

# ASYMMETRIC STRUCTURE OF AN ISOLATED FLUX ARCH

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(Received 27 July, 1993)

**Abstract.** This paper extends two-dimensional model of symmetric magnetostatic flux arches confined in stratified atmospheres (Zhang and Hu, 1992, 1993) to asymmetric models. Numerical results show that the flux structure is influenced greatly by the boundary condition of magnetic field, the force-free factor, the atmospheric pressure distribution and the position of footpoints (especially the width ratio of outlet to entrance, which differs from symmetric case).

## 1. Introduction

High resolution observations using a magnetic field telescope discovered fine structures in the solar photosphere magnetic field. It is believed that the fundamental magnetic field structure in the stellar atmosphere is the isolated flux tube confined in the background atmosphere. Furthermore, space observations in EUV show the loop structure in the chromosphere and corona.

Therefore, to construct the flux tube model based on the boundary condition at photosphere, it is necessary for understanding magnetic processes in astrophysics. Two sorts of models, the returned isolated flux tube and the unreturned isolated flux tube in the atmosphere can be studied. A consistent model of the returned isolated flux tube has been developed by perturbation theory (see for example, Hu, 1987, 1989; Parker, 1979; Zhang and Hu, 1992), and the results gave quantitatively the influence of the atmosphere on the structure of magnetic field. However, the weak non-linear approximation has strong limitations when compared to the real cases, and a numerical simulation of finite element method was developed for the complete non-linear problem of the symmetric isolated flux tube (Zhang and Hu, 1993). In previous papers, the isolated flux tubes were assumed axisymmetric, that is the configuration of magnetic flux tube is symmetric to a symmetric plane which is perpendicular to a direction of solar radius. It seems that the symmetric condition should be improved, and we will discuss more general cases of the non-symmetric model in the present paper.

We present the physical model in the following section at first. Section 3 will give numerical simulations which place emphasis on asymmetric property of the flux arch. The last section gives a discussion and conclusions.

## 2. Physical Model

We discuss a two-dimensional magnetic arch as shown in Figure 1, where the  $z$ -axis is adopted outward and assume  $\partial/\partial y = 0$ . The inside field is assumed to

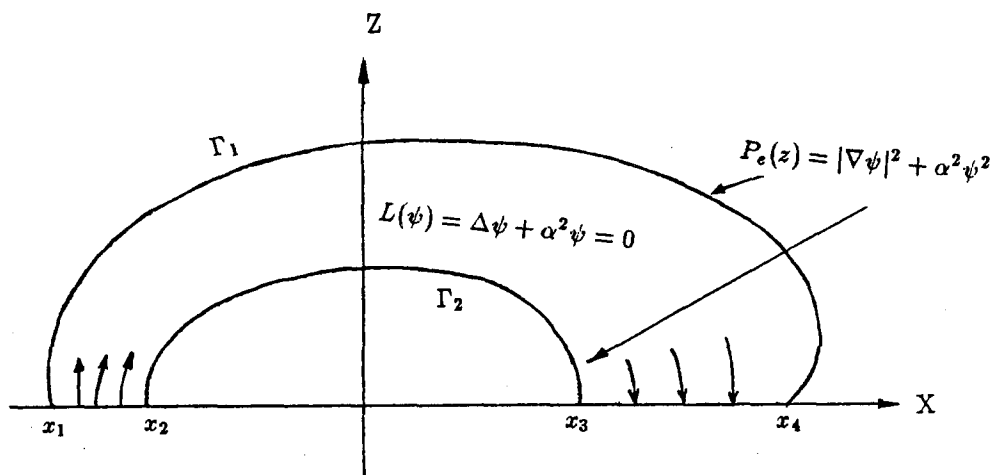


Fig. 1. Model of magnetic flux arch.

be force-free. The magnetic flux arch is confined by the solar atmosphere, and the total pressures at both sides of the free boundaries  $\Gamma_1$  or  $\Gamma_2$  balance. The detailed equations and boundary condition are given elsewhere (Zhang and Hu, 1992, 1993). The asymmetric model permits  $x_3 \neq -x_2$ ,  $x_4 \neq -x_1$  and  $|x_4 - x_3| \neq |x_1 - x_2|$ .

