J. Phys. D: Appl. Phys. 27 (1994) 1781–1785. Printed in the UK

# The positive floating potential of high-temperature particles in plasma

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Received 22 September 1993, in final form 28 March 1994

**Abstract.** It has been predicted that the floating potential of particles in plasma may become positive when the particle surface temperature is high enough, but, to our knowledge, no positive floating potential has been obtained yet. In the present paper the floating potential theory of high-temperature particles in plasma is developed to cover the positive potential range for the first time, and a general approximate analytical formula for the positive floating potential with a thin plasma sheath and subsonic plasma flow is derived from the new model recently proposed by the authors. The results show that when the floating potential is positive, the net flux of charge incident on the particle approaches a constant similar to the 'electron saturation' phenomena in the case of the electric probes.

## 1. Introduction

In recent years, much effort has been concentrated on the study of the interaction between particles and plasma because of the wide applications of plasmas in the preparation of new materials (including superconducting materials, nano materials and high-performance ceramics), enhancement of material surface performance, plasma spraying, synthesis of fine and ultrafine powders, etc [1–3], and considerable progress has been reported on particle charging and heat transfer in plasmas [4–16].

As is well known, when particulate matter is immersed in plasma, a negative potential relative to the surrounding plasma, which is called floating potential, will exist on its surface due to the great difference in thermal velocity between electrons and ions, and particle charging is very important in determining the particle drag and heat transfer [4–16]. Previous works [4–6] studied the heat transfer of a spherical particle in the free molecular flow region with experimental and analytical methods incorporating the effects of electrons and ions in a partly ionized plasma, and the differences in particle charging and heat transfer for metallic and non-metallic spherical particles was firstly discussed by Chen and He [7] under the condition of a thin plasma sheath; the effects of a thick plasma sheath [8] and particle shape [9] were studied by Gnedovets and Uglov. An analytic model for plasma flow around a spherical particle including rarefaction and particle charging effects was given by Chang and Pfender [10] using a two-temperature (i.e. $T_e \neq T_i$) approach. The sheath structure and the electrostatic force on an isolated particle in low-pressure discharges were studied by Daugherty *et al* [11, 12]. Recently, the effect of thermal electron emission on particle charging and heat transfer was noted and considered by Gnedovets and Uglov [13] and Chen and Chen [14], however in their papers the counter effect of particle charing on thermal electron emission was not considered and the original Richardson formula was adopted, thus their analyses were limited to within the range of a negative floating potential.

In fact, the floating potential on the particle surface is generally several volts [5–16] and the corresponding static electric field intensity around the particle can then reach $10^7$–$10^8$ V m$^{-1}$ under the condition of a thin plasma sheath—thus the Schottky effect (effect of the electric field on the thermal electron emission through increasing or decreasing the material's work function) should be important. A new model, including thermal electron emission and the Schottky effect, has been proposed, and some meaningful results about particle charging and heat transfer were obtained [15, 16].

In this paper, the model proposed previously [15] is developed further to cover the positive floating potential as well as the negative potential for high-temperature spherical particles with a thin plasma sheath. The equations for determining the positive floating potential are obtained for the first time, from which a general analytic formula is derived. The results show that when the floating potential is positive, the net charge flux incident on the particle surface is independent of the particle surface temperature and rapidly approaches a constant determined by plasma conditions, as does the electron saturation current collected by the electric probe when a positive bias is applied.

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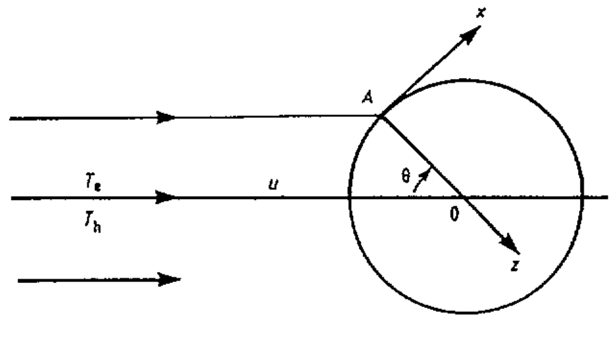


**Figure 1.** The coordinate system and the plasma conditions.

## 2. Physical model

When thermal electron emission is taken into consideration, the charge flux for a given particle–plasma system includes the contribution of two parts. One is the fluxes of electrons and ions incident on the particle surface from the plasma and the other is the flux of thermal electrons emitted from the particle surface to the plasma.

