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EXPERIMENTAL LAWS OF CRATERING FOR HYPERVELOCITY IMPACTS OF SPHERICAL PROJECTILES INTO THICK TARGET

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Summary It is shown in this paper that the laws of cratering in a thick target under hypervelocity impact by a spherical projectile can be approximately expressed by the so-called iso-deviation law and a 2/3 power law. Moreover, hypervelocity impact should be characterized by the isotropic expansion of a crater. In the special case, when the projectile and target are of the same material, the laws mentioned above reduce to the result of a semi-spherical crater and the energy criterion. Generally speaking, a semi-spherical crater and the energy criterion are both approximations, which only take projectile density and target strength into account, and can be used for a rough estimation on the order of magnitude. The inconsistency in various fitted power laws in the literature was also clarified and explained in the paper.

1. INTRODUCTION

The normal impact of a sphere onto a semi-infinite target has been a typical problem in hypervelocity impact phenomena. Herrmann and Wilbeck [1] have summarized the essential mechanical processes and phenomena in this area. No doubt, cratering on a thick target is the fundamental and most significant feature. Obviously, the scales of a crater characterize the damage of the target. Hence, the study on hypervelocity impact should provide an appropriate approach for the determination of cratering parameters. However, it seems to be too difficult for mathematical analysis to meet this need. This is mainly due to the extremely large deformation involved in the phenomenon, which usually induce complicated aspects, such as splashing, melting as well as vaporization. Any theoretical analysis of the phenomenon would unavoidably include sophisticated equations of state or constitutive relations, which should cover all these material behaviors. Numerical simulations usually adopt simplified material models, and therefore computational experiments can be carried out beyond laboratory techniques. Moreover, the numerical approach can reveal the transient and internal processes of cratering, by changing individual material parameters. However, the simulation results should be justified through comparison with experimental data. In this sense, the information obtained by experimental observation is of most significance; whereas the reliability of numerical studies is bound to be parallel to the physical understanding of the phenomenon concerned. In the case of cratering in thick targets under hypervelocity impact general rules inferred by analysis of experimental data should play the most important role. Although this viewpoint has long been accepted, the induced formulations still suffer from severe inconsistencies.

The data obtained in our own laboratory have already been reported [2]. The intention of the present paper is to investigate those experimental expressions, given in the literature or by ourselves, and try to look for the essential and intrinsic law of cratering.

2. DIMENSIONLESS FORMULATION

The governing parameters involved in the problem are

$$d_p, v, \{\mu_p\}, \{\mu_t\}$$

where $\{\mu\}$ denotes material parameters, subscripts p and t refer to projectile and target,

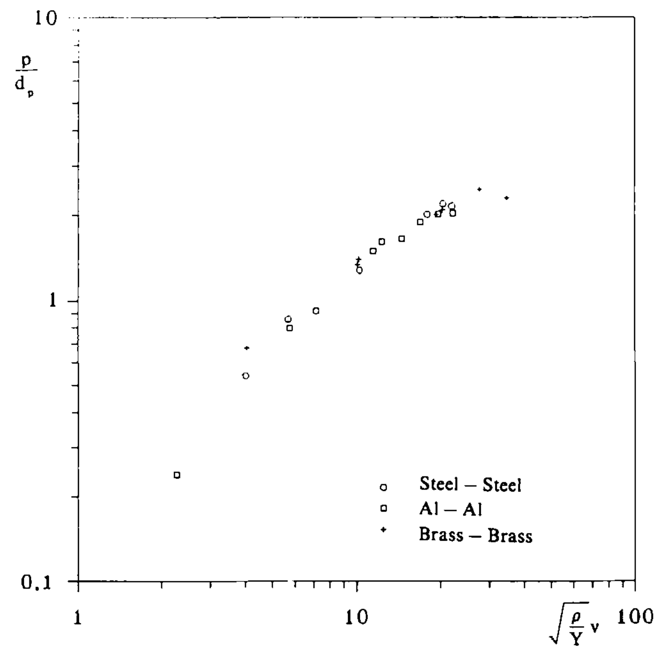

FIG. 1.

