DYNAMIC INTERACTION OF AN ELASTIC CONTAINER WITH FLUID

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The vibration analysis of an elastic container with partially filled fluid was investigated in this paper. The container is made of a thin cylinder and two circular plates at the ends. The axis of the cylinder is in the horizontal direction. It is difficult to solve this problem because the complex system is not axially symmetric. The equations of motion for this system were derived. An incompressible and ideal fluid model is used in the present work. Solutions of the equations were obtained by the generalized variational method. The solution was expressed in a series of normalized generalized Fourier's functions. This series converged rapidly, and so its approximate solution was obtained with high precision. The agreement of the calculated values with the experimental result is good. It should be mentioned that with our method, the computer time is less than that with the finite-element method.

1. Introduction

The problem of the vibration of an elastic container with partially filled fluid is common in various fields of industry, for example, the liquid propellent rocket, the oil tank of aircraft, the large oil tank on the ground, and the oil tank truck. Some containers of nuclear reaction are subjected to various types of excitation. The exact solution for the coupled fluid-structure system is very difficult.

In previous studies, most authors assumed that the wall of the container is rigid. It is a sloshing problem, in which only the motion of the fluid was investigated. This method is suitable for a thick wall of the container. Moeseev and Petrov [1] calculated the natural frequencies of a finite bulk of liquid by a numerical method. When the wall of the container is very thin, the wall is deformed under the hydrodynamic force so that the elasticity of the container should be considered. Some methods for the calculation of the frequencies of the elastic container were investigated in [2-4]. In recent years, most of the published papers are concerned with the axial symmetry of the container. It is helpful to solve problems by an analytical method or the finite-element method.

The finite-element method is always used in the solution of coupled structures and a fluid system. Akkas et al. [5] used the program SAPIV, in which the fluid element was treated as an elastic solid element with a negligible shear modulus and used a special value for bulk modulus. But spurious modes were obtained in the calculated results. The author of [6] and other authors used a penalty coefficient for the irrotationality of displacement in the fluid and showed that these spurious modes were determined by the type of mesh. Consequently it is not possible to separate the real and the spurious modes by a simple inspection of the values of the frequencies. It may be separated by the value of the derivative of the square of the frequencies with respect to the penalty coefficient.

In the present paper vibration analysis for an elastic container with partially filled fluid is investigated by an analytical method. The container is made of a thin cylinder and two circular plates at the ends. The axis of the cylinder is in the horizontal direction. It is difficult to solve this problem because the coupled system is not axially symmetric, and the liquid in the container has a free surface. It is almost impossible to find a potential function for this irregular bulk of liquid which satisfies all the boundary conditions.

In this paper we also assume that the fluid is incompressible, ideal, and without surface wave, that is, it is a linear system. The equations of motion for this coupled system

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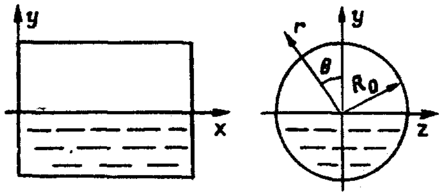

Fig. 1. Scheme of the container.

were derived by the generalized variational method. The solution was expressed in a series of normalized generalized Fourier functions. This series converges rapidly, so that an approximate solution with high precision may be obtained. The agreement of the calculated values with the experimental results is good.

2. Governing Equations

The coupled system consists of three parts: 1) two elastic circular plates at the supported ends; 2) a thin cylindrical shell; and 3) a finite bulk of liquid, shown in Fig. 1. We shall find the functional of each part by the variational method [7].