Magnetic flux is conserved, which requires

$$\int_{x_1}^{x_2} \psi_0 dx = \int_{x_3}^{x_4} \psi_0 dx \quad (1)$$

where  $\psi_0(x) = \psi(x, z_0)$  is the boundary value of magnetic stream function at the photosphere boundary  $z = z_0$ .

The asymmetric problem of an isolated flux arch is a non-linear problem with free boundaries, where the configuration of boundaries  $\Gamma_1$  and  $\Gamma_2$  are to be determined together with the internal solution for given outside pressure distribution. The strong non-linear property comes from the boundary conditions at determined boundaries  $\Gamma_1$  and  $\Gamma_2$ , they are given as

$$|\Delta\psi|^2 + \alpha^2\psi^2 = P_e(z), \quad (\mathbf{r}(x, y, z) \in \Gamma_i, i = 1, 2) \quad (2)$$

where the force-free factor is

$$\alpha = B_y/\psi. \quad (3)$$

The constant force-free factor  $\alpha$  in isolated models is assumed for simplification. Here, we use the model of a stratified atmosphere, i.e. the pressure distribution

outside the flux tube is not influenced by the magnetic field inside the flux tube, that is

$$P_e(z) = P_e(z_0) \exp \left( - \int_{z_0}^z \frac{g dz}{RT_e} \right), \quad (4)$$

where  $RT_e/g = H$  is defined as the atmospheric scale height.

### 3. Numerical Simulation

The finite element method is used here for numerical simulation. The arch is divided into 160 iso-parameter elements with 189 nodes in the present model. The flux arch emerges from one side and returns into the other side of the convective region, in ranges  $x_1 \leq x \leq x_2$  and  $x_3 \leq x \leq x_4$  respectively. To study the asymmetric features of the asymmetric arch and its magnetic field, the quantities are made non-dimensional and we concentrate our attention on studying sensitive parameters which influence the isolated arch.

The emergence condition on the  $x_3 - x_4$  outlet is determined by the influx condition on the  $x_1 - x_2$  entrance plane, the total pressure conservation (at points  $x_3, x_4$ ) and the flux conservation condition. The entrance condition is given according to the total pressure condition at  $x_1$  and  $x_2$ . In addition, points  $x_1$  and  $x_4$  should be on the same magnetic field line. So do the points  $x_2$  and  $x_3$ . Several typical photospheric boundary conditions are shown in Figure 2.

#### 3.1. ATMOSPHERIC SCALE HEIGHT $H$

Here we consider an isothermal model  $T_e = \text{constant}$  and therefore have a constant atmospheric scale height  $H = RT_e/g$  in Equation (4). Figures 3a, 3b and 3c show the cases of scale height  $H = 1, 0.8$  and  $0.6$  respectively for fixed footpoint positions  $x_1 = -1, x_2 = -0.8, x_3 = 0.8, x_4 = 1.1$  and the force-free factor  $\alpha = 1$ . The emerging field is given in Figures 2a and 2b. The results show clearly that the arch expands more with a smaller scale height  $H$  due to less confining pressure with the maximum position  $(X, Z) = (0.025, 0.643), (0.021, 0.74)$  and  $(0.016, 1.003)$ , respectively, where  $X|_{\text{top}} = 0$  in the symmetric case. The asymmetric configuration displayed here is due to the fact that, for a lower  $H$ , the outside pressure will decrease, and the magnetic pressure to confine the arch should also be cut down to keep balance. Thus we have a weaker magnetic field  $B^2$  on the boundary  $\Gamma_1$  and  $\Gamma_2$ , the top point should move toward left and the arch will expand. Its magnetic energy change will be discussed later in Section 3.3.