respectively, d_p and v are the diameter of a spherical projectile and the impact velocity, respectively. Our experimental data concerned with like-material impacts, i.e. aluminum–aluminum ($\rho = 2.7 \text{ g cm}^{-3}$, $Y = 30 \text{ kgf mm}^{-2}$), steel–steel ($\rho = 7.8 \text{ g cm}^{-3}$, $Y = 80 \text{ kgf mm}^{-2}$) and brass–brass ($\rho = 8.5 \text{ g cm}^{-3}$, $Y = 30 \text{ kgf mm}^{-2}$), are correlated in dimensionless form

$$\frac{p}{d_p} = \Phi_1\left(\sqrt{\frac{\rho}{Y}}v\right) \tag{1}$$

where p is the depth of a crater, ρ is the density and Y the dynamic yield strength of the material. The result is shown in the log–log diagram in Fig. 1. It seems to us that correlation (1) is a proper approximation in the experimental range for like-material impacts, namely, density and strength appear to be the most significant among all the material parameters related to impact cratering. Here, elastic parameters are not important because of extremely large deformations occurring in hypervelocity impacts.

For the unlike-material impacts, the above concept can be extrapolated as

$$p = \varphi(d_p, v, \rho_p, Y_p, \rho_t, Y_t)$$

or in dimensionless form

$$\frac{p}{d_p} = \Phi\left(\sqrt{\frac{\rho_p}{Y_t}}v, \frac{\rho_p}{\rho_t}, \frac{Y_p}{Y_t}\right). \tag{2}$$

Generally speaking, various dimensionless combinations can be taken to form the independent variables. However, their sequence here in the correlation (2) has implied our understanding on the importance of these dimensionless parameters. For material parameters, both ρ_p and Y_t are of great significance, ρ_t is the second and Y_p is the least important parameter. The reason for this will be elaborated later.

The reduced case of $\rho_p = \rho_t$ and $Y_p = Y_t$, leads to

$$\Phi\left(\sqrt{\frac{\rho_p}{Y_t}}v, 1, 1\right) = \Phi_1\left(\sqrt{\frac{\rho_p}{Y_t}}v\right).$$

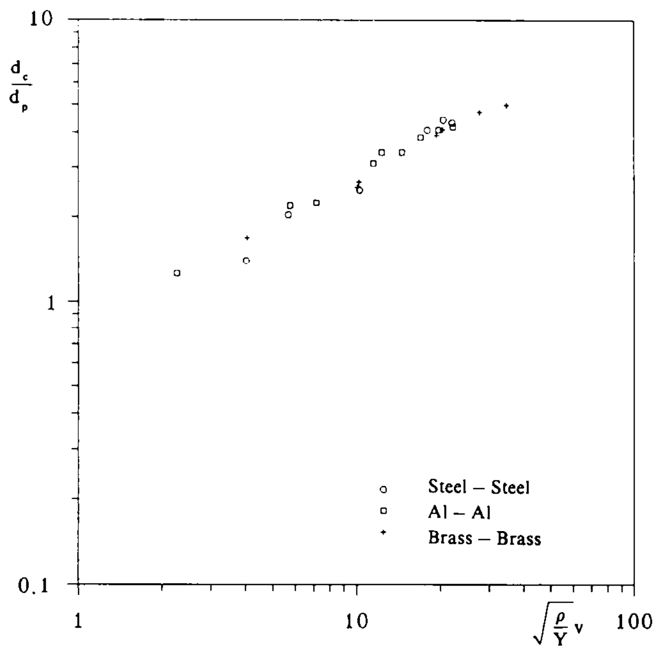

FIG. 2.

Similarly, the dimensionless diameter of a crater in the two cases can be written as

$$\frac{d_c}{d_p} = \Psi_1\left(\sqrt{\frac{\rho}{Y}}v\right)$$

and

$$\frac{d_c}{d_p} = \Psi\left(\sqrt{\frac{\rho_p}{Y_t}}v, \frac{\rho_p}{\rho_t}, \frac{Y_p}{Y_t}\right)$$

respectively, where d_c is the crater diameter. The data are shown in Fig. 2.

3. SEMI-SPHERICAL CRATER AND ENERGY CRITERION

Previous studies have put forward two fundamental concepts [1].