1. The functional of the two circular plates in a polar coordinate system is

$$\pi_i = \frac{1}{2}\int_{t_1}^{t_2}\iint_{\sigma_i} D_i\Big\{(\nabla^2 u_i)^2 - 2(1-\nu_i)\Big[\frac{\partial^2 u_i}{\partial r^2}\Big(\frac{1}{r}\frac{\partial u_i}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_i}{\partial\theta^2}\Big) - $$
$$-\Big(\frac{1}{r}\frac{\partial^2 u_i}{\partial r\partial\theta} - \frac{1}{r^2}\frac{\partial^2 u_i}{\partial\theta^2}\Big)^2\Big]\Big\} r\,dr\,d\theta\,dt -$$
$$-\int_{t_1}^{t_2}\iint_{\sigma_i'}\rho'\frac{\partial^2\psi}{\partial t^2}u_i r\,dr\,d\theta\,dt - \frac{1}{2}\int_{t_1}^{t_2}\iint_{\sigma_i}\rho_i h_i\Big(\frac{\partial u_i}{\partial t}\Big)^2 r\,dr\,d\theta\,dt +$$
$$+\int_{t_1}^{t_2}\int_{C_i} D_i\Big(\frac{\partial}{\partial r}(\nabla^2 u_i) + (1-\nu_i)\frac{1}{r}\frac{\partial^2}{\partial r\partial\theta}\Big(\frac{1}{r}\frac{\partial u_i}{\partial\theta}\Big)(u_i - \bar{u}_i)\Big] R_0\,d\theta\,dt -$$
$$-\int_{t_1}^{t_2}\int_{C_i} D_i\Big[\frac{\partial^2 u_i}{\partial r^2} + \nu_i\Big(\frac{1}{r}\frac{\partial u_i}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_i}{\partial\theta^2}\Big)\Big]\Big(\frac{\partial u_i}{\partial r} - \frac{\partial\bar{u}_i}{\partial r}\Big) R_0\,d\theta\,dt \qquad (1)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2};\quad i = 1, 2.$$

2. The functional of a cylindrical shell is

$$\pi_3 = \frac{1}{2}\int_{t_1}^{t_2}\iint_{\sigma_3} D_0\Big\{\Big[\Big(\frac{\partial u}{\partial\xi}\Big)^2 + \Big(\frac{\partial v}{\partial\theta} + W\Big)^2 + 2\nu_3\frac{\partial u}{\partial\xi}\Big(\frac{\partial v}{\partial\theta} + W\Big) +$$
$$+\frac{1-\nu_3}{2}\Big(\frac{\partial u}{\partial\theta} + \frac{\partial v}{\partial\xi}\Big)^2\Big] + \delta_0\Big[\Big(\frac{\partial^2 w}{\partial\xi^2}\Big)^2 + \Big(\frac{\partial^2 w}{\partial\theta^2}\Big)^2 + 2\nu_3\frac{\partial^2 w}{\partial\xi^2}\frac{\partial^2 w}{\partial\theta^2} +$$
$$+2(1-\nu_3)\Big(\frac{\partial^2 w}{\partial\xi\partial\theta}\Big)^2\Big]\Big\} d\xi\,d\theta\,dt - \int_{t_1}^{t_2}\iint_{\sigma_3'}\rho'\frac{\partial^2\psi}{\partial t^2} w R_0\,d\xi\,d\theta\,dt -$$

$$-\frac{1}{2}\int_{t_1}^{t_2}\int_{\sigma_3}\int\rho_3 h_3\left(\frac{\partial w}{\partial t}\right)^2 R_0^2 d\xi d\theta dt-\int_{t_1}^{t_2}\int_{C_3} D_0\left[\frac{\partial u}{\partial \xi}+\nu_3\left(\frac{\partial v}{\partial \theta}+w\right)\right]\times$$

$$\times(u-\bar{u})\,d\theta dt-\int_{t_1}^{t_2}\int_{C_3}\frac{1-\nu_3}{2}D_0\left(\frac{\partial u}{\partial \theta}+\frac{\partial v}{\partial \xi}\right)(v-\bar{v})\,d\theta dt+$$

$$+\int_{t_1}^{t_2}\int_{C_3} D_0\delta_0\left[\frac{\partial}{\partial \xi}\left(\frac{\partial^2 w}{\partial \xi^2}+\nu_3\frac{\partial^2 w}{\partial \theta^2}\right)\right](w-\bar{w})\,d\theta dt-\int_{t_1}^{t_2}\int_{C_3} D_0\delta_0\left(\frac{\partial^2 w}{\partial \xi^2}+\nu_3\frac{\partial^2 w}{\partial \theta^2}\right)\left(\frac{\partial w}{\partial \xi}-\frac{\partial \bar{w}}{\partial \theta}\right)d\theta dt \tag{2}$$

where

$$\xi=\frac{x}{R_0},\quad \delta_0=\frac{h_3^2}{12R_0^2}.$$

In deriving the strain energy for the cylindrical shell, the theory of Flugge was used.

