AIRFLOW OVER A MOUNTAIN AND THE CONVECTIVE BOUNDARY LAYER

QI YING* and ZHOU JINGNAN
Institute of Mechanics, Chinese Academy of Sciences, Beijing 100080, China

and

FU BAOPU
Department of Atmospheric Sciences, Nanjing University, Nanjing, China

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Abstract. In this paper, the analytical model coupling the convective boundary layer (CBL) with the free atmosphere developed by Qi and Fu (1992) is improved. And by this improved model, the interaction between airflow over a mountain and the CBL is further discussed. The conclusions demonstrate: (1) The perturbation potential temperatures in the free atmosphere can counteract the effect of orographic thermal forcing through entraining and mixing in the CBL. If $\bar{u}_M > \bar{u}_F$, the feedback of the perturbation potential temperatures in the free atmosphere is more important than orographic thermal forcing, which promotes the effect of interfacial waves. If $\bar{u}_M < \bar{u}_F$, orographic thermal forcing is more important, which makes the interfacial height and the topographic height identical in phase, and the horizontal speeds are a maximum at the top of the mountain. (2) The internal gravity waves propagating vertically in the free atmosphere cause a strong downslope wind to become established above the lee slope in the CBL and result in the hydraulic jump at the top of the CBL. (3) With the CBL deepening, the interfacial gravity waves induced by the potential temperature jump at the top of the CBL cause the airflow in the CBL to be subcritical.

Introduction

There have been extensive studies of boundary-layer flows over microscale terrain in the past twenty years (Taylor and Teunissen, 1987; Carruthers and Hunt, 1990). However, boundary-layer flow over mesoscale terrain greatly differs dynamically from that over microscale terrain. For example, the internal gravity waves induced by mesoscale mountains may propagate vertically in a stably stratified airflow (Gill, 1982), and the effect of thermally-induced horizontal hydrostatic pressure gradients is also apparently prominent (Taylor and Gent, 1980). Thus whether the theory on boundary-layer flows over microscale terrain is suitable for that over mesoscale terrain is still to be discussed. Therefore, many researchers have studied the interaction between airflow over a mountain and the atmospheric boundary layer by mesoscale numerical models with detailed parameterized schemes for the atmospheric boundary layer (Qi and Fu, 1993; Knight, 1992). But numerical

* Present Address: Atmospheric Environment Service, 4905 Dufferin Street, Downsview, Ontario, Canada M3H 5T4.

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simulations often have a poor physical picture in the theoretical analysis, so Qi and Fu (1992) attempted to discover the rule of the interaction between mesoscale terrain flows and the atmospheric boundary layer by developing an analytical model coupling CBL with the free atmosphere. However, in the work of Qi and Fu (1992), the potential-temperature jump at the top of the CBL was ignored and in order to stress the effect of orographic thermal forcing, potential-temperature advection was neglected. In this paper, the analytical model is improved and the potential-temperature jump at the top of the CBL is considered in the modified model. Some valuable results are obtained by this modified model.

2. Modified Coupling Model

2.1. Basic Equations

For simplicity, the model mountain is presumed to be uniform in the y direction. It is assumed that hydrostatic equilibrium is satisfied. Then the basic equations are

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho_0}\frac{\partial P}{\partial x} + fv + \frac{\partial(-\overline{u'w'})}{\partial z} \tag{1}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + w\frac{\partial v}{\partial z} = f(u_g - u) + \frac{\partial(-\overline{v'w'})}{\partial z} \tag{2}$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + w\frac{\partial \theta}{\partial z} = -\frac{\partial\overline{\theta'w';}}{\partial z} \tag{2}$$

$$0 = -\frac{1}{\rho_0}\frac{\partial P}{\partial z} + \frac{\theta}{\theta_0}\cdot g \tag{4}$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0\,, \tag{5}$$

where $\overline{u'w'}$, $\overline{v'w'}$, $\overline{\theta'w'}$ are the vertical turbulent fluxes of momentum and potential temperature in the atmospheric boundary layer, respectively. In this paper, the atmospheric boundary layer is taken to be a well-mixed CBL. The model atmosphere is divided into the CBL and the free atmosphere (as shown in Figure 1). The variables with subscript "M" are those in the CBL and the variables with subscript "F" are those in the free atmosphere.

(1) Perturbation equations in the CBL
In the well-mixed CBL, $\partial u_M/\partial z = \partial v_M/\partial z = \partial\theta_M/\partial z = 0$. Equations (1)–(5) are vertically integrated over the whole CBL:

$$\frac{\mathrm{d}u_M}{\mathrm{d}t} = \left\langle -\frac{1}{\rho_0}\frac{\partial P_m}{\partial x}\right\rangle + fv_M - \frac{(\overline{u'w'})_i - (\overline{u'w'})_0}{d} \tag{6}$$

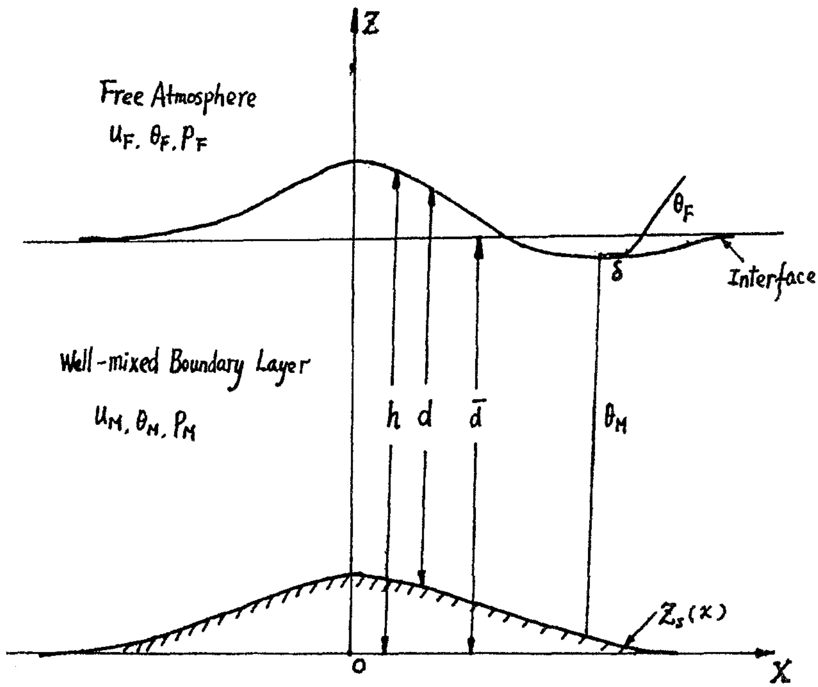

Fig. 1. The model vertical structure.

