A Momentum Theorem of Impact Forces Owing to Breaking Waves*

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The impact force owing to breaking waves is one of the important wave loadings exerted on the ocean structures, especially the parts nearby mean sea level (MSL). Since the 1950s, people have proposed a great many methods to calculate the impact forces exerted on vertical piles. Early experiential formulae were based on the modified Morison equation according to the experiments by Ross and Hall[1] with a drag coefficient 2.5 times larger for shallow water breakers on slender and cylindrical piles. In 1966, the Von Karman entry theory was applied to the problem. The model only reflected the features of impact forces exerted on the piles due to breaking waves to a certain extent, while the pressure distributions were different from the experiment. In recent years, Y. B. Li, Honda and Sawaragi carried out similar works[3－5]. Tarimoto[6] studied the case for slant piles. In 1985, Kjeldsen[7] attempted to explore the relation between breaking wave forces and ocean wave factors. However, the impact force depends on varieties of parameters such as incidence wave steepness, breaking point, bottom slopeness, relative pile radius, etc., and no law has been found for it. Hence, the present state of breaking wave study remains at a semi-theoretical and semi-experiential stage.

The original difficulty lies in that there is no approach to deciding the flow field induced by breaking waves. In the 1970s, people made a breakthrough in the computation of breaking waves since Longuet-Higgins' work[8]. Based on the flow field by Li and Chen[9], we propose a momentum theorem for the first time and obtain the impact force on the circular cylinder in this note. The study is merely restricted to plunging breaking waves due to its maximum wave loading among the others.

1 Momentum Theorem

In this note, we endeavour to determine the breaking wave force stemming from the most fundamental law in mechanics so that the formula becomes more general and less experiential than ever. For this reason, we assume

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(i) Long crest assumption. It means that the breaking waves are assumed infinitely long in the lateral direction so as to reduce the problem to a two-dimensional one. Generally speaking, the hypothesis keeps valid for piles with a smaller diameter compared with the wavelength around 100 meters.

(ii) Frozen flow field assumption. It means that once the piles are impacted, the wave shape and flow field will be kept unvaried, and a mass of water body will move forward as a whole at the same speed as that of the particle which touches the piles first. It is because what we concern most is the maximum impact force at the beginning. As is known, the velocity in flow field induced by breakers is of the order of phase velocity c and the acceleration is of the order of gravity g. The process appears so short (1/100 s or so) that no considerable changes of flow field are detected during the process due to $g\Delta t \ll c$. As for the impact force at the later stage, it seems to be secondary to the former and will not arouse much errors.

(iii) Total loss of normal momentum assumption. It means that the existence of the piles hinders the plunging of moving water particles so as to transform all the normal momentum into the impact force with viscosity neglected.

On the basis of the above-mentioned assumptions, we are able to derive a formula for breaker impact forces. It is seen from Fig.1 that as the water in the thin layer of dy plunges to the pile, the normal momentum loss of water nearby p turns out to be

$$\rho u^2(z,y)\cos^2\theta\, dz\, dy\ , \tag{1}$$

where $\cos\theta=\sqrt{(R^2-z^2)}\ /R$. Hence, the total impact force is

$$F(t)=\int_0^{\eta} dy \int_{-w(t)}^{w(t)} \rho u^2 \cos^2\theta\ dz\ , \tag{2}$$

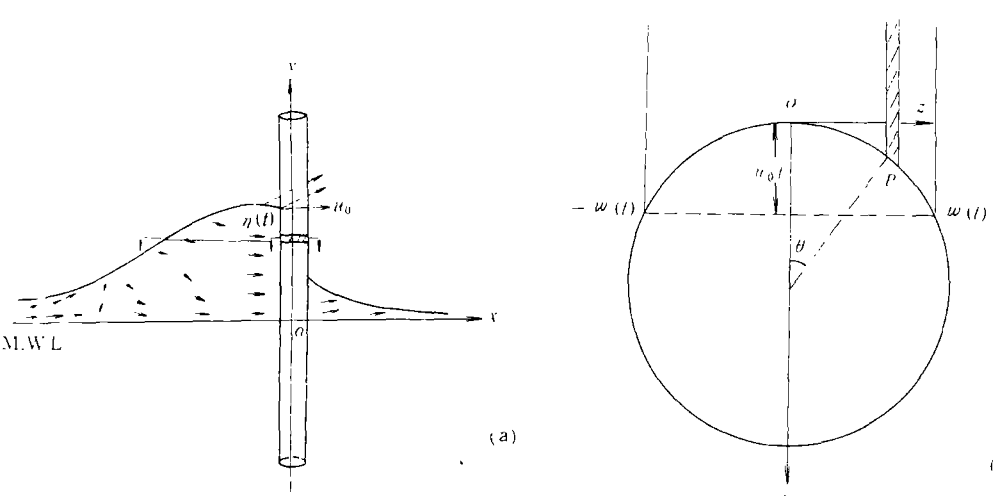

Fig. 1. Vertical pile in flows induced by breakers.

in which ρ is the density of water; u, the horizontal velocity component; η, the wave elevation; R, the radius of piles; $w(t)$ is half chord of part of the pile which is surrounded by water; y and z are vertical and lateral coordinates, respectively.