First, the ratio of crater depth p to diameter d_c was suggested to approach 1/2 with increasing impact velocity. Namely, the crater tends to become semi-spherical. Figure 3 shows the data obtained in our laboratory. In the literature, similar figures were used to illustrate the hypervelocity effect, and, moreover, a semi-spherical crater was taken as a feature of hypervelocity impact. Accordingly,

$$\frac{p}{d_c} = \frac{1}{2} \quad \text{or} \quad \frac{V_c}{V_p} = 4\left(\frac{p}{d_p}\right)^3 \tag{3}$$

is used as the identification of hypervelocity impact, where V_c and V_p are volumes of crater and sphere projectile, respectively.

The second concept assumes that the crater volume becomes proportional to the kinetic energy of a projectile but inversely proportional to the strength of a target at sufficiently high impact velocities. The dimensionless formulation is

$$\frac{V_c Y_t}{V_p \rho_p v^2} = \text{const.} \tag{4}$$

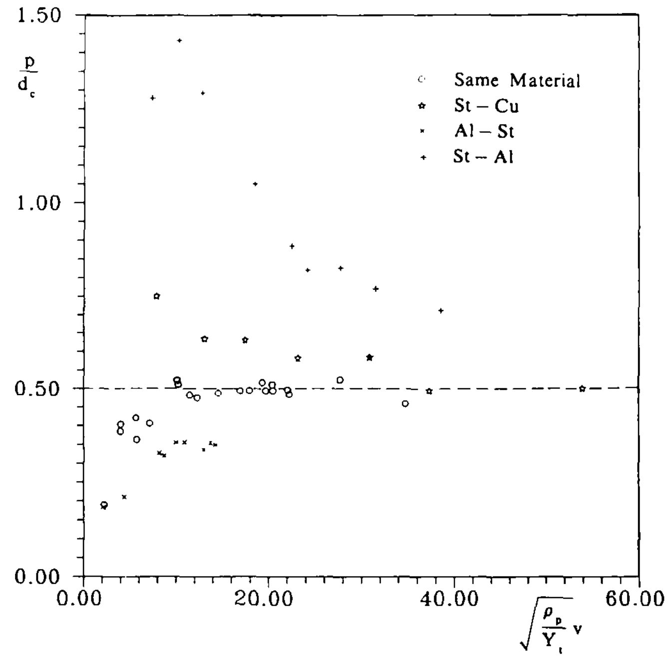

FIG. 3.

The equivalence of this concept is that there exists a single projectile parameter governing the crater volume, i.e. kinetic energy ($\frac{1}{2}V_p\rho_p v^2$). Hence $V_c = f(V_p\rho_p v^2, Y_t, \rho_t)$. In accordance with the Π theorem in dimensional analysis, formula (4) can be easily deduced. V_cY_t can be regarded as the effective energy dissipated during cratering, so (4) is simply a linear energy relation and may be called an energy criterion.

Substitution of (4) into (3) gives

$$\frac{p}{d_p} = K\left(\sqrt{\frac{\rho_p}{Y_t}}v\right)^{2/3} \tag{5}$$

where K is a constant. Obviously, a semi-spherical crater and the energy criterion constitute a rough model for determining the crater size. Compared with formula (2), expression (5) includes only the first argument of (2) and neglects the others. In addition, a simple power function is presumed to hold for the undetermined function in (2).

Looking back to experimental data, unfortunately, a semi-spherical crater and the energy criterion are limited approximations. They hold well for like-material impacts, but not for unlike-material impacts. In conclusion, the law of semi-spherical crater and energy criterion can be used to describe the cratering while the materials of target and projectile are the same. However, they can not be straightforwardly extrapolated to other cases. A satisfactory unified law is still required.

4. ISO-DEVIATION LAW AND ISOTROPIC EXPANSION

Usually, a semi-spherical crater is established according to those curves, similar to that in Fig. 3. Indeed, when the materials of the targets and the projectiles are fixed for each set of tests with increasing impact velocity, penetration p/d_c does approach 1/2. However, it is evident that most of the data within the experimental range fall far away from the asymptote $p/d_c = 1/2$, and hence do not follow the assumption of a semi-spherical crater. Generally speaking, when the density and strength of the projectile are lower than those of the target, the crater is shallow; otherwise the crater is deep, and even much deeper than a semi-spherical crater.