$$\frac{dv_M}{dt} = f(u_g - u_M) - \frac{(\overline{v'w'})_i - (\overline{v'w'})_0}{d} \tag{7}$$

$$\frac{d\theta_M}{dt} = -\frac{(\overline{\theta'w'})_i - (\overline{\theta'w'})_0}{d} \tag{8}$$

$$\left\langle -\frac{1}{\rho_0}\frac{\partial P_M}{\partial x}\right\rangle = -\frac{1}{\rho_0}\frac{\partial P_M}{\partial x}\bigg|_h + \frac{gd}{2\theta_0}\frac{\partial \theta_M}{\partial x} \tag{9}$$

$$w_M|_h = w_M|_{z_s} - d\,\frac{\partial u_M}{\partial x}, \tag{10}$$

where $d/dt = \partial/\partial t + u_M\,\partial/\partial x$, and $\langle\,\rangle$ indicates the average vertically integrated value in the CBL. The subscript "0", "i" respectively indicate the values at the surface and the top of the CBL.

The variation of the CBL height can be expressed as (Stull, 1988)

$$\frac{dh}{dt} = w_e + w_M|_h \tag{11}$$

where w_e is the speed of entrainment; it must be >0.

$$w_e = -\frac{(\overline{\theta' w'})_i}{\delta},$$

where δ is the potential-temperature jump at the top of the CBL,

$$\delta = \theta_F|_h - \theta_M \tag{12}$$

or

$$\frac{\mathrm{d}\delta}{\mathrm{d}t} = \frac{\partial \theta_F}{\partial z}\frac{\mathrm{d}h}{\mathrm{d}t} + \left.\frac{\mathrm{d}\theta_F}{\mathrm{d}t}\right|_h - \frac{\mathrm{d}\theta_M}{\mathrm{d}t}. \tag{12'}$$

We use Equation (12′) in the present model. This differs from the approach adopted by Nieuwstadt and Glendening (1989) who assumed that the equation

$$\frac{\mathrm{d}\delta}{\mathrm{d}t} = \frac{\partial \theta_F}{\partial z} w_e - \frac{\mathrm{d}\theta_M}{\mathrm{d}t}$$

would apply as in the case of a uniform underlying surface (Tennekes, 1973). We consider this as incorrect since it ignores movements of the CBL top caused by vertical velocities within the CBL.

For simplicity of analysis, each variable is assumed to be composed of two parts: an average quantity over a uniform underlying surface (with superscript "−") and a perturbation quantity induced by the mountain (with superscript "′"), i.e.,

$$\theta_M = \bar{\theta}_M + \gamma z_s + \theta'_M$$

$$u_m = \bar{u}_m + u'_M$$

$$v_M = \bar{v}_M + v'_M$$

$$d = \bar{d} + d'$$

$$h = \bar{d} + z_s + d' \quad (h' = z_s + d')$$

$$\delta = \bar{\delta} + \delta'$$

$$w_e = \bar{w}_e + w'_e,$$

where $\gamma = \partial\bar{\theta}_F/\partial z$. Average quantities satisfy the following equations,

$$\frac{\partial \bar{\theta}_M}{\partial t} = -\frac{(\overline{\theta' w'})_i - (\overline{\theta' w'})_0}{\bar{d}}$$

$$\frac{\partial \bar{d}}{\partial t} = \bar{w}_e$$

$$\bar{\delta} = \bar{\theta}_F|_{\bar{d}} - \bar{\theta}_M$$

$$\bar{w}_e = -\frac{(\overline{\theta' w'})_i}{\bar{\delta}}$$

$$\frac{\partial \bar{u}_M}{\partial t} = -\frac{1}{\rho_0}\frac{\partial \bar{P}_M}{\partial x}\bigg|_{\bar{d}} + f\bar{v}_M - \frac{\overline{(u'w')}_i - \overline{(u'w')}_0}{\bar{d}}$$

$$\frac{\partial \bar{v}_M}{\partial t} = f(u_g - \bar{u}_M) - \frac{\overline{(v'w')}_i - \overline{(v'w')}_0}{\bar{d}} \ .$$

It is assumed that the perturbation quantities are much smaller than their average quantities and that the heating of the mountain surface is distributed uniformly. The Rossby number is presumed to be great enough to neglect Coriolis force. The momentum friction for the perturbation quantities in the CBL is neglected (Nieuwstadt and Glendening, 1989). Then the perturbation equations can be deduced as following,

$$\frac{Du'_M}{Dt} = -\frac{1}{\rho_0}\frac{\partial P'_M}{\partial x}\bigg|_h + \frac{g\gamma\bar{d}}{2\theta_0}\frac{\partial z_s}{\partial x} + \frac{g\bar{d}}{2\theta_0}\frac{\partial \theta'_M}{\partial x} \tag{13}$$

$$\frac{D\theta'_M}{Dt} = -\gamma u_M\frac{\partial z_s}{\partial x} + \frac{\overline{(\theta'w')}_i - \overline{(\theta'w')}_0}{\bar{d}^2}\cdot d' \tag{14}$$

$$\frac{Dd'}{Dt} = -\bar{d}\frac{\partial u'_M}{\partial x} + w'_e \tag{15}$$

$$\delta' = \theta'_F|_h - \theta'_M + \gamma d' \tag{16}$$

$$w'_e = \frac{\overline{(\theta'w')}_i}{\bar{\delta}^2}\cdot\delta' \ , \tag{17}$$

in which $D/Dt = \partial/\partial t + \bar{u}_M\,\partial/\partial x$ and it has been assumed that $w_M|_{z_s} = \bar{u}_M\,\partial z_s/\partial x$.

(2) Perturbation equations in the free atmosphere

The prevailing wind is assumed to be vertically uniform in the free atmosphere. The increase of average potential temperature with height is assumed to be a constant, $\bar{\theta}_F = \theta_0 + \gamma z$. Then the perturbation equations in the free atmosphere are

$$\frac{\partial u'_F}{\partial t} + \bar{u}_F\frac{\partial u'_F}{\partial x} = -\frac{1}{\rho_0}\frac{\partial P'_F}{\partial x} \tag{18}$$

$$\frac{\partial \theta'_F}{\partial t} + \bar{u}_F\frac{\partial \theta'_F}{\partial x} = -\gamma w'_F \tag{19}$$

$$-\frac{1}{\rho_0}\frac{\partial P'_F}{\partial z} + \frac{\theta'_F}{\theta_0}g = 0 \tag{20}$$

$$\frac{\partial u'_F}{\partial x} + \frac{\partial w'_F}{\partial z} = 0\,. \tag{21}$$

(3) Coupling conditions
It is assumed that the kinematic and dynamic continuity conditions are satisfied at the interface between the CBL and the free atmosphere.

$$w'_F|_h = w'_M|_h \tag{22}$$

$$P'_F|_h = P'_M|_h\,, \tag{23}$$

or Equation (23) can be replaced by the following equation;

$$-\frac{1}{\rho_0}\frac{\partial P'_F}{\partial x}\bigg|_h = -\frac{1}{\rho_0}\frac{\partial P'_M}{\partial x} + \frac{g\bar{\delta}}{\theta_0}\frac{\partial h}{\partial x}\,. \tag{23'}$$

Equations (13)–(23) therefore make up a group of basic equations coupling the CBL with the free atmosphere.

