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ON THE VALIDITY OF THE SLIT ISLANDS ANALYSIS IN THE MEASURE OF FRACTAL DIMENSION OF FRACTURE SURFACES

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Slit islands analysis (SIA) for the measure of the fractal dimension of fracture surfaces introduced by Mandelbrot et al. [1] has been widely used in fractography of fracture surfaces. At the same time, the validity of fractal dimension measured with SIA method has still been in dispute [2-6]. This method is based on the following relation

$$P(\delta)^{1/D} = \alpha(\delta) \quad A(\delta)^{1/2} \tag{1}$$

where P and A are the perimeter and the area of islands which is formed when a specimen plated with nickel is polished parallel to the plane of fracture individually, δ is the yardstick used for measuring, D+1 is the fractal dimension of the fracture surface.

In the case of standard shapes (D=1), the ratio α(δ) between perimeter and area is a dimensionless constant. As an example, for a circle, $\alpha = 2\sqrt{\pi}$. However, for nonstandard or fractal shapes, the ratio α(δ) is obviously related to the yardstick δ, that is

$$\alpha(\delta) = \frac{[P(\delta)]^{1/D}}{[A(\delta)]^{1/2}} \tag{2}$$

According to fractal theory [7], the length of a fractal curve is

$$P(\delta) = N(\delta) \quad \delta = \left(\frac{\delta}{P_0}\right)^{-D} \delta = P_0^D \quad \delta^{1-D} \tag{3}$$

where N(δ) is the number measured with the yardstck δ, P_0 is the characteristic length of the fractal curve. Here P_0 is the length of the initiator. It is worth noting that P(δ) in (3) is the measured perimeter and its dimension is still [length]. In the application of fractal geometry, the physical dimension should remain unchangeable.

Combining (2) and (3), we obtain

$$\alpha(\delta) = \frac{P_0 \delta^{(1-D)\ /D}}{[A(\delta)]^{1/2}} \tag{4}$$

When we use the SIA method to measure the fractal dimension of fracture surfaces, the yardstick δ is a fixed value. Intuitively, the ratio α(δ) seems to relate to the size of islands. In fact, Mandelbrot has shown that the ratio α(δ) is independent of the shapes and sizes of fractals [7].

Suppose that we have a series of Koch islands with different sizes in a section surface. For the *i*th Koch island (see Fig. 1), its size can be determined by its initial lateral length a. Then its perimeter and area are individually

$$P(\delta) = 3(4\delta)^n a \tag{5}$$

$$A(\delta) = \frac{\sqrt{3}}{4}\left\{1 + \frac{3}{5}(1 - (4\delta^2)^n)\right\}a^2 \tag{6}$$

where n is the number of self-similar levels related to the yardstck δ, here, $\delta=(1/3)^m$. The initial perimeter P_0 of the *i*th Koch island is equal to 3a. From (4), we have

$$\alpha(\delta) = \frac{P_0 \delta^{(1-D)\ /D}}{[A(\delta)]^{1/2}} = \frac{3\delta^{(1-D)\ /D}}{\sqrt{\frac{\sqrt{3}}{4}\{1 + \frac{3}{5}(1 - (4\delta^2)^n)\}}} \tag{7}$$

It is obvious that the ratio α(δ) is only dependent on the yardstick δ, and independent of the size of the islands. At the same time, we also see that the ratio α(δ) is dependent on the self-similar level, that is, the scaling range when the yardstick δ is fixed.

Generally speaking, the value of area A(δ) increases much slower than that of its perimeter P(δ) as the yardstick δ decreases. In most cases, the A(δ) almost tends to a constant if small δ is chosen or there is a certain range of scaling. Then

$$A(\delta) \sim \langle d \rangle^2 \tag{8}$$

where <d> is the average diameter of the island. Therefore, the relation (1) can be rewritten as

$$P(\delta)^{1/D} \sim \langle d \rangle \tag{9}$$

Recently, Mu et al. [8] pointed out a similar method, perimeter and maximum-diameter relation, which can well estimate the fractal dimension of fracture surfaces. In order to ensure the validity of (9), a certain scaling range is needed. In our opinion, the scaling range must be over two orders of magnitude.

According to the discussion above, we can see that the statistical self-similarity of the islands in fracture section is a prerequisite condition. However, it is often very difficult to evidence the self-similarity *a priori*. Considering the anisotropy of microstructures in materials and outer loading, it is possible that the fracture surfaces are self-affine [9], rather than self-similar. There is now sample evidence that fracture surfaces possess statistically self-affine fractal properties [10,11]. Therefore, the neglect of this geometrical feature can also result in various disputes about the true value of the fractal dimension D, and even obtain the claims that D can take any of two or more values for the same fractal [12].

In conclusion, the fractal dimension of fracture surfaces can be well estimated by the SIA method if the fracture surface is statistically self-similar and a certain range of length exists. At the same time, an equivalent and easily measured method, perimeter and average-diameter relation, is given.

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REFERENCES

[1] B.B. Mandelbrot, D.E. Passoja and A.J. Paullay, *Nature* 308 (1984) 721-722.

[2] E. Underwood and K. Banerji, *Materials Science and Engineering* 80 (1986) 1-14.

[3] C.S. Pande, L.E. Richards, N. Louat, B.D. Dempsey and A.J.Schwoeble, *Acta Metallurgica* 35 (1987) 1633-1637.

[4] C.W. Lung and Z.Q. Mu, *Physics Review B*, 38 (1988) 11781-11784.

[5] L.V. Meisel, *Journal of Physics D: Applied Physics* 24 (1991) 942-952.

[6] W. Lei and B. Chen, *International Journal of Fracture* 65 (1994) R29-R39.

[7] B.B. Mandelbrot, *The Fractal Geometry of Nature*, Freeman, San Francisco (1982).

[8] Z.Q. Mu, C.W. Lung, Y. Kang and Q.Y. Long, *Physical Review B*, 48 (1993) 7679-7681.

[9] B.B. Mandelbrot, *Physica Scripta* 32 (1985) 257-260.

[10] K.J. Maloy, A. Hansen and E.L. Hinrichsen, *Physical Review Letters* 68 (1992) 213-215.

[11] J.P. Bouchaud, E. Bouchaud, G. Lapasset and J. Planes, *Physical Review Letters* 71 (1993) 2240-2243.

[12] B.B. Mandelbrot, *Fractals* 1 (1993) 117-123.

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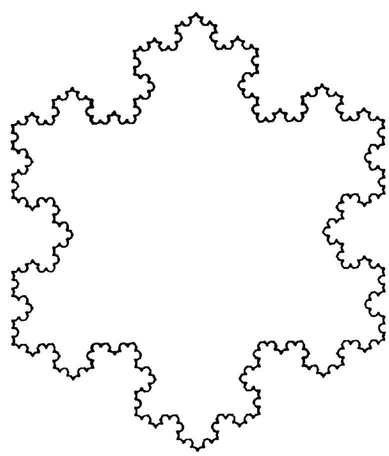

Figure 1. The triadic Koch island.