#### 3.2. INFLUENCE OF RELATIVE WIDTH OF FOOTPOINTS

Here we define the relative width ratio as emergence width to entrance width

$$\Delta = \left| \frac{x_4 - x_3}{x_2 - x_1} \right|. \quad (5)$$

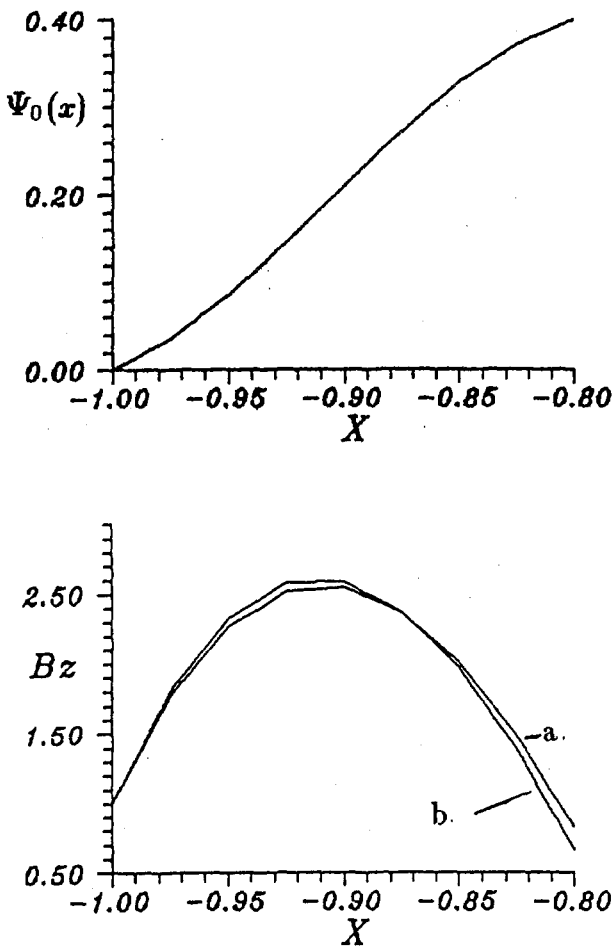


Fig. 2. Different photosphere boundary conditions; with  $B_x(x, z_0) = \psi_0(x)$ . (2a - top; 2b - bottom).

For the fixed position  $x_1 = -1.0$ ,  $x_2 = -0.8$  and  $x_4 = 1.0$ , we change the position of  $x_3$  to be 0.84, 0.8 and 0.734 associated with  $\Delta = 0.8, 1.0$ , and 1.33 respectively. Here, we fix  $H = 0.8$  and  $\alpha^2 = 1$ . The related entrance conditions are shown in Figures 2a and 2b. The results shown in Figures 4a, 4b and 4c that as  $x_3$  decreases, i.e.  $\Delta$  increases for fixed  $|x_1 - x_2|$ , the arch will go down with the position of top point  $(X, Z) = (0.007, 0.76)$ ,  $(0.00, 0.742)$  and  $(-0.01, 0.706)$  respectively, with a little leftshift from the symmetric case. Here, the solar magnetic flux or the internal magnetic pressure will increase for the increasing width  $|x_4 - x_3|$ , henceforth the external pressure will have more confinement on the arch. Furthermore, we can also change the position of  $x_1$ ,  $x_2$  or  $x_4$ . Thus the relative position, i.e. the width

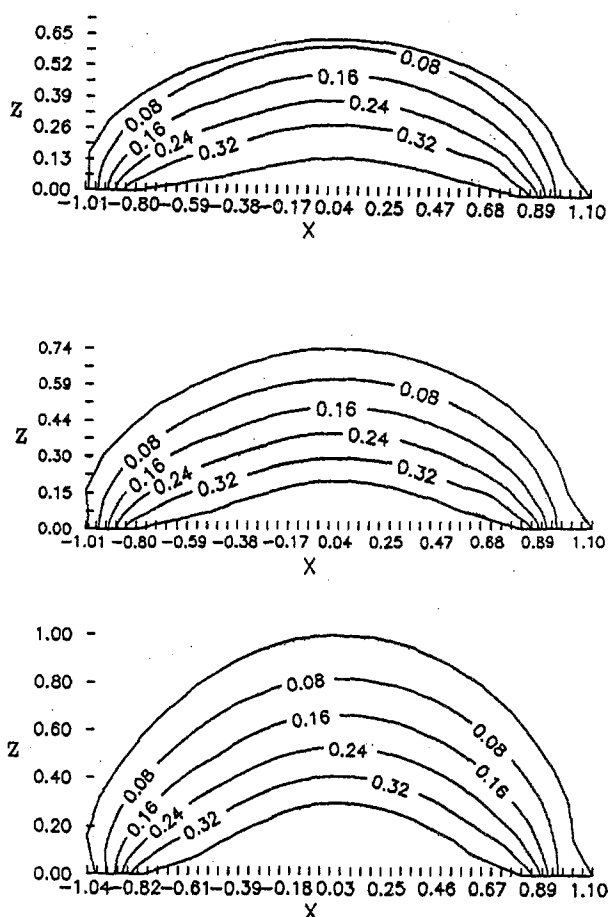


Fig. 3. Magnetic arch configurations a (top), b (middle), c (bottom) for  $x_3 = 0.84, 0.8, 0.734$ , respectively.