2.2. Reduction of the basic equations

It is assumed that the horizontal characteristic scale of the mountain $L \sim 10^4$ m, $\bar{u}_M$, $\bar{u}_F$ are of the same order of magnitude with a geostrophic wind $u_g \sim 10$ m/s. The local variation with time is mainly affected by sunshine, so $\tau = 86400\text{ s} \sim 10^4$ s.

$$O\left(\frac{\partial}{\partial t}\right) = \frac{1}{\tau} \ll O\left(\bar{u}_M\frac{\partial}{\partial x}\right) = \frac{u_g}{L}$$

$$O\left(\frac{\partial}{\partial t}\right) = \frac{1}{\tau} \ll O\left(\bar{u}_F\frac{\partial}{\partial x}\right) = \frac{u_g}{L}\,,$$

so the time tendency terms can be ignored in the perturbation equations. From coupling condition (22) and Equation (11), we deduce

$$w'_F|_h = \frac{Dh'}{Dt} - w'_e = \bar{u}_M\frac{\partial d'}{\partial x} + \bar{u}_M\frac{\partial z_s}{\partial x} - w'_e\,. \tag{24}$$

From Equations (14), (19), (16), (24) and (17), neglecting the time tendency terms and the small quantity $\partial\theta'_F/\partial z\,|_h\, Dh/Dt$, we can deduce

$$\bar{u}_M\frac{\partial \delta'}{\partial x} = \frac{\bar{u}_M\gamma(\overline{\theta' w'})_i}{\bar{u}_F\bar{\delta}^2}\cdot\delta' - \left[\frac{(\overline{\theta' w'})_i - (\overline{\theta' w'})_0}{\bar{d}^2}\cdot d' + \frac{(\bar{u}_M - \bar{u}_F)\bar{u}_M\gamma}{\bar{u}_F}\left(\frac{\partial d'}{\partial x} + \frac{\partial z_s}{\partial x}\right)\right].$$

(I) (II) (III) (25)

In the CBL, $O(\partial\bar{\theta}_M/\partial t) = \gamma\bar{d}/\tau$, $O(\bar{w}_e) = \bar{d}/\tau$, $O(\bar{\delta}) = c/(1+2c)\,\gamma\bar{d}$, where $c = -(\overline{\theta' w'})_i/(\overline{\theta' w'})_0$. The orders of magnitudes of term (I) and term (II) in Equation (25) are

$$O(\mathrm{I}) = O\left(\frac{u_g}{L}\delta'\right)$$

$$O(\mathrm{II}) = O\left(\frac{\bar{w}_e}{\bar{\delta}}\gamma\delta'\right) = O\left(\frac{1+2c}{c}\cdot\frac{\delta'}{\tau}\right).$$

In term (III),

$$O\left(\frac{(\overline{\theta' w'})_i - (\overline{\theta' w'})_0}{\bar{d}^2}d'\right) = O\left(\frac{\partial\bar{\theta}_M}{\partial t}\frac{d'}{\bar{d}}\right) = O\left(\frac{\gamma d'}{\tau}\right) = O(\gamma d' \times 10^{-4})$$

$$O\left(\gamma\left(\frac{\partial d'}{\partial x} + \frac{\partial z_s}{\partial x}\right)\right) = 0\left(\frac{\gamma}{L}d'\right) = O(\gamma d' \times 10^{-4}).$$

Because $O(\bar{u}_F) = O(\bar{u}_M)$,

$$O\left(\frac{(\overline{\theta' w'})_i - (\overline{\theta' w'})_0}{\bar{d}^2}d'\right) \gg O\left(\frac{(\bar{u}_M - \bar{u}_F)\bar{u}_M\gamma}{\bar{u}_F}\left(\frac{\partial d'}{\partial x} + \frac{\partial z_s}{\partial x}\right)\right)$$

$$O(\mathrm{III}) = O\left(\frac{\partial\bar{\theta}_M}{\partial t}\frac{d'}{\bar{d}}\right) = O\left(\frac{\gamma d'}{\tau}\right).$$

In Equation (25), the solution of δ' would be an exponential distribution if term (III) were neglected. This is not reasonable in the physical sense, so term (III) in Equation (25) must be a large term. If $O(\mathrm{III}) \sim O(\mathrm{II})$, then $O(\delta') \sim O(c/(1+2c)\,\gamma d')$; if $O(\mathrm{III}) \sim O(\mathrm{I})$, then $O(\delta') \sim O(L/u_g\tau\,\gamma d')$. Since $c \approx 0.2 \sim 10^{-1}$, $O(L/u_g\tau) = 10^{-1}$, we can always get

$$\delta' \ll \gamma d'$$

$$w_e' = \frac{\bar{w}_e}{\bar{\delta}}\cdot\delta' \sim \frac{1+2c}{\gamma c}\cdot\frac{\delta'}{\tau} \ll \frac{u_g}{L}d'.$$

Accordingly, Equations (15) and (16) may be simplified to

$$\bar{u}_M\frac{\partial d'}{\partial x} = -\bar{d}\frac{\partial u_M'}{\partial x} \tag{26}$$

$$\theta_F'|_h - \theta_M' + \gamma d' = 0. \tag{27}$$

It is inferred from Equations (23′), (27) and (13) that

$$\bar{u}_M\frac{\partial u_M'}{\partial x} = -\Delta_1\frac{1}{\rho_0}\frac{\partial P_F'}{\partial x}\bigg|_h + \Delta_2\frac{g\bar{d}}{2\theta_0}\frac{\partial\theta_F'}{\partial x}\bigg|_h + \Delta_3\frac{g\gamma\bar{d}}{2\theta_0}\frac{\partial h'}{\partial x} - \Delta_4\frac{g\bar{\delta}}{\partial_0}\frac{\partial h'}{\partial x}, \tag{28}$$

where Δ_1, Δ_2, Δ_3, Δ_4 are trace coefficients. The term with Δ_1 reflects the feedback of forced perturbation pressures in the free atmosphere to the CBL, which is substantially the feedback of the internal gravity waves forced in the free atmosphere. The term with Δ_2 reflects the feedback of the perturbation potential temperatures entrained into the CBL from the free atmosphere. The term with Δ_3 reflects the thermal forcing induced by the horizontal gradients at the top of the CBL, which is substantially the role of the thermal forcing source caused by the horizontal difference of mountain heights. The term with Δ_4 reflects the restoring force caused by the potential-temperature jump at the top of the CBL, which is substantially the role of the interface gravity waves.

