\varepsilon \ln\frac{r_c}{L}\right) \\ &C_0 = \frac{\mu b}{(1-\nu)(1-\beta^2)} \frac{1}{\sqrt{2\pi r_c}} \frac{1}{k_{\rm IIe}} \end{aligned}\right\} \tag{25}$$

where $k_{\rm IIe}$ is the critical stress intensity factor for dislocation emission from the crack tip. The fracture criterion for the interfacial crack is given by

$$\left(k_{\rm I}^{\rm tip}\right)^2 + \left(k_{\rm II}^{\rm tip}\right)^2 = \left(k_{\rm Ic}\right)^2 \tag{26}$$

where $k_{\rm Ic}$ is the fracture toughness for the "pure mode I" crack defined according to the present new mechanics model.

Let

$$\text{tg}\psi = \frac{k_{\rm II}}{k_{\rm I}} \qquad \rho = \frac{k_{\rm Ic}}{k_{\rm IIe}} \tag{27}$$

For given ρ and ψ, one can easily get the number N and the critical value of $\frac{k_{\rm I}}{k_{\rm IIe}}$ from Eqs.(24) and (26). The critical energy release rate G_c is given by

$$G_c = \frac{(1-\nu)(k_{\rm I}^2 + k_{\rm II}^2)}{2\mu \cos h^2 \pi\varepsilon} \tag{28}$$

The experiment data of interfacial toughness under mixed mode condition for epoxy/glass system measured by Liechti and Chai[7] are shown in Fig.5. It is found the toughness curve has a minimum value of $G_{\min} = 2\gamma_s = 4\,\text{J/m}^2$ at $\hat{\psi} = 16°$.

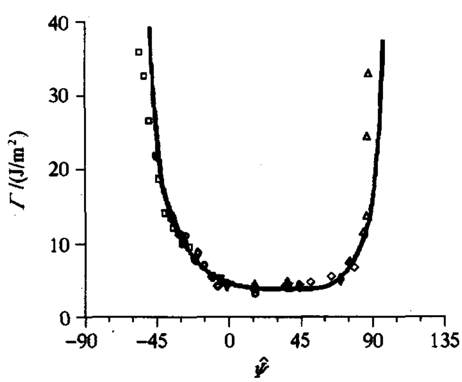

Fig.5 Present theoretical prediction of the fracture toughness and comparison with experiment data on an epoxy/glass interface system by Liechti and Chai[7]

Using above formulas, we can obtain following material and geometry parameters

$$\left.\begin{aligned} &\Delta_m = 2.32\,\mathrm{nm} \qquad b = h = 2.5\Delta_m = 5.8\,\mathrm{nm} \\ &L = 6.93b = 40.2\,\mathrm{nm} \\ &\hat{\psi} = \psi + \varepsilon \ln\frac{\hat{L}}{L} = \psi + 43.5^\circ,\ \hat{L} = 12.7\,\mathrm{mm} \\ &\rho = \frac{k_{\mathrm{Ic}}}{k_{\mathrm{IIc}}} = \sqrt{\frac{2\gamma_s}{\gamma_{us}}} = \sqrt{\frac{\sigma_{\max}\Delta_m e\pi}{\tau_{\max} b}} = 2.89 \end{aligned}\right\} \tag{29}$$

Only the parameter r_c keeps unknown. After adjustments, we find that the theoretical toughness curve with parameter $r_c = 6.59\,\mu\mathrm{m}$ gives excellent prediction, as compared with the experiment data of Liechti and Chai[7]. The theoretical results are also shown in Fig.5 with solid line, meanwhile the experiment data are denoted by "$\circ, \triangle, \square, \diamondsuit$".

5 CONCLUSION

A new mechanics model for the interfacial crack is proposed in this paper based on the concept of Peierls dislocation. The new model dispels naturally the unrealistic behaviors such as the oscillatory singularity of the stress fields and the penetration of the crack faces.

Incorporating to the new model, a lenth scale and a generalized stress intensity factor k are introduced which can characterize the intrinsic features of the interface crack.

Using the generalized stress intensity factor k, a fracture criterion for predicting the interface toughness is developed, which takes into account for the shielding effects of the emitted dislocations. The theoretical predictions are in good agreement with the experiment data by Liechti and Chai[7].

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