ratio and the entrance width together, influenced the shape of the arch. It is different from the symmetric case in which only the entrance width plays a role.

### 3.3. INFLUENCE OF PHOTOSPHERIC MAGNETIC FIELD DISTRIBUTION

We have different arch configurations corresponding to different influx magnetic field. The arch will rise and expand owing to the reduced flux profile during the ratio  $B_x/B_z$  decreases. This is similar to the symmetric case. We consider the

TABLE I

	Figure							
	3a	3b	3c	4a	4b	4c	5	3b
$E_{xz}$	0.830	0.806	0.764	0.992	0.854	0.791	0.785	0.806
$E_y (\times 10^{-2})$	3.939	4.307	5.345	4.545	4.345	4.046	12.3	4.307
$\Delta E (\times 10^{-2})$	4.747	5.329	6.998	4.583	5.089	5.118	15.4	5.329
$z_{\max}$	0.643	0.745	0.001	0.76	0.742	0.706	0.80	0.745
Note	$H = 1.0, 0.8, 0.6$			$x_3 = 0.84, 0.8, 0.734$			$\alpha^2 = 2.5, 1.0$	

In each group all other parameters are the same.

Here,  $E_{xz} = \int (B_x^2 + B_z^2) d\tau$ ,  $E_y = \int_\tau B_y^2 d\tau$ ,  $\Delta E = E_y / E_{xz}$ .

magnetic energy

$$E = \int_\tau B^2 d\tau, \quad (6)$$

where  $\tau$  is the volume of the flux arch. We are especially interested in the azimuthal field and define the ratio of transverse magnetic energy to longitudinal magnetic energy as

$$\Delta E = \frac{\int_\tau B_y^2 d\tau}{\int_\tau (B_x^2 + B_z^2) d\tau}. \quad (7)$$

In our calculation for a twisted flux arch model, we notice that  $\Delta E$  is increased for more expanding configuration (under similar conditions) as given in Table I. The transverse magnetic energy  $\int_\tau B_y^2 d\tau$  is easy to be released in solar atmosphere. The larger  $\Delta E$  becomes unstable easily (for the same  $\alpha$ ). Here, the azimuthal field  $B_y$  plays an important role. When  $\Delta E$  is larger than critical in number, the transverse magnetic energy may be released, this may be explained as being the energy resource for the solar activity.

#### 3.4. FORCE-FREE FACTOR

The force-free factor  $\alpha$  is determined by the total pressure conservation condition and related to the azimuthal field  $B_y$  directly. The case of  $\alpha^2 = 2.5$  is given in Figure 5 for  $H = 1.0$ ,  $x_1 = -1.0$ ,  $x_2 = -0.8$ ,  $x_3 = 0.8$  and  $x_4 = 1.1$  for the boundary condition given in Figures 2a and 2b. Compared with Figure 3b with  $\alpha^2 = 1$ , the configuration of the flux arch in Figure 6 rises relatively higher upward. Since  $\alpha = B_y/\psi$ , the increase of  $\alpha$  will make larger transverse magnetic energy under similar conditions. In Figure 3b,  $\Delta E = 0.0534$ ,  $E_{xz} = \int_\tau (B_x^2 + B_z^2) d\tau = 0.806$ ,  $E_y = \int_\tau B_y^2 d\tau = 0.043$ , but we have  $\Delta E = 0.1564$ ,  $E_{xz} = 0.7854$  and