According to Equations (26), (22) and (8), we can deduce

$$\bar{u}_M \frac{\partial d'}{\partial x} = w'_F|_h - \bar{u}_M \frac{\partial z_s}{\partial x}$$

or

$$w'_F|_h = \bar{u}_M \frac{\partial h'}{\partial x} . \tag{29}$$

From Equations (29) and (19), ignoring the time tendency term, we can get

$$\left.\frac{\partial \theta'_F}{\partial x}\right|_h = -\gamma \frac{\bar{u}_M}{\bar{u}_F} \frac{\partial h'}{\partial x} .$$

It can be shown from this equation that the two terms with Δ_2 and Δ_3 in Equation (28) counteract each other. This means that the perturbation potential temperatures of air entrained into the CBL from the free atmosphere counteract the effect of the orographic thermal forcing. If $\bar{u}_M > \bar{u}_F$, the term with Δ_2 is greater than the term with Δ_3; that implies that the feedback of the perturbation potential temperatures in the free atmosphere caused by entraining and mixing from the CBL is more important than the orographic thermal forcing caused by turbulent transfer. In this case, the sum of the term with Δ_2 and the term with Δ_3 is of the same sign as the term with Δ_4. If $\bar{u}_M < \bar{u}_F$, the term with Δ_3 is greater than the term with Δ_2; that implies that the orographic thermal forcing caused by turbulent transfer is more important. In this case, the sum of the terms with Δ_2 and Δ_3 is of opposite sign to the term with Δ_4. For simplicity, assuming $\bar{u}_F = \bar{u}_M = \bar{u}$, only the effects of the feedbacks of the internal gravity waves in the free atmosphere (term with Δ_1) and the interface gravity waves at the top of the CBL (term with Δ_4) are left in the perturbation wind field in the CBL.

From Equations (18) and (21), we can infer

$$-\frac{1}{\rho_0} \frac{\partial P'_F}{\partial x} = -\bar{u} \frac{\partial w'_F}{\partial z} . \tag{30}$$

From Equations (10), (22), (30) and (28), we can deduce

$$(1 - \Delta_4 \mathrm{Fr}^{-2}) w'_F|_h = \Delta_1 \bar{d} \left.\frac{\partial w'_F}{\partial z}\right|_h + \bar{u}\frac{\partial z_s}{\partial x}, \tag{31}$$

in which $\mathrm{Fr}^2 = \theta_0 \bar{u}^2 / g\bar{\delta}\,\bar{d}$.

From Equations (18)–(21), we can also deduce

$$\frac{\partial^2 w'_F}{\partial z^2} + l^2 w'_F = 0, \tag{32}$$

in which $l^2 = g\gamma/\theta_0 \bar{u}^2$.

It is assumed that the upper boundary of the free atmosphere is a radiation boundary. Equations (31)–(32) together with this condition make it possible to obtain definite solutions to w'_F in the linear system coupling the CBL with the free atmosphere.

3. Solutions over an Isolated Mountain

In this section, we shall solve the problem in the linear coupled system over a typical isolated mountain. The topographic function is taken to be

$$z_s(x) = \frac{ab^2}{b^2 + x^2}$$

where $O(a) = 10^2$ m, $O(b) = 10^4$ m. We have $O(a/H_F) = 10^{-1}$, where $H_F = ((g/\theta_0)(\partial\bar{\theta}_e/\partial z)/\bar{u}_F^2)^{1/2} \sim 10^3$ m is the vertical characteristic scale of the free atmosphere; so the nonlinear effect of the interface between the CBL and free atmosphere can be neglected. This means that the lower boundary condition (31) can be written as

$$(1 - \Delta_4 \mathrm{Fr}^{-2}) w'_F|_{\bar{d}} = \Delta_1 \bar{d} \left.\frac{\partial w'_F}{\partial z}\right|_{\bar{d}} + \bar{u}\frac{\partial z_s}{\partial x}. \tag{33}$$

Equations (32) and (33) are Fourier transformed for x:

$$\frac{\partial^2 \hat{w}'_F}{\partial z^2} + l^2 \hat{w}'_F = 0 \tag{34}$$

$$(1 - \Delta_4 \mathrm{Fr}^{-2}) \hat{w}'_F|_{\bar{d}} = \Delta_1 \bar{d} \left.\frac{\partial \hat{w}'_F}{\partial z}\right|_{\bar{d}} + \bar{u}_i k \hat{z}_s. \tag{35}$$

The general solution to Equation (34) is

$$\hat{w}'_F = \chi_1 e^{ilz} + \chi_2 e^{-ilz}. \tag{36}$$

The upper boundary is presumed to be a radiation boundary, so $\chi_2 = 0$ (Durran, 1986). Bringing (36) into (35), we get

$$\chi_1 = \frac{\bar{u}ik\hat{z}_s e^{il\bar{d}}}{(1 - \Delta_4 \mathrm{Fr}^{-2}) - \Delta_1 il\bar{d}} \; .$$

Therefore

$$\hat{w}'_F = \frac{\bar{u}(-l\bar{d}\,k\Delta_1 + ik(1 - Fr^{-2}\Delta_4)]}{(1 - \Delta_4 \mathrm{Fr}^{-2})^2 + (\Delta_1 l\bar{d})^2}\; \hat{z}_s e^{i(z-\bar{d})} \, . \tag{37}$$

(37) is Fourier convert-transformed,

$$\begin{aligned} w'_F &= \bar{u}\mathrm{Re}\left(\int_0^\infty \frac{-\Delta_1 l\bar{d}\,k + ik(1 - Fr^{-2}\Delta_4)}{(1 - \Delta_4 \mathrm{Fr}^{-2})^2 + (\Delta_1 l\bar{d})^2}\; abe^{-bk+i(lz - l\bar{d} + kx)}\,\mathrm{d}k \right) \\ &= \frac{\bar{u}}{(1 - \Delta_4 \mathrm{Fr}^{-2})^2 + (\Delta_1 l\bar{d})^2} \left[\frac{(x^2 - b^2)ab}{(b^2 + x^2)^2} \sin(lz - l\bar{d} + \chi) - \right. \\ &\quad \left. - \frac{2ab^2 x}{(b^2 + x^2)^2} \cos(lz - l\bar{d} + \psi) \right], \end{aligned} \tag{38}$$

where $\psi = \Delta_1 l\bar{d}/(1 - \Delta_4 \mathrm{Fr}^{-2})$.