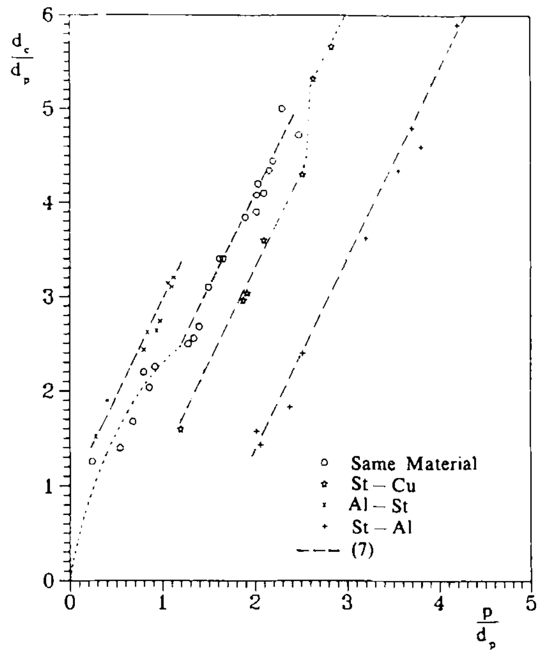

FIG. 4.

Perhaps, the intuition of idealized semi-spherical crater leads people to correlate data in the form of p/d_c. We propose that the law of crater shape should be traced better by plotting p versus d_c (see Fig. 4). Each set of data can be fitted by a straight line with the same slope of two for higher impact velocities,

$$\frac{d_c}{d_p} = 2\frac{p}{d_p} + A\left(\frac{\rho_p}{\rho_t}, \frac{Y_p}{Y_t}\right) \tag{6}$$

where A is a dimensionless intercept of the line depending on the material parameters. This reveals that for a set of experimental data with fixed projectile and target materials, craters expand isotropically with increasing impact velocity as soon as the velocity is high enough, namely, the deviation to semi-sphere $(d_c - 2p)/d_p$ remains unchanged. Now, we term this as an iso-deviation law or isotropic expansion. Accordingly, instead of the assumption of semi-spherical crater, it would be better to adopt the iso-deviation as the identification of hypervelocity impact. Obviously, a semi-spherical crater is merely a special case for $A = 0$. When the impact velocity increases, the crater will expand, and hence the deviation A becomes insignificant in (6) and then $p/d_c \to 1/2$. Evidently, the iso-deviation law also reveals the aforementioned tendency to a semi-spherical crater.

The dependence of A on the material parameters has the following features:

when $\rho_p/\rho_t = 1$ and $Y_p/Y_t = 1$, $A = 0$ (semi-spherical crater);
when $\rho_p/\rho_t < 1$ and $Y_p/Y_t < 1$, $A > 0$ (shallow crater);
when $\rho_p/\rho_t > 1$ and $Y_p/Y_t > 1$, $A < 0$ (deep crater).

The data fitting gives the following expression

$$A\left(\frac{\rho_p}{\rho_t}, \frac{Y_p}{Y_t}\right) = 1.4\left[1 - \left(\frac{\rho_p}{\rho_t}\right)^{0.73}\left(\frac{Y_p}{Y_t}\right)^{0.31}\right] \tag{7}$$

or simply:

$$A\left(\frac{\rho_p}{\rho_t}, \frac{Y_p}{Y_t}\right) = 1.4\left[1 - \left(\frac{\rho_p}{\rho_t}\right)^{2/3}\left(\frac{Y_p}{Y_t}\right)^{1/3}\right]. \quad (8)$$

The exact formulation of deviation A needs more experimental data and further investigation is required.