3. The functional of the fluid is

$$\pi_4=\frac{1}{2}\int_{t_1}^{t_2}\int_{\Omega}\int\int\left[\left(\frac{\partial \Phi}{\partial x}\right)^2+\left(\frac{\partial \Phi}{\partial y}\right)^2+\left(\frac{\partial \Phi}{\partial z}\right)^2\right]dxdydzdt-$$

$$-\int_{t_1}^{t_2}\int_{\sigma_i'}\int\sum_{i=1}^{2}\left[\frac{\partial \Phi}{\partial x}-\frac{\partial u_i}{\partial t}\right]\Phi dydzdt-\int_{t_1}^{t_2}\int_{\sigma_3'}\int\left(\frac{\partial \Phi}{\partial r}-\right.$$

$$\left.-\frac{\partial w}{\partial t}\right)\Phi R_0 dxd\theta dt-\int_{t_1}^{t_2}\int_{\sigma}\int\frac{\partial \Phi}{\partial y}(\Phi-\bar{\Phi})\,dxdzdt. \tag{3}$$

The total functional of the coupled system is

$$\pi=\pi_1+\pi_2+\pi_3+\pi_4. \tag{4}$$

3. Solution of Equations

When the functional of the coupled system was obtained, we solved the equations by the Ritz method. An approximation to the displacement function was assumed as follows:

$$u_1=\sum_{m_1}^{m_1'}\sum_{n_1}^{n_1'}U_{m_1n_1}\left[J_{n_1}\left(\frac{\lambda_{m_1n_1}}{R_0}r\right)-\frac{J_{n_1}(\lambda_{m_1n_1})}{I_{n_1}(\lambda_{m_1n_1})}I_{n_1}\left(\frac{\lambda_{m_1n_1}}{R_0}r\right)\right]\cos n_1\theta\sin\omega t+u_1', \tag{5}$$

$$u_2=\sum_{m_2}^{m_2'}\sum_{n_2}^{n_2'}U_{m_2n_2}\left[J_{n_2}\left(\frac{\lambda_{m_2n_2}}{R_0}r\right)-\frac{J_{n_2}(\lambda_{m_2n_2})}{I_{n_2}(\lambda_{m_2n_2})}I_{n_2}\left(\frac{\lambda_{m_2n_2}}{R_0}r\right)\right]\times$$

$$\times\cos n_2\theta\sin\omega t+u_2', \tag{6}$$

$$u=\sum_{m_3}^{m_3'}\sum_{n_3}^{n_3'}U_{m_3n_3}\left[\bar{U}\left(\frac{\lambda_{m_3}}{l}x\right)-\frac{\bar{u}(\lambda_{m_3})}{\bar{v}(\lambda_{m_3})}\bar{v}\left(\frac{\lambda_{m_3}}{l}x\right)\right]\cos n_3\theta\sin\omega t+u', \tag{7}$$

$$v=\sum_{m_3}^{m_3'}\sum_{n_3}^{n_3'}V_{m_3n_3}\left[\bar{U}\left(\frac{\lambda_{m_3}}{l}x\right)-\frac{\bar{U}(\lambda_{m_3})}{\bar{V}(\lambda_{m_3})}\bar{V}\left(\frac{\lambda_{m_3}}{l}x\right)\right]\sin n_3\theta\sin\omega t+v', \tag{8}$$

$$w = \sum_{m_3}^{m_3'} \sum_{n_3}^{n_3'} W_{m_3 n_3} \left[\bar{U}\left(\frac{\lambda_{m_3}}{l} x\right) - \frac{\bar{U}(\lambda_{m_3})}{\bar{V}(\lambda_{m_3})} V\left(\frac{\lambda_{m_3}}{l} x\right) \right] \cos n_3 \theta \sin \omega t + w', \tag{9}$$

$$\psi = \sum_{m}^{m'} \sum_{n}^{n'} \psi_{mn} J_n \left(\frac{m\pi r}{l}\right) \cosh \left(\frac{m\pi r}{l}\right) \cos n\theta \sin \omega t + \psi', \tag{10}$$

which satisfy the fixed supported boundary condition, where

$$\bar{U}(\eta) = \frac{1}{2}(\cosh \eta - \cos \eta); \quad \bar{V}(\eta) = \frac{1}{2}(\sinh \eta - \sin \eta).$$

$\lambda_{m_1 n_1}$ is m_1-th root of the following equation:

$$J_{n_1}(\lambda)[I_{n_1-1}(\lambda) + I_{n_1+1}(\lambda)] - I_{n_1}(\lambda)[J_{n_1-1}(\lambda) - J_{n_1+1}(\lambda)] = 0;$$