From (38), we can also obtain the horizontal perturbation speeds in the free atmosphere and the CBL and the interface height between the CBL and free atmosphere:

$$\begin{aligned} u'_F &= \frac{\bar{u}}{(1 - \Delta_4 \mathrm{Fr}^{-2})^2 + (\Delta_1 l\bar{d})^2} \left[\frac{abxl}{b^2 + x^2} \cos(lz - l\bar{d} + \psi) + \right. \\ &\quad \left. + \frac{ab^2 l}{b^2 + x^2} \sin(lz - l\bar{d} + \psi) \right] \end{aligned} \tag{39}$$

$$\begin{aligned} u'_M &= \frac{\bar{u}ab^2}{\bar{d}(b^2 + x^2)} + \frac{\bar{u}}{\bar{d}[(1 - \Delta_4 \mathrm{Fr}^{-2})^2 + (\Delta_1 l\bar{d})^2]} \left(\frac{abx}{b^2 + x^2} \sin\psi - \right. \\ &\quad \left. - \frac{ab^2}{b^2 + x^2} \cos\psi \right) \end{aligned} \tag{40}$$

$$h' = \frac{1}{(1 - \Delta_4 \mathrm{Fr}^{-2})^2 + (\Delta_1 l\bar{d})^2} \left(\frac{ab^2}{b^2 + x^2} \cos\psi - \frac{abx}{b^2 + x^2} \sin\psi \right). \tag{41}$$

Equations (38)–(41) are the solutions over an isolated mountain.

4. Interaction between the CBL and Airflow over a Mountain

In order to discuss the physical picture of the interaction between the CBL and airflow over a mountain, we take $\bar{u} = 7$ m/s, $\gamma = 0.0033$ k/m, $a = 10^2$ m, $b = 10^4$ m,

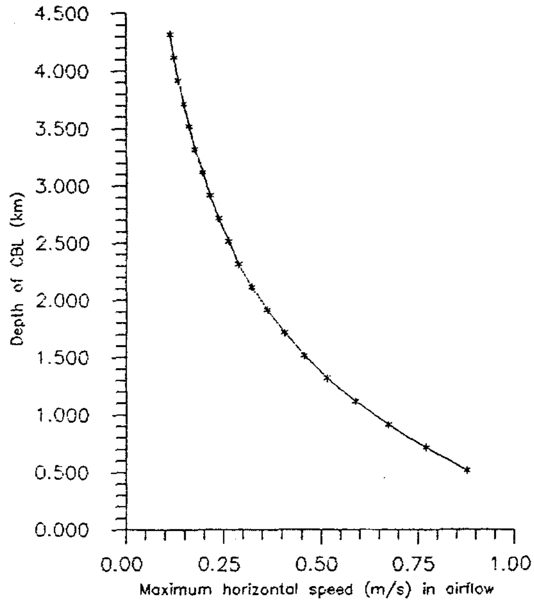

Fig. 2. The variation of maximum horizontal perturbation speeds with CBL depth.

$g = 9.8\,\mathrm{m\,s^{-2}}$, $\theta_0 = 289\,\mathrm{k}$, $\bar{\delta} = c/(1+2c)\,\gamma\bar{d}$, $c = 0.2$. Figure 2 demonstrates the maximum perturbation speed's variation with CBL depth in the whole perturbation wind field. This figure shows that the perturbation speeds in the airflow over a mountain decrease with CBL deepening. This shows that the existence of the CBL restrains the effect of the orographic perturbation in the airflow over the mountain. This result is similar to that in the case of inviscid flow with prescribed temperature structure (Gill, 1982), so neutural stratification caused by turbulent mixing in the CBL is a very important factor. Queney (1948) discussed inviscid fluid of uniform N and $\bar{u}$ over a similar isolated mountain and showed that the horizontal perturbation speeds at the lowest level of the fluid are $u(x, 0) = abx/(b^2 + x^2)\,\bar{u}l$ when the mountain is wide enough to satisfy hydrostatic equilibrium but when Coriolis force can be ignored, that is, there is a maximum perturbation speed at $x = b$ on the lee slope. In the present model, it can be also found from Equation (39) that if $\Delta_4 = 0$, i.e., neglecting the feedback of the interfacial waves, and in the limit as $\bar{d}$ approaches 0, the location of maximum speed is $x = b$, which is the same as Queney (1948)'s results. However, when the lowest part of the inviscid fluid is considered to be the CBL, the solid line in Figure 3 shows that the maximum perturbation speed at the lowest level of the free atmosphere gradually moves upstream from the lee slope with the CBL deepening. Therefore, the existence of a CBL not only suppresses the effect of orographic forcing but changes the phases of the internal gravity waves propagating vertically in the free atmosphere. In the CBL, Figure 4 shows that a strong downslope wind occurs above the lee slope

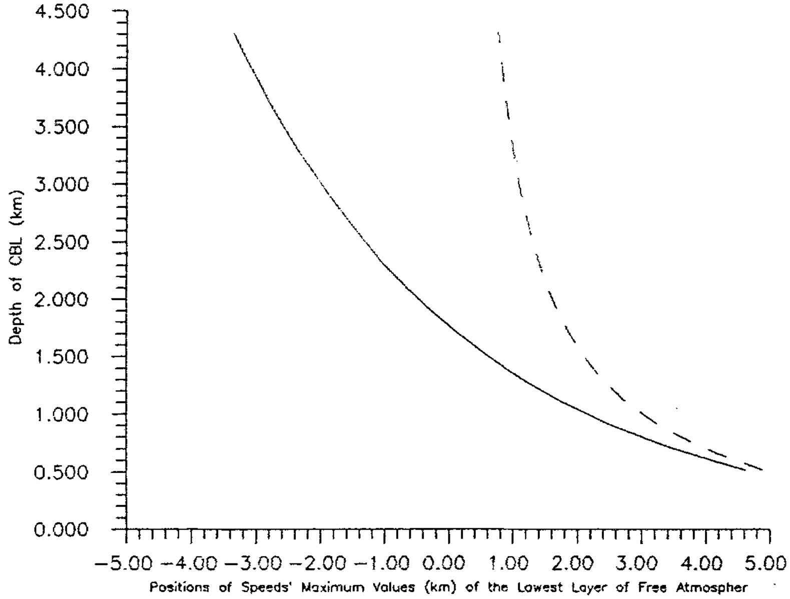

Fig. 3. The variation in position of maximum horizontal perturbation speeds at the lowest level of the free atmosphere with CBL depth. -----, the case of only internal gravity wave feedback. ——, the case of both internal gravity waves and interfacial waves' feedbacks.