If the flow field of breaking waves $u(x,y)$ is given, from (i) and (ii) we derive

$$u(z,y)=\bar{u}\left(R-\sqrt{R^2-z^2}-u_0t,\ y\right), \quad (3)$$

and

$$w(t)=\sqrt{2Ru_0t-u_0^2t^2}, \quad (4)$$

where u is the velocity of the particle that touches the pile first, thus

$$F(t)=\int_0^{\eta(t)}dy\int_{-w(t)}^{w(t)}\rho\bar{u}^2\cos^2\theta\,dz \quad (5)$$

which can be made dimensionless by a factor $2\pi\rho g\eta_{max}D/k$. Provided the breaker flow field is given, it is not difficult to compute time process of dimensionless breaker impact force.

2 Results and Discussion

Fig.2 shows the time series of breaker impact force for $ak=0.42$ and 0.44, both curves display salient impulse type, and the rise duration is rather short. Furthermore, the steeper the amplitude, the earlier the curve rise and the larger the impact force, which can be attributed to the fact that such kind of waves evolve faster, its maximum velocity and acceleration are larger and the water tongue at the front is longer. The ascending and

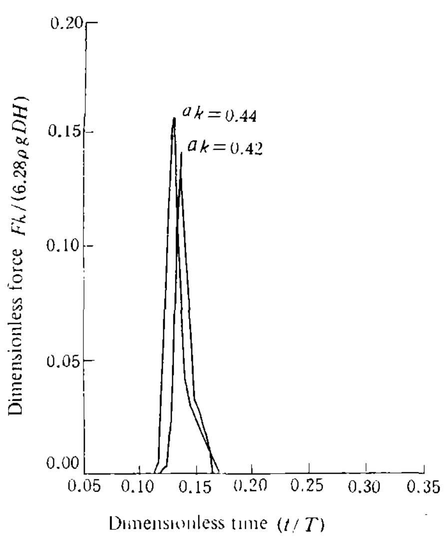

Fig.2. Time series of breaking wave force for different amplitudes.

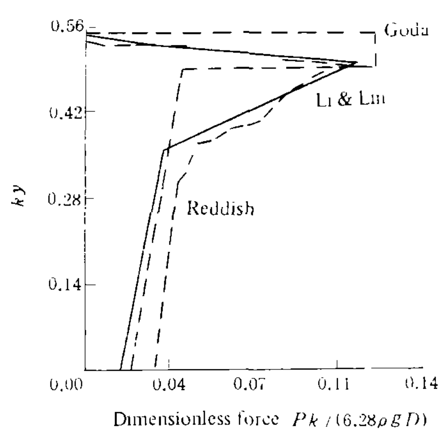

Fig.3. Vertical distribution of breaking wave force.

descending durations are 0.1—0.05 s and 0.1—0.2 s, respectively according to Ref.[3]. The theory is basically in accord with Li's experiment. The vertical distribution of impact force is given in Fig.3, from which we find a triangular profile, namely, the maximum force does not occur at the crest. According to the momentum theorem, the impact force relies on the water mass multiplied by velocity. Although the velocity is larger at the crest, a small mass still reduces the impact force to a certain extent. Compared with Goda's result, ours is more reasonable and consistent with Reddish's experiment quantitatively. Hence, the results derived from the momentum theorem are fairly reliable.

In Fig. 4 we compare the forces with or without (ii). It is found that the rising part of the force almost coincides with each other and the descending part deviates a little bit with the descending duration, for the former being 5% shorter than the latter. The situation further and indirectly verifies the momentum theorem we have just proposed.

As for statistical analysis of the breaker force and the so called "cusion effects", we will study them in the near future.

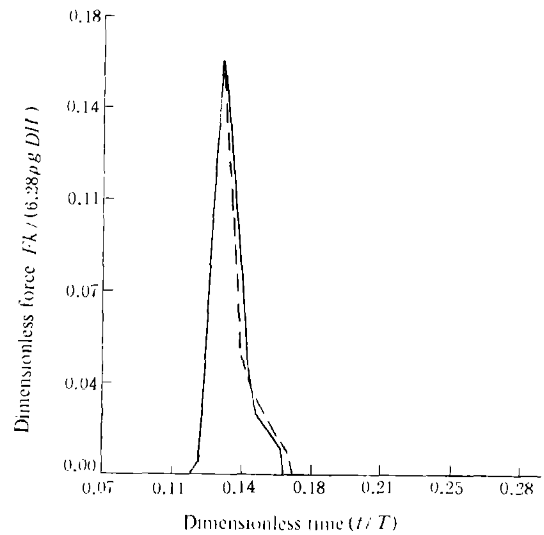

Fig. 4. Breaking wave force in frozen and transient flow field cases.

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