### 2.1. Fluxes of electrons and ions incident on the particle surface

When the particle diameter is much less than the average free path length of the plasma species and much greater than the Debye length (thin plasma sheath), the velocity distribution of electrons and ions incident on the particle surface ($v_z > 0$) can be expressed as [4–10, 13–16]

$$f_j^- = \frac{n_i}{(2\pi k_B T_j/m_j)^{3/2}} \times \exp\left(-\frac{(\boldsymbol{v}-\boldsymbol{u})\cdot(\boldsymbol{v}-\boldsymbol{u})}{2k_B T_j/m_j}\right) \qquad j = i, e \tag{1}$$

where the subscript j denotes electrons (j = e) and ions (j = i), $n$, $T$ and $m$ are the number density, temperature and mass respectively, $k_B$ is the Boltzmann constant, $\boldsymbol{v}$ is the thermal motion velocity of the plasma and $\boldsymbol{u}$ is the relative velocity between the particles and the plasma, as shown in figure 1. The MKS units are adopted in the present paper.

For spherical particles, the local flux of electrons and ions incident on the particle surface is defined by

$$\begin{aligned}\psi_j(\theta) &= \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{A}^{\infty} v_z f_j^- \, dv_z \\ &= \frac{1}{4} n_j \overline{v}_j \left[ \exp - \left(\frac{A - u\cos\theta}{\sqrt{2k_B T_j/m_j}}\right)^2 \right. \\ &\quad \left. + 2s_j \cos\theta \int_{A - u\cos\theta/\sqrt{2k_B T_j/m_j}}^{+\infty} \exp^{-t^2} \, dt \right]\end{aligned} \tag{2}$$

where $\overline{v}_j = \sqrt{8k_B T_j/\pi m_j}$ is the mean thermal motion velocity of electrons and ions, $s_j = u/\sqrt{2k_B T_j/m_j}$ is the speed ratio, for subsonic flow ($s_i \leqslant 1$) $s_e$ can be approximately set the value of zero.

$$A = \begin{cases} 0 & j = i \\ \sqrt{-2e\phi_f/m_e} & j = e \end{cases}$$

for $\phi_f \leqslant 0$ and

$$A = \begin{cases} \sqrt{2e\phi_f/m_i} & j = i \\ 0 & j = e \end{cases}$$

for $\phi_f > 0$, i.e. electrons with velocity less than $\sqrt{-2e\phi_f/m_e}$ for $\phi_f \leqslant 0$ and ions with velocity less than $\sqrt{2e\phi_f/m_i}$ for $\phi_f > 0$ cannot reach the particle surface due to the floating potential barrier. Here $e$ is the elementary charge and $\phi_f$ is the floating potential.

Substituting $A$ into equation (2) gives

for $\phi_f \leqslant 0$

$$\begin{cases} \psi_i = \frac{1}{4} n_i \overline{v}_i \{\exp(-s_i^2 \cos^2\theta) + \square t\pi s_i \\ \quad \times \cos\theta[1 + \mathrm{erf}(s_i \cos\theta)]\} \\ \psi_e = \frac{1}{4} n_e \overline{v}_e \exp(e\phi_f/k_B T_e) \end{cases} \tag{3}$$

for $\phi_f > 0$

$$\begin{cases} \psi_i = \frac{1}{4} n_i \overline{v}_i \{\exp-(c_f - s_i\cos\theta)^2 + \sqrt{\pi} s_i \\ \quad \times \cos\theta[1 + \mathrm{erf}(s_i\cos\theta - c_f)]\} \\ \psi_e = \frac{1}{4} n_e \overline{v}_e \end{cases} \tag{4}$$

where $c_f = \sqrt{e\phi_f/k_B T_i}$. Because the metallic particle is unipotential, the charge fluxes averaged over the whole sphere surface are defined as

$$\overline{\psi}_j = \frac{1}{2}\int_0^{\pi} \psi_j \sin\theta \, d\theta.$$

Then, for $\phi_f \leqslant 0$

$$\begin{cases} \overline{\psi}_i = \frac{1}{8} n_i \overline{v}_i \left\{ \exp(-s_i^2) + \frac{\sqrt{\pi}}{2s_i}(1 + 2s_i^2)\,\mathrm{erf}(s_i)] \right\} \\ \overline{\psi}_e = \frac{1}{4} n_e \overline{v}_e \exp(e\phi_f/k_B T_e) \end{cases} \tag{5}$$