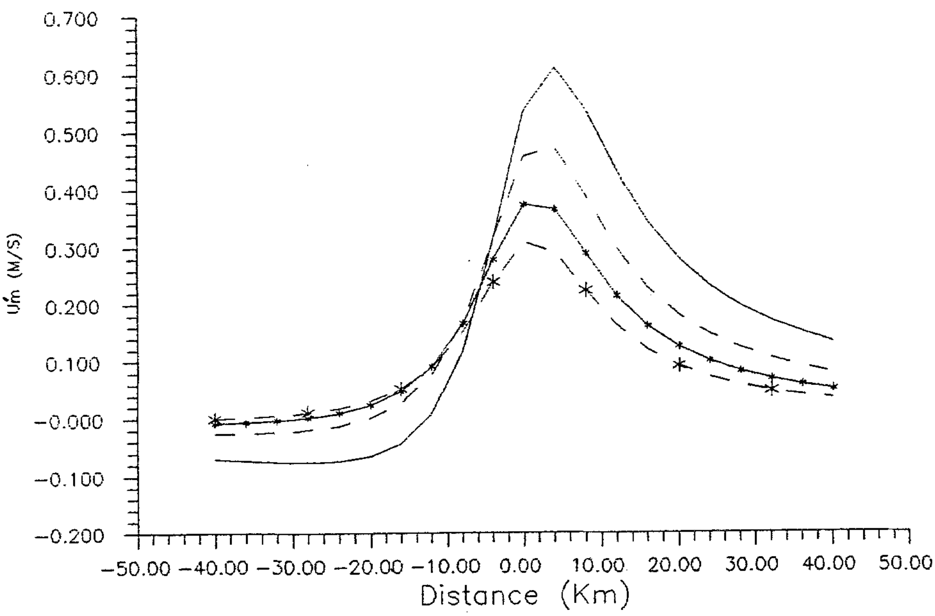

Fig. 4. Variation of horizontal perturbation speeds in the CBL with horizontal distance (x) under the condition of extensive feedbacks of internal gravity waves and interfacial waves. ——, $\bar{d}$ = 1.0 km; -*-*-*-*-, $\bar{d}$ = 2.0 km; -----, $\bar{d}$ = 1.5 km; -*---*-, $\bar{d}$ = 2.5 km.

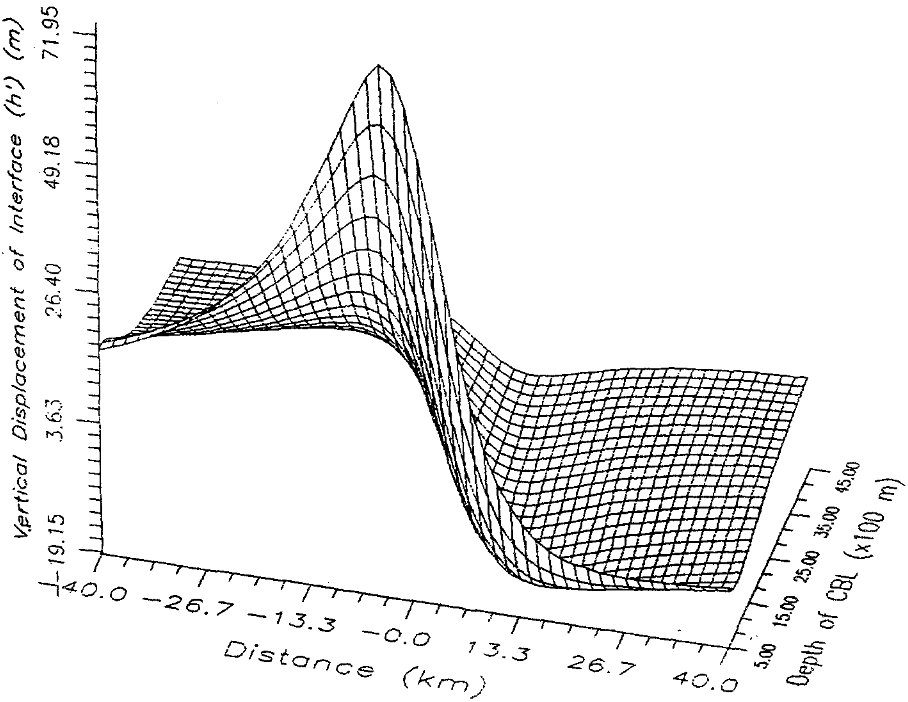

Fig. 5. Variation of the displacement of the interface between the CBL and the free atmosphere with CBL depth.

because of orographic forcing, and the downslope wind gradually moves towards the top of the mountain with the CBL deepening. The interface between the CBL and the free atmosphere varies correspondingly; as shown in Figure 5, the hydraulic jump of the interface gradually weakens with CBL deepening, finally changing into a subcritical flow.

According to the discussion above, internal gravity waves in the free atmosphere and the interfacial waves at the top of the CBL are the two most important characteristics which reflect the interaction between the airflow over a mountain and the CBL. In order to further discuss the rule of the interaction, we shall analyse the actions of the two characteristic factors.

(1) *Action of interfacial waves*

In Equation (31), we take $\Delta_1 = 0$, $\Delta_4 = 1$ and then amalgamate Equations (29) and (31):

$$(1 - Fr^{-2})\frac{\partial h'}{\partial x} = \frac{\partial z_s}{\partial x}. \tag{42}$$

This formula is similar to Durran (1990)'s formula (4.33). It can be seen from (42) that if Fr > 1, the airflow in the CBL is supercritical; if Fr < 1, the airflow in the