λ_{m_3} is m_3-th root of the following equation:

$$1 - \cosh \lambda \cos \lambda = 0.$$

Here $J_{n_1}(\lambda)$, $I_{n_1}(\lambda)$ are Bessel functions of the first kind and the Bessel function with imaginary argument, respectively. u_1', u_2', u', w', and ψ' are the odd terms of the functions u_1, u_2, u, w, and ψ, and v' is the even term of the function V. Calculation shows that the effect of these terms on the result is very small, so that only the first term is taken. It is convenient for calculation to use m_1', m_2', m_3', m', n_1', n_2', n_3', n', k. Substituting Eqs. (5), (6), (7), (8), (9), and (10) in Eq. (4), we obtain the following equation by the variational method:

$$\begin{bmatrix} A^u & A^{uv} & A^{uw} & & & \\ A^{vu} & A^v & A^{vw} & & & \\ A^{wu} & A^{wv} & A^w & & & \\ & & & A^{u_1} & & \\ & & & & A^{u_2} & \\ & & & & & 0 \end{bmatrix} \begin{bmatrix} \tilde{u}' \\ \tilde{v}' \\ \tilde{w}' \\ \tilde{u}_1' \\ \tilde{u}_2' \\ \tilde{\psi}' \end{bmatrix} - \omega^2 \begin{bmatrix} B^u & & & & & B^{u\Phi} \\ & B^v & & & & B^{v\Phi} \\ & & B^w & & & B^{w\Phi} \\ & & & B^{u_1} & & B^{u_1\Phi} \\ & & & & B^{u_2} & B^{u_2\Phi} \\ B^{u\Phi} & B^{\Phi v} & B^{\Phi w} & B^{\Phi u_1} & B^{\Phi u_2} & B^{\Phi} \end{bmatrix} \begin{bmatrix} \tilde{u}' \\ \tilde{v}' \\ \tilde{w}' \\ \tilde{u}_1' \\ \tilde{u}_2' \\ \tilde{\psi}' \end{bmatrix} = 0, \tag{11}$$

where $\tilde{u}'$, $\tilde{v}'$, $\tilde{w}'$, $\tilde{u}_1'$, $\tilde{u}_2'$, and $\tilde{\psi}'$ are column matrices with K elements. A^u, A^{uv}, A^{uw}, A^v, A^{vw}, A^{u_1}, A^{u_2} and B^u, B^v, B^w, B^{u_1}, B^{u_2}, B^{Φ}, $B^{u\Phi}$, $B^{v\Phi}$, $B^{w\Phi}$, $B^{u_1\Phi}$, $B^{u_2\Phi}$, $B^{\Phi u_1}$, $B^{\Phi u_2}$, $B^{\Phi w}$ are submatrices with $K^2 \times K^2$ elements.

By using the symmetry of the boundary condition, Eq. (11) can be simplified as follows:

$$
\begin{bmatrix}
A^{u} & A^{uv} & A^{uw} & & \\
A^{vu} & A^{v} & A^{vw} & & \\
A^{wu} & A^{wv} & A^{w} & & \\
 & & & A^{u_1} & \\
 & & & & A^{u_2}
\end{bmatrix}
\begin{bmatrix}
\tilde{u}' \\ \tilde{v}' \\ \tilde{w}' \\ \tilde{u}_1 \\ \tilde{u}_2
\end{bmatrix}
- \omega^2
\begin{bmatrix}
B^{u} - B^{u\Phi} B^{\Phi-1} B^{\Phi u} & -B^{u\Phi} B^{\Phi-1} B^{\Phi v} & -B^{u\Phi} B^{\Phi-1} B^{\Phi w} & -B^{u\Phi} B^{\Phi-1} B^{\Phi u_1} & -B^{u\Phi} B^{\Phi-1} B^{\Phi u_2} \\
-B^{v\Phi} B^{\Phi-1} B^{\Phi u} & B^{v} - B^{v\Phi} B^{\Phi-1} B^{\Phi v} & -B^{v\Phi} B^{\Phi-1} B^{\Phi w} & -B^{v\Phi} B^{\Phi-1} B^{\Phi u_1} & -B^{v\Phi} B^{\Phi-1} B^{\Phi u_2} \\
-B^{w\Phi} B^{\Phi-1} B^{\Phi u} & -B^{w\Phi} B^{\Phi-1} B^{\Phi v} & B^{w} - B^{w\Phi} B^{\Phi-1} B^{\Phi w} & -B^{w\Phi} B^{\Phi-1} B^{\Phi u_1} & -B^{w\Phi} B^{\Phi-1} B^{\Phi u_2} \\
-B^{u_1\Phi} B^{\Phi-1} B^{\Phi u} & -B^{u_1\Phi} B^{\Phi-1} B^{\Phi v} & -B^{u_1\Phi} B^{\Phi-1} B^{\Phi w} & B^{u_1} - B^{u_1\Phi} B^{\Phi-1} B^{\Phi u_1} & -B^{u_1\Phi} B^{\Phi-1} B^{\Phi u_2} \\
-B^{u_2\Phi} B^{\Phi-1} B^{\Phi u} & -B^{u_2\Phi} B^{\Phi-1} B^{\Phi v} & -B^{u_2\Phi} B^{\Phi-1} B^{\Phi w} & -B^{u_2\Phi} B^{\Phi-1} B^{\Phi u_1} & B^{u_2} - B^{u_2\Phi} B^{\Phi-1} B^{\Phi u_2}
\end{bmatrix}
\begin{bmatrix}
\tilde{u}' \\ \tilde{v}' \\ \tilde{w}' \\ \tilde{u}_1' \\ \tilde{u}_2'
\end{bmatrix}
= 0. \tag{12}
$$