for $\phi_f > 0$

$$\begin{cases} \overline{\psi}_i = \frac{1}{8} n_i \overline{v}_i \left\{ \frac{\sqrt{\pi}}{4s_i}(1 + 2s_i^2 - 2c_f^2) \right. \\ \quad \times [\mathrm{erf}(s_i - c_f) + \mathrm{erf}(s_i + c_f)] + \frac{s_i - c_f}{2s_i} \\ \quad \left. \times \exp[-(s_i + c_f)^2] + \frac{s_i + c_f}{2s_i} \exp[-(s_i - c_f)^2] \right\} \\ \overline{\psi}_e = \frac{1}{4} n_e \overline{v}_e. \end{cases} \tag{6}$$

### 2.2. The thermal electron emission and Schottky effect

When the particle temperature is sufficiently high, the thermal electron emission will greatly influence particle charging and heat transfer from plasma to particles [13–16]. If the Schottky effect is included, the flux of thermal emitting electrons $\psi_{em}$ can be calculated by the Richardson formula

$$\psi_{em} = \frac{A_R T_s^2}{e} \exp\left[-\frac{W_E}{k_B T_s}\right] \tag{7}$$

where $A_R$ is the Richardson constant, $T_s$ is the particle surface temperature, $W_E = W + \Delta W$ is the work function of particle amaterial corrected by the Schottky effect. $\Delta W$ is the decrease ($\phi_f < 0$) or increase ($\phi_f > 0$) of the work function due to Schottky effect [17, 18]. Here $W_E$, $W$ and $\Delta W$ are all expressed in Joule

$$\Delta W = e\left(\frac{eE}{4\pi\varepsilon_0}\right)^{1/2} = \begin{cases} -e^{3/2}/\sqrt{4\pi\varepsilon_0}\sqrt{|E|} & \phi_f \leqslant 0 \\ e^{3/2}/\sqrt{4\pi\varepsilon_0}\sqrt{|E|} & \phi_f > 0 \end{cases} \tag{8}$$

where $\varepsilon_0$ is the permittivity of vacuum, $E = \sqrt{2}\phi_f/\lambda_D$ takes the value of the electric field intensity on the particle surface and the Debye length $\lambda_D = \sqrt{\varepsilon_0 k_B T_e/e^2 n_e}$ with $T_e$ in K and $n_e$ in $m^{-3}$.

### 2.3. Determination of the floating potential

With the initial charging process neglected, the floating potential can be considered as steady and determined by equations (5)–(8) and the following equation

$$\overline{\psi}_e = \overline{\psi}_i + \psi_{em}. \tag{9}$$

## 3. Results and discussions

So far, there are three models to describe particle charging in a plasma under the condition of a thin plasma sheath. Model A, adopted in [5–10], neglects the thermal electron emission; model B, adopted in [13] and [14], considers the thermal electron emission but neglects the Schottky effect; model C, proposed by the authors in [15] and [16] and developed in the present paper, considers both the thermal electron emission and Schottky effect.

For model A, the floating potential of particles is determined only by balancing the fluxes of electrons and ions incident on the particle surface and equation (9) is simplified as $\overline{\psi}_e = \overline{\psi}_i$. Then

$$\phi_f = \frac{k_B T_e}{e}\ln\sqrt{\frac{m_e T_i}{m_i T_e}} + \frac{k_B T_e}{e}\ln \times\left[\frac{1}{2}\exp(-s_i^2) + \frac{\sqrt{\pi}}{4s_i}(1+2s_i^2)\,\mathrm{erf}(s_i)\right]$$

which is always negative and independent of the particle surface temperature.

When the thermal electron emission is taken into consideration, the floating potential should be determined by balancing the fluxes of electrons and ions incident on the particle surface and the thermal electron flux emitting from the particle. It can be predicted that there must be a critical particle surface temperature $T_{crit}$—when $T_s > T_{crit}$ the thermal electron emission is so strong that the floating potential $\phi_f$ should become positive. However, for model B,