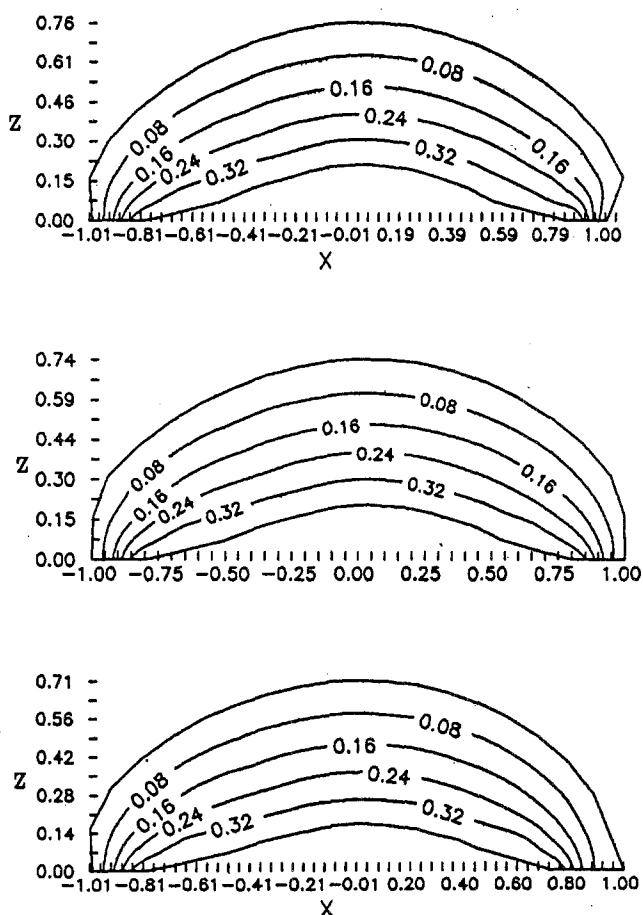


Fig. 4. Magnetic arch configurations a (top), b (middle), c (bottom) for  $H = 0.6, 0.8, 1.0$  respectively.

$E_y = 0.1228$  in Figure 5 with  $\alpha^2 = 2.5$ . We can conclude that the force-free factor is rather important in understanding the magnetic energy release for solar activity.

#### 4. Discussion and Conclusions

Numerical simulations in the present paper are effective for analyzing the equilibrium of confined isolated magnetic arch. Here we extend the symmetric case in the previous paper to the nonsymmetric situation and obtain more general conclu-

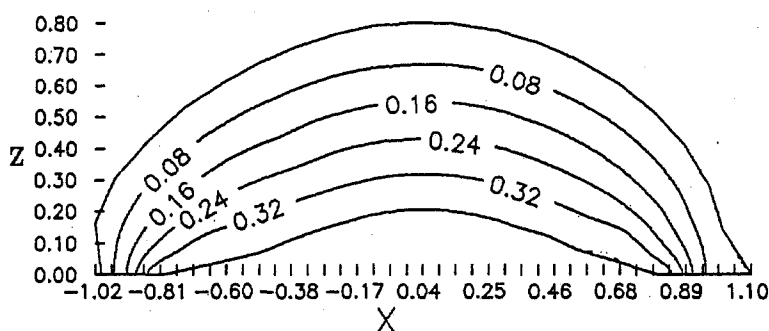


Fig. 5. Magnetic arch configuration for  $\alpha^2 = 2.5$ .

sions. The preliminary results for the asymmetric properties are given. However, the asymmetry discussed in this paper is not large enough due to the difficulty of numerical method. A larger asymmetry will introduce a complex magnetic configuration, which cannot be explained by a clear physical picture. This problem comes from either the physical instability or the calculation method. It should be analyzed carefully in the next step.

In contrast to the continuous model, the conservation condition of the total pressure at free boundary surface is what we have mainly considered. The magnetostatic structure of the flux arch depends on many factors such as the atmosphere scale height, the photosphere magnetic field and the force-free factor, etc. For the asymmetric flux arch, the relative position of footpoints, not only the entrance width or outlet width but also the width ratio, have influences. In addition, we considered the magnetic energy produced in the arch. The azimuthal field is extremely important. These considerations may be helpful for us to understanding the eruption of the solar flare.

This paper has used various approximations for simplification. Practical magnetic flux arches are not static and are much more complicated. The more complex configuration and larger asymmetric properties should be discussed further. By using the isolated model, we can consider flow phenomenon and other problems in future.

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