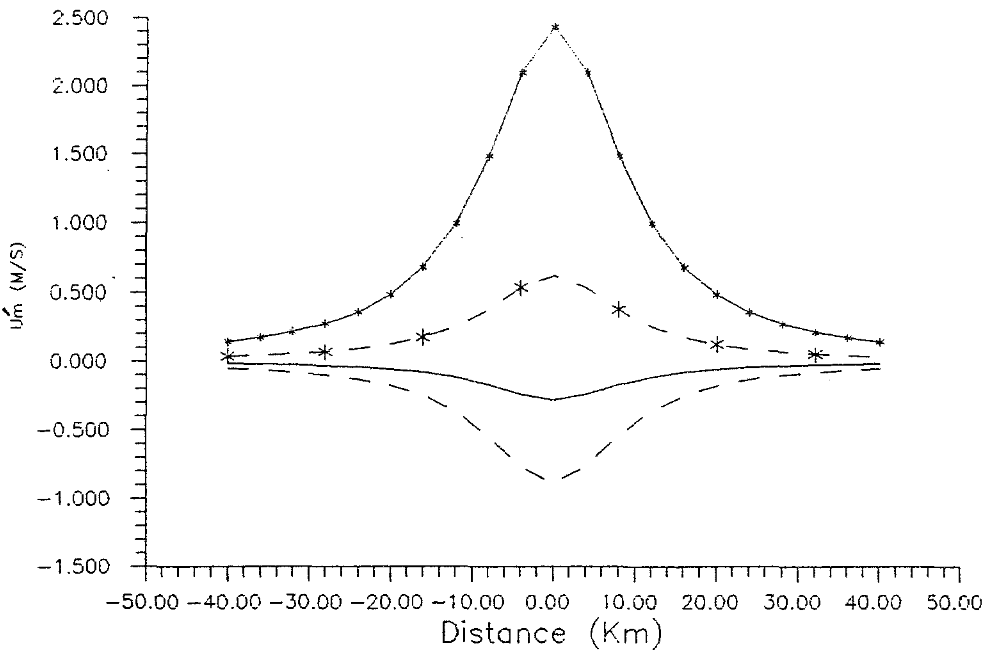

Fig. 6. The same as in Figure 4, except for the case of only interfacial gravity wave feedback.

CBL is a subcritical flow and $\bar{d}_c = \theta_0 \bar{u}^2 / g\bar{\delta}$ or $\bar{d}_c = [\theta_0 \bar{u}^2 (1 + 2c)/g\gamma c]^{1/2}$ is a critical depth of the CBL. Therefore, in the case of a small orographic perturbation (that is, a linear approximation can be presumed), the interfacial waves make the CBL either a supercritical or a subcritical flow. And the interfacial waves can not cause a hydraulic jump in the CBL unless nonlinear advection is included and the depth of the CBL is near the critical depth according to Long (1954)'s conclusions.

In Figure 6, when $\bar{d} < \bar{d}_c$, the perturbation speed at the top of the mountain is a minimum, that is, the airflow is a supercritical flow; when $\bar{d} > \bar{d}_c$, perturbation speed at the top of the mountain is a maximum, that is, the airflow is subcritical. In the free atmosphere (as shown in Figure 7), the phases of the internal gravity waves are identical with Queney (1948)'s result when the airflow in the CBL is supercritical ($\bar{d} < \bar{d}_c$); conversely, when the airflow in the CBL is subcritical ($\bar{d} > \bar{d}_c$), the phases of the internal gravity waves are opposite to Queney (1948)'s result.

(2) ACTION OF INTERNAL GRAVITY WAVES

In Equation (31), we take $\Delta_1 = 1$ and $\Delta_4 = 0$. Figure 8 is the distribution of the perturbation speeds in the CBL, induced only by the feedback of the internal gravity waves. It can be seen from Figure 8 that the feedback of the internal gravity waves always makes a very strong downslope wind appear above the lee slope and the maximum speed of the downslope wind gradually moves towards the top of the mountain with the CBL deepening. This distribution is quite similar

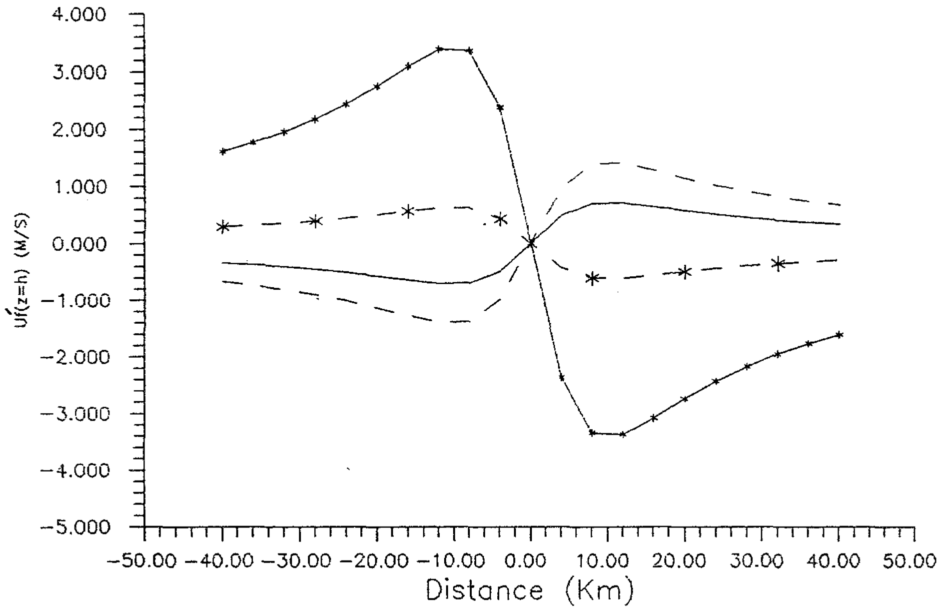

Fig. 7. The same as in Figure 6, except for the variation of horizontal perturbation speed at the lowest level of the free atmosphere with horizontal distance (x).

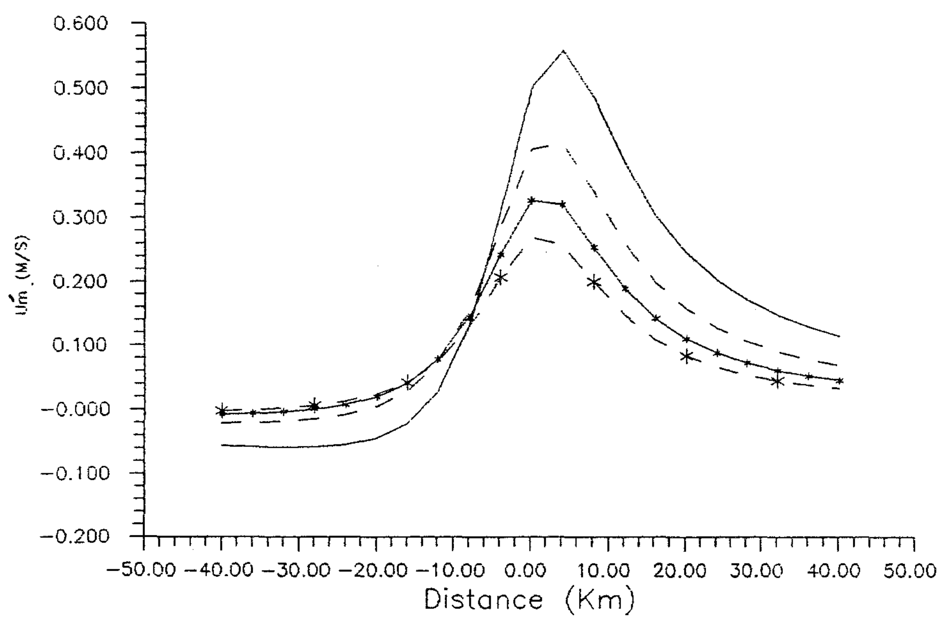

Fig. 8. The same as in Figure 4, except for the case of only internal gravity wave feedback.