5. "$V_p\rho_p^2v^2$" LAW OR 2/3 POWER LAW

The iso-deviation law is just a relation of the crater depth to the diameter, and does not reveal the effect of impact velocity (or the primary dimensionless parameter) on the crater. Now, let us study this aspect of the phenomenon by examining Figs 5–7, which show the experimental relations of crater depth, diameter and volume to projectile velocity in a dimensionless form. Here the volume of a crater is assumed approximately to be

$$\frac{V_c}{V_p} = \frac{p}{d_p}\left(\frac{d_c}{d_p}\right)^2. \quad (9)$$

According to Fig. 7, it is clear that the error of the energy criterion ranges over $\pm 75\%$, so it can only be used to evaluate the order of magnitude of the crater size. It appears too rough to adopt as a quantitative approximation. We also found that the relation of crater depth with impact velocity in Fig. 5 can be easily fitted as follows to reveal approximately the effect of ρ_p/ρ_t:

$$\frac{p}{d_p} = 0.27\left(\frac{\rho_p}{\rho_t}\right)^{1/3}\left(\sqrt{\frac{\rho_p}{Y_t}}v\right)^{2/3} = 0.27\left(\frac{\rho_p}{\rho_t}\right)^{2/3}\left(\sqrt{\frac{\rho_t}{Y_t}}v\right)^{2/3} = 0.27\left(\frac{\rho_p v}{\sqrt{\rho_t Y_t}}\right)^{2/3}. \quad (10)$$

Figure 8 represents the comparison between the experimental data and the fitting formula (10). It indicates that under hypervelocity impact, namely the impact velocity is high enough that the crater may be described by iso-deviation law (as compared with Fig. 4), the

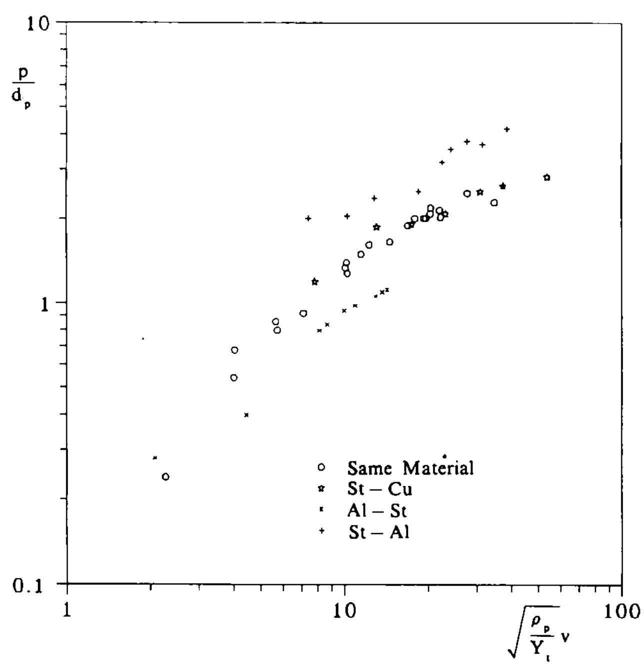

FIG. 5.

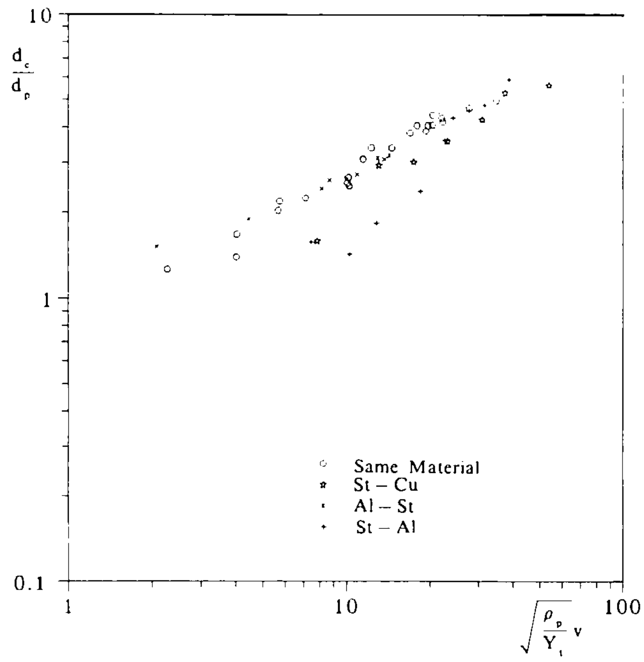

FIG. 6.

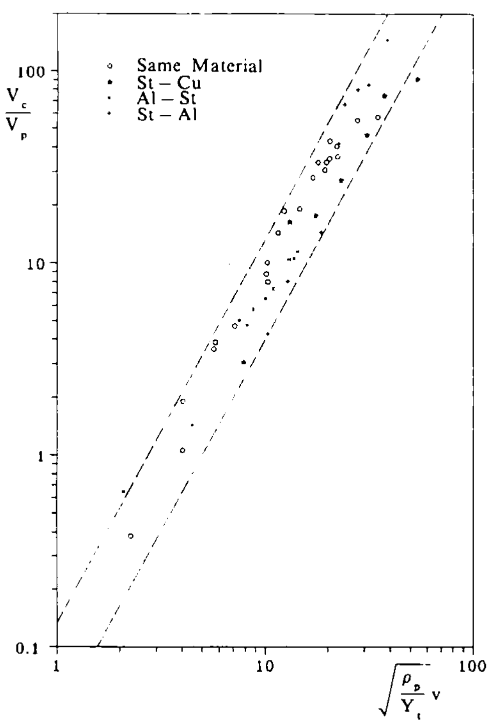

FIG. 7.

compound projectile parameter governing uniquely the crater depth is $V_p\rho_p^2v^2$, which is neither kinetic energy nor momentum of a projectile. More directly, the depth of a crater is proportional to $(\rho_p v)^{2/3}$, i.e. 2/3 power of specific momentum of a projectile, therefore, according to dimensional analysis inversely proportional to $(\rho_t Y_t)^{1/3}$. We call it the "$V_p\rho_p^2v^2$" law or 2/3 power law.

As a matter of fact, this law has been discovered by numerous investigators [1]. Herrmann and Jones [3] collected over 1700 data obtained in 15 laboratories and established a

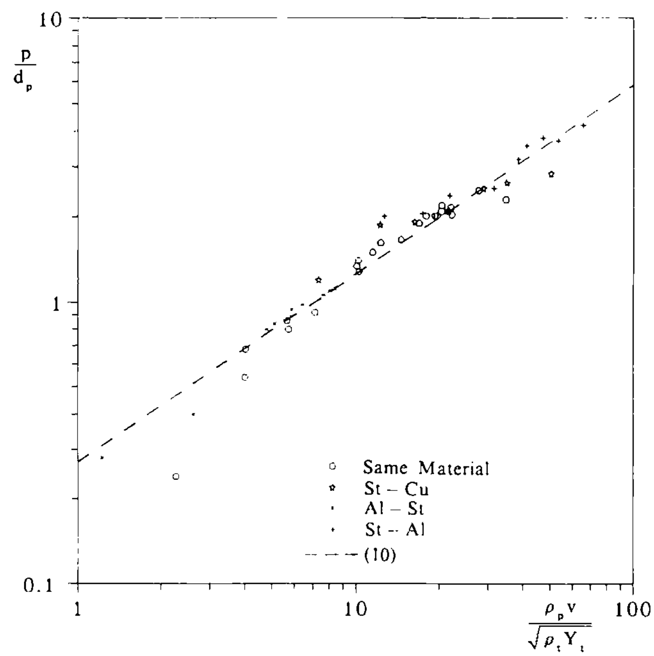

Fig. 8.

Table 1

Authors	p–ρ_p	p–v	V_c–ρ_p	V_c–v	Quoted from
(a) Summers and Charters (1958)	2/3	2/3			Ref. [1]
(b) Charters and Summers (1959)	2/3	2/3			Ref. [1]
(c) Summers (1959)	2/3	2/3	3/2	2	Ref. [4]
(d) Herrmann and Jones (1961)	2/3	2/3	5/3†	2	Ref. [3]
(e) Christman and Gehring (1966)	2/3	2/3			Ref. [5]
(f) Goodman and Liles (1963)	0.7	2/3	3/2	2	Ref. [6]
(g) Eichelberger and Gehring (1962)	{1/3}	2/3	{1}	2	Ref. [7]
(h) Loeffler *et al.* (1963)	{1/2}	2/3			Ref. [8]
(i) Bruce (1961)	{1/2}	{2/3}	3/2‡	2	Ref. [4]
(j) Sorenson (1965)	{0.45}	{0.56}	1.35	1.69	Ref. [9]

† In Table 2 of Ref. [1] it was 3/2 by mistake. According to the original Ref. [3] it should be 5/3.
‡ In Table 2 of Ref. [1] it was 2/3 by mistake. According to the original Ref. [4] it should be 3/2.

fitting law as

$$\frac{p}{d_p} = 0.36\left(\frac{\rho_p}{\rho_t}\right)^{2/3}\left(\sqrt{\frac{\rho_t}{H_t}}\,v\right)^{2/3}$$

where strength of target was replaced by the Brinell hardness H or the static yield strength S. If the following simple correlations exist

$$H = 3.6S,\ Y = 1.5S$$

the two fittings will be in good consistency.

However, some authors use different exponents. To establish 2/3 power law as a universal law, these contradictions should be satisfactorily explained. We list here in Table 1 some so-called experimental exponents for p, including those in Table 1 of Ref. [1], together with V_c in order to clarify the contradictions.

If the difference in the fitting exponents was due to different data, the power law would not be a proper formulation for the phenomenon and would only be valid in a limited case. Generally speaking, scatter in experimental data will lead to certain scatter in the fitting parameters even for an exact fitting form. Clearly, if the values of a fitting parameter given by different authors are within the scatter range, they should be considered as consistent and the obtained fitting parameter should be universal; otherwise the proposed fitting form would be invalid. In the concerned case, ρ_p/ρ_t ranges from 1/5 to 5, hence the variation of its logarithm is about 1.4. Provided the scatter of a fitting exponent is taken to be from 2/3 to 0.7, given by (f), the fitted data should be within a relative error of 6% (i.e. $\pm 3\%$). Suppose the absolute error of the fitting exponent is ± 0.05, then the relative error of the fitted data is about $\pm 8.4\%$ (see Appendix). As for another argument $\sqrt{(\rho_t/Y_t)}\,v$ (some authors used the nondimensional impact velocity v/c_t instead, where c_t is the sound velocity of the target), the variation range of its logarithm is roughly 1.4 too. Therefore, it is reasonable to assume the accuracy of the exponent is at decile, so the fitting law of p from (f) is actually in agreement with the 2/3 power law. Now, we should examine the last four cases, which seem to be different from the 2/3 power law.

The authors in (g) in Table 1 gave the exact formulation of a semi-spherical crater and the energy criterion, as discussed in Section 3. In the case of like-material impacts, iso-deviation law and the 2/3 power law reduce to the semi-spherical crater law and the energy criterion. In this special case, the 1/3 power of ρ_p shows only the joint effect of densities of the projectile and the target, but not the individual effect of projectile density itself. In fact, it is the sum of the 2/3 power of the projectile density and $(-1/3)$ power of the target density, and is really not inconsistent with the 2/3 power law.

For (h) [8], the proposed 1/2 power was actually a compromise between the 2/3 and 1/3 powers in different references. The misunderstanding for the 1/3 power has been clarified in the last paragraph.

The author in (i) thought that the power laws for p and V_c obtained directly from experimental data, e.g. by (c), did not agree with the assumption of a semi-spherical crater, and therefore he suggested that a power law for a crater depth was deduced from that for crater volume and the assumption of a semi-spherical crater [4] according to expression (3). Sorenson in (j) repeated the same argument as (i) [9]. In short, the power laws for crater depth by (i) and (j) were not fitted directly to experimental data but were just deductions from the power laws for crater volume and the assumption of a semi-spherical crater, (3).

If we discard the exponents, in { } in Table 1, which were either from deduction or only a misunderstanding from the special cases of like-material impacts and not directly based on experimental data, we can conclude the discussion as follows.

1. The 2/3 power law is general and proper fitting for crater depth.
2. Crater volume can not be fitted by a simple power law properly.

It has been confirmed that the 2/3 power law is a proper representation of the crater depth. If the crater volume could be fitted by a power law, the crater diameter could also be fitted in the same way, but on the contrary, Figs 6 and 7 show that it is quite impossible to fit all data by a family of parallel straight lines. In fact, the formula (6) also indicates that it is unsuitable to fit the crater diameter by a power law. Some authors [3] have pointed out that power fittings for crater volume are much rougher than those for crater depth and the fitting powers obtained are inconsistent. Data in Fig. 7 show a rough $(\pm 75\%)$ average slope of 2 (± 0.35), as shown in Table 1.

3. The assumption of a semi-spherical crater does not hold in general.

6. PHYSICAL IMPLICATION

Now, what enlightenments about the cratering process can we obtain from the derived laws? The iso-deviation law describes the loci of terminals of cratering processes in the plane of d_c–p. This law unveils the feature of isotropic expansion of craters, at relatively high impact velocities. Generally speaking, locus of terminals of processes cannot be confused with the processes itself. However, this feature of isotropic expansion demonstrated by terminal craters of cratering processes should be considered as the characteristics of cratering in the later stage of the process, because it is unimaginable that only the craters just in the terminal have the feature of isotropic expansion. In other words, the plot of d_c–p of terminal craters may be supposed to represent the cratering processes at least in the later stage.

If so, the major material parameters affecting the crater size are density and strength. One might think that the initial stage of cratering should be affected by the shock wave, therefore by the wave impedances of the materials. However, any cratering process should rapidly go into a stage, in which the density and strength dominate the process. The iso-deviation law characterizes an isotropic expansion stage in cratering, and the 2/3 power law implies that the strength of a projectile might not affect the development of the crater in this stage. It can be imagined that the projectile at high pressure manifests itself as a fluid. With the fluidized projectile the development of the crater is not affected by the strength of the projectile. The distribution of the fluidized projectile on the surface of the crater gradually becomes isotropic, and then the crater expands isotropically. But the crater in the earlier stage, in which cratering is heterogeneous, is not semi-spherical in general. The deviation A, which remains constant during the later isotropic expansion stage, is just the reflection of the influence on cratering in the earlier stage. If the impact velocity is high enough that the cratering process comes into the isotropic expansion stage, the impact process is defined as hypervelocity impact.

7. CLOSING REMARKS

1. Experimental laws of crater under hypervelocity impact can be expressed by an iso-deviation law and a 2/3 power law. All so called empirical "facts", such as a semi-spherical crater and the energy criterion in a special case, the tendency to semi-spherical crater as well as inconsistencies in the power law of crater volume etc., are just representations of the iso-deviation law and the 2/3 power law.

2. In the special case of like-material impacts, the two laws reduce to a semi-spherical crater and the energy criterion. These two reduced "laws" keep the first argument of the function only and can be used for an estimation of the order of magnitude of the crater size.

3. Density and strength of materials are the most significant material parameters in cratering under hypervelocity impact. It is generally believed that at very high impact velocity, melting and vaporization of materials may play an unavoidable effect [10]. This has not been taken into account in this paper. In addition, some experiments indicate that p/d_p may be still dependent on d_p [11], hence the geometry similarity might be invalid. This implies that the rate effect of materials on cratering exists. This is also not considered in this paper.

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APPENDIX

Experimental data of y depending on x, which ranges over the interval (a,b), would be fitted by a power function $y = kx^{\alpha}$. In the log–log diagram, this is a straight line fitting, and the exponent α is the slope of the straight line. Scatter in the experimental data should

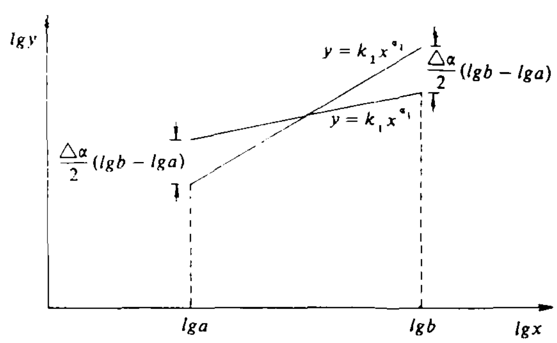

FIG. A1.

lead to scatter in the exponent as (see Fig. A1)

$$\frac{\Delta\alpha}{2}(\log b - \log a) = \log\left(1 + \frac{\Delta y}{y}\right)$$

where $\Delta\alpha$ is the scatter range of the exponent (i.e. $\pm$ absolute error), $\Delta y/y$ is the relative scatter range of the data.