$$\psi_{em} = \frac{A_R T_s^2}{e}\exp\left(-\frac{W}{k_B T_s}\right)$$

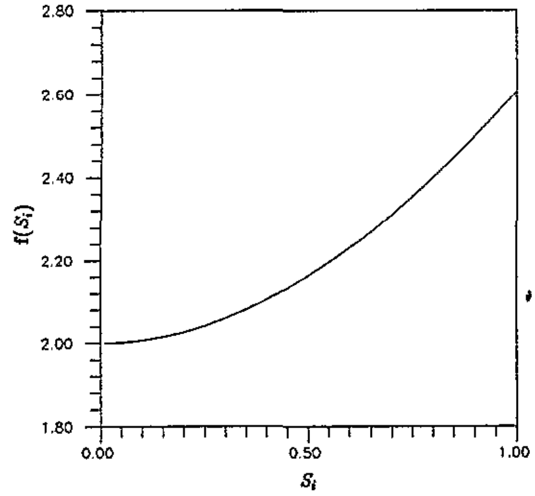


**Figure 2.** Variation of $f(s_i)$ with $s_i$ for subsonic flow.

increases monotonically with $T_s$ and when $T_s > T_{crit}$ it is always greater than $\overline{\psi}_e - \overline{\psi}_i$, i.e. the charge fluxes to and from the particle surface cannot reach a equilibrium, thus no e $\phi_f$ and the increase of $\psi_{em}$ is restricted by the Schottky effect. As a result, it is possible to obtain a proper positive floating potential $\phi_f$ to satisfy equation (9). When $\phi_f > 0$

$$\overline{\psi}_i|_{\phi_f>0} < \overline{\psi}_i|_{\phi_f=0} = \frac{1}{8}n_i\overline{v}_i\left(\exp(-s_i^2) + \frac{\sqrt{\pi}}{2s_i}\mathrm{erf}(s_i)\right) = \frac{1}{8}n_i\overline{v}_i f(s_i) \tag{10}$$

the variation of $f(s_i)$ with $s_i$ from 0 to 1 is shown in figure 2. Obviously, $\overline{\psi}_i|_{\phi_f>0} < \frac{3}{8}n_i\overline{v}_i$ then

$$\left.\frac{\overline{\psi}_i}{\overline{\psi}_e}\right|_{\phi_f>0} < \frac{\frac{3}{8}n_i\overline{v}_i}{\frac{1}{4}n_e\overline{v}_e} = \frac{3}{2}\sqrt{\frac{m_e T_i}{m_i T_e}} \ll 1.$$

Thus, equation (9) can be simplified as $\overline{\psi}_e = \psi_{em}$ for $s_i \leqslant 1$, which leads to

$$\phi_f = \frac{4\pi\varepsilon_0}{e}\frac{\lambda_D}{\sqrt{2}}\left[\frac{k_B T_s}{e}\left(-\frac{W}{k_B T_s} + \ln\frac{4A_R T_s^2}{e n_e \overline{v}_e}\right)\right]^2. \tag{11}$$

This formula is valid for particles with a thin plasma sheath and it is obvious that the positive floating potential is independent of $s_i$ when $s_i \leqslant 1$.

As an example, the floating potential of a spherical tungsten particle in a stationary atmospherical thermal equilibrium Ar plasma ($T_i = T_e = 10^4$ K, $s_i = 0$) is calculated for different surface temperature from 1000 K to 5900 K and shown in figure 3. The floating potential keeps constant for different particle temperature as calculated by model A (curve A). According to model B and model C, the absolute value of $\phi_f$ decreases with increasing $T_s$ when $\phi_f$ is negtive (the full curve B is the result of model B and the broken curve C is that of model C). The critical particle temperature $T_{crit}$ is about 4930 K. When $T_s > T_{crit}$, model B is invalid, while a rapid increase of the positive floating potential

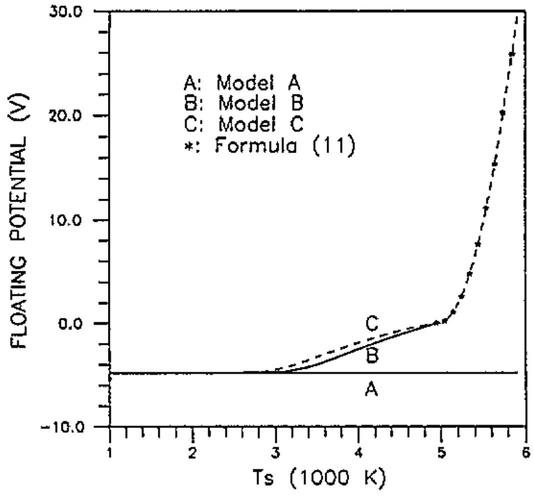


**Figure 3.** The floating potential against particle surface temperature as calculated by different models. A, model A; B, model B; C, model C; $*$, formula (11).

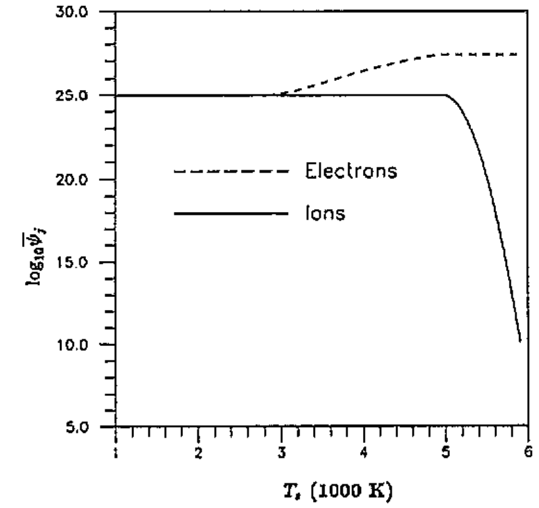


**Figure 4.** Variation of the fluxes of electrons (broken curve) and ions (full curve) with the particle surface temperature.

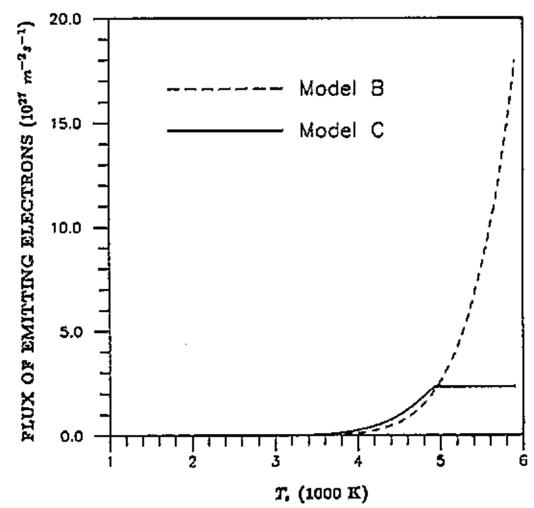


**Figure 5.** The thermal electron flux emitting from the particle as calculated by model B (broken curve) and model C (full curve).

is predicted by model C when $T_s$ increases. On the other hand, the positive floating potential calculated by the approximate formula (11) is also drawn in figure 3 '$*$', which coincides with the results obtained from the accurate equation (9) so well that the difference can hardly be discerned in figure.

The fluxes of electrons and ions incident on the particle surface and the thermal electrons flux emitted from the particle are shown in figures 4 and 5 respectively. It is obvious that when $T_s > T_{crit}$, the flux of electrons incident on the particle surface reaches a constant $\frac{1}{4}n_e\overline{v}_e$. It can also be derived from equations (6) and (10) that the net charge flux incident on the particle surface is approximately a constant $\frac{1}{4}n_e\overline{v}_e$, $(\overline{\psi}_e - \overline{\psi}_i \approx \overline{\psi}_e = \frac{1}{4}n_e\overline{v}_e)$, when the floating potential is positive. This result is asimilar to that obtained in the classic electric probe theory, where the current flowing athrough the probe approaches a constant which is called the electron saturation current and determined by the plasma conditions when a positive bias is applied [19, 20]. Similarly, when $\phi_f$ is positive, the flux of thermoemitting electrons also keeps a constant $\frac{1}{4}n_e\overline{v}_e$ value due to the Schottky effect instead of a rapidly increasing function of $T_s$ as calculated by model B.

## 4. Conclusions

The main conclusions to be drawn from this work are as follows.

(1) When the particle temperature is sufficiently high in the plasma, the floating potential of particles may become positive, and so far the positive floating potential value can be obtained only by the model previously proposed by the authors and developed further in this paper.

(2) A general approximate analytical formula is derived to calculate the positive floating potential with a thin plasma sheath, and the floating potential, when positive, is independent of the relative velocity between particles and plasma for subsonic flow.

(3) When the floating potential is positive, the emitting thermal electron flux is approximately a constant $\frac{1}{4}n_e\overline{v}_e$ instead of a rapidly increasing function of the particle surface temperature as predicted by previous theories, and the net charge flux incident on the particle approaches a constant $\frac{1}{4}n_e\overline{v}_e$ detemined by the plasma condition. This is similar to the 'electron saturation' phenomena in the classic theory of electric probes when a positive bias is applied.

## Acknowledgment

This work has been supported by the National Natural Science Foundation of China.

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