This is a characteristic equation for the coupled system, in which a mass matrix of fluid is added. It also shows that the elements of this matrix not only depend on the bulk of fluid, but also on the shape of the interface of solid with fluid, and on the free surface of fluid.

Equation (12) is rewritten in the following form:

$$
Ax - \omega^2 Bx = 0, \tag{13}
$$

where B = B' + B",

$$
B' = \begin{bmatrix}
B^{u} & & & & \\
 & B^{v} & & & \\
 & & B^{w} & & \\
 & & & B^{u_1} & \\
 & & & & B^{u_2}
\end{bmatrix};
$$

$$
B'' = \begin{bmatrix}
B^{u\Phi} B^{\Phi-1} B^{\Phi u} & B^{u\Phi} B^{\Phi-1} B^{\Phi v} & B^{u\Phi} B^{\Phi-1} B^{\Phi w} & B^{u\Phi} B^{\Phi-1} B^{\Phi u_1} & B^{u\Phi} B^{\Phi-1} B^{\Phi u_2} \\
B^{v\Phi} B^{\Phi-1} B^{\Phi u} & B^{v\Phi} B^{\Phi-1} B^{\Phi v} & B^{v\Phi} B^{\Phi-1} B^{\Phi w} & B^{v\Phi} B^{\Phi-1} B^{\Phi u_1} & B^{v\Phi} B^{\Phi-1} B^{\Phi u_2} \\
B^{w\Phi} B^{\Phi-1} B^{\Phi u} & B^{w\Phi} B^{\Phi-1} B^{\Phi v} & B^{w\Phi} B^{\Phi-1} B^{\Phi w} & B^{w\Phi} B^{\Phi-1} B^{\Phi u_1} & B^{w\Phi} B^{\Phi-1} B^{\Phi u_2} \\
B^{u_1\Phi} B^{\Phi-1} B^{\Phi u} & B^{u_1\Phi} B^{\Phi-1} B^{\Phi v} & B^{u_1\Phi} B^{\Phi-1} B^{\Phi w} & B^{u_1\Phi} B^{\Phi-1} B^{\Phi u_1} & B^{u_1\Phi} B^{\Phi-1} B^{\Phi u_2} \\
B^{u_2\Phi} B^{\Phi-1} B^{\Phi u} & B^{u_2\Phi} B^{\Phi-1} B^{\Phi v} & B^{u_2\Phi} B^{\Phi-1} B^{\Phi w} & B^{u_2\Phi} B^{\Phi-1} B^{\Phi u_1} & B^{u_2\Phi} B^{\Phi-1} B^{\Phi u_2}
\end{bmatrix}.
$$

Here B' is the mass matrix of the container without fluid, and B" is the matrix of the additional coupled mass, which reflects the changing part of the frequency and mode in the coupled system. We can calculate the natural frequencies of this system by the previous equation.

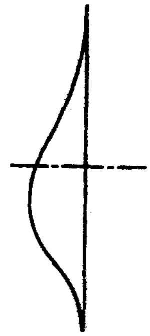

Fig. 2. Mode of the plate.

4. Calculation of the Response of the Forced Vibration

The forced vibration of the container filled with fluid is investigated. This container was excited by a harmonic displacement $\bar{\bar{u}} = \bar{\bar{u}}' \sin pt$ in the axial direction. Then the generalized variational functionals of this system are

$$\pi_1' = \pi_1 - \int_{t_1}^{t_2}\iint_{\sigma_1} \rho_1 h_1 u_1 \frac{\partial^2 u}{\partial t^2} R_0 dr d\theta dt; \tag{14}$$

$$\pi_2' = \pi_2 - \int_{t_1}^{t_2}\iint_{\sigma_2} \rho_2 h_2 u_2 \frac{\partial^2 u}{\partial t^2} R_0 dr d\theta dt; \tag{15}$$

$$\pi_3' = \pi_3 - \int_{t_1}^{t_2}\iint_{\sigma_3} \rho_3 h_3 u \frac{\partial^2 u}{\partial t^2} R_0^2 d\xi d\theta dt; \tag{16}$$

$$\pi_4' = \pi_4 - \int_{t_1}^{t_2}\iiint_{\Omega} \frac{\partial \Phi}{\partial x} \frac{\partial^2 u}{\partial t^2} dx dy dz dt. \tag{17}$$

The total functional is

$$\pi' = \pi_1' + \pi_2' + \pi_3' + \pi_4'. \tag{18}$$

Substituting Eqs. (5), (6), (7), (8), (9), and (10) in Eq. (18), the following equation was obtained by variation:

$$\begin{bmatrix} A^u & A^{uv} & A^{uw} & & & \\ A^{vu} & A^v & A^{vw} & & & \\ A^{wu} & A^{wv} & A^w & & & \\ & & & A^{u_1} & & \\ & & & & A^{u_2} & \\ & & & & & 0 \end{bmatrix} \begin{bmatrix} \tilde{u}' \\ \tilde{v}' \\ \tilde{w}' \\ \tilde{u}_1' \\ \tilde{u}_2' \\ \tilde{\psi}' \end{bmatrix} - \omega^2 \begin{bmatrix} B^u & & & & & B^{u\Phi} \\ & B^v & & & & B^{v\Phi} \\ & & B^w & & & B^{w\Phi} \\ & & & B^{u_1} & & B^{u_1\Phi} \\ & & & & B^{u_2} & B^{u_2\Phi} \\ B^{u\Phi} & B^{\Phi v} & B^{\Phi w} & B^{\Phi u_1} & B^{\Phi u_2} & B^{\Phi} \end{bmatrix} \begin{bmatrix} \tilde{u}' \\ \tilde{v}' \\ \tilde{w}' \\ \tilde{u}_1' \\ \tilde{u}_2' \\ \tilde{\psi}' \end{bmatrix} = \begin{bmatrix} \tilde{Q}_u' \\ \tilde{Q}_v' \\ \tilde{Q}_w' \\ \tilde{Q}_{u_1}' \\ \tilde{Q}_{u_2}' \\ \tilde{Q}_\psi' \end{bmatrix} \tag{19}$$

TABLE 1. Value of Natural Frequencies in Hz

Result	Frequency	
	f_1	f_2
Calculated	48.33	68.66
Experimental	49	70

This equation is simplified in the following form through repeated calculation of the matrix:

$$A\tilde{x} - \omega^2\tilde{x} = \tilde{Q}, \tag{20}$$

where

$$\tilde{Q} = \tilde{Q}' + \tilde{Q}''; \ \tilde{Q}' = [\tilde{Q}_{u_1} \ \tilde{Q}'_v \ \tilde{Q}'_w \ \tilde{Q}'_{u_1} \ \tilde{Q}'_{u_2}]^T;$$

$$\tilde{Q}'' = [B^{u\Phi}B^{\Phi-1}\tilde{Q}'_\psi \quad B^{v\Phi}B^{\Phi-1}\tilde{Q}'_\psi \quad B^{w\Phi}B^{\Phi-1}\tilde{Q}'_\psi \quad B^{u_1\Phi}B^{\Phi-1}\tilde{Q}'_\psi \quad B^{u_2\Phi}B^{\Phi-1}\tilde{Q}'_\psi]^T.$$

A numerical algorithm was implemented for the solution of Eqs. (13) and (20) in this work. By using this algorithm program the frequency and response of the container partially filled with fluid can be calculated.

5. Numerical Results and Discussion

The container is made of aluminum with length L = 400 mm, radius R_0 = 150 mm, and thickness of the wall $h_1 = h_2 = h_3$ = 1.8 mm. The container is filled with one-half bulk of fluid. In this case the calculation was implemented by using the present procedure. The calculated results of the first and second frequencies are given in Table 1, in which experimental results are also given [8]. It should be noted that this result is in satisfactory agreement with the experiments. It can be concluded that the analytical