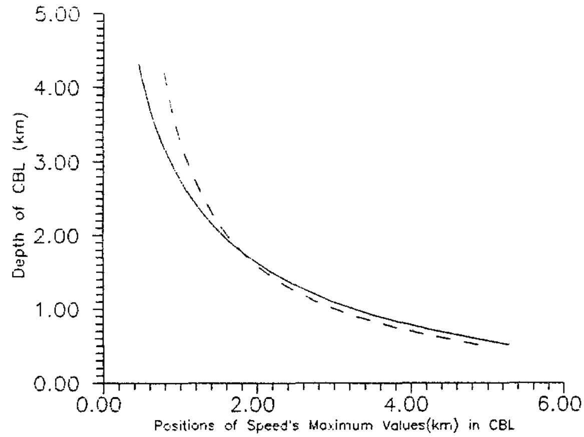

Fig. 9. The same as in Figure 3, except for the variation in position of the maximum horizontal perturbation speed in the CBL with CBL depth.

to that in Figure 4. However, by contrasting the solid line with the dashed line in Figure 9, that with CBL deepening, the maximum speed of the downslope wind moves toward the mountain top under the role of only internal gravity waves' feedback slower than under the roles of both the internal gravity waves and interfacial waves' feedbacks. In the free atmosphere, the rules of the maximum speeds' moving at the lowest level of the free atmosphere towards the upstream are also different in the two cases of only the internal gravity waves' feedback and both the internal gravity waves and interfacial waves' feedbacks. It can be seen from contrasting the solid line with the dash line in Figure 3 that under the role of only feedback for the internal gravity waves; the phases of the internal gravity waves finally become different from Queney (1948)'s result by $\pi/2$, while under the role of both internal gravity waves and feedback of interfacial waves, the phases of the internal gravity waves finally become opposite to Queney (1948)'s result.

To sum up, the strong downslope wind above the lee slope in the CBL and the hydraulic jump at the top of the CBL are mainly caused by the feedback of internal gravity waves in the free atmosphere; with CBL deepening, the airflow in the CBL finally evolves into subcritical flow and the phases of the internal gravity waves in the free atmosphere finally become opposite to Queney (1948)'s result.

Finally, we must point out that since the terms with Δ_2 and with Δ_3 differ only in their coefficients from the term with Δ_4 in Equation (28), only the case of $\bar{u}_M = \bar{u}_F$ is discussed. In this case, the effects of the internal gravity waves and interfacial waves are similar to those when turbulent entrainment and mixing are neglected except that due to the development of mixing and entrainment in the CBL, there exists a relationship between $\bar{\delta}$ and $\bar{d}$. This relationship demonstrates that the effect of the interfacial waves at the top of the CBL becomes more and more obvious with CBL deepening. In addition, the roles of turbulent entrainment mixing can be further analysed by the present model.

In Equation (28), assuming $\Delta_1 = 0$, from Equations (10), (19), (22), (28) and (29), we can get

$$\left[1 - \Delta_4 \frac{g\bar{\delta}\bar{d}}{\theta_0 \bar{u}_M^2} + \left(\Delta_3 - \Delta_2 \frac{\bar{u}_M}{\bar{u}_F}\right) \frac{g\gamma\bar{d}^2}{2\theta_0 \bar{u}_M^2}\right] \frac{\partial h'}{\partial x} = \frac{\partial z_s}{\partial x}.$$

When $\bar{u}_M/\bar{u}_F > 1$, the effect of the term with Δ_2 is more important than that of the term with Δ_3; and the sum of the terms with Δ_2 and Δ_3 is of the same sign as the term with Δ_4. Therefore, the feedback of the perturbation potential temperatures in the free atmosphere caused by turbulent mixing and entrainment promotes the development of interfacial waves; with CBL deepening, it can rapidly make the flow field in the CBL evolve into subcritical flow. When $\bar{u}_M/\bar{u}_F < 1$, the term with Δ_3 is more important; and the sum of the terms with Δ_2 and Δ_3, is of opposite sign to the term with Δ_4. And if $(\bar{u}_F - \bar{u}_M)/2\bar{u}_F > \bar{\delta}/\gamma\bar{d} = c/(1 + 2c)$, the orographic thermal forcing caused by the turbulent heat transfer restrains the effect of the interfacial waves and makes the interface at the top of the CBL become of the same phase as the shape of the mountain and the maximum horizontal speed appears over the top of the mountain.

5. Conclusions

In this paper, Qi and Fu (1992)'s linear analytical model coupling the CBL with the free atmosphere is improved. By this modified model, it is further proved that the CBL suppresses the effect of orographic dynamical forcing on the airflow over a mountain and it is also demonstrated that the CBL can change the phases of the internal gravity waves propagating vertically in the free atmosphere. In addition, some new results are obtained as follows:

(1) Perturbation potential temperatures in the free atmosphere can counteract the effect of orographic thermal forcing with the help of turbulent entrainment and mixing of the CBL.

(2) Under the condition of a linear approximation, the interfacial gravity waves induced by the potential-temperature jump at the top of the CBL make the CBL either a supercritical flow ($\bar{d} < \bar{d}_c$) or a subcritical flow ($\bar{d} > \bar{d}_c$). Only when the CBL' s depth is near the critical depth $\bar{d}_c$ and nonlinear advection is considered, do the interfacial waves lead to the occurrence of a hydraulic jump in the CBL. With the CBL deepening, the interfacial gravity waves cause the airflow in the CBL to evolve into a subcritical flow.

(3) The internal gravity waves propagating vertically in the free atmosphere, by the perturbation pressures, exert a feedback influence on the wind field in the CBL. This effect causes a strong downslope wind to form above the lee slope in the CBL and results in the hydraulic jump at the top of the CBL.

(4) If $\bar{u}_M > \bar{u}_F$, feedback of the perturbation potential-temperatures in the free atmosphere is more important than orographic thermal forcing, which promotes the effect of interfacial waves. If $\bar{u}_M < \bar{u}_F$, orographic thermal forcing is more important, which makes the interfacial and the topographic heights identical in phase and the horizontal speeds become a maximum at the top of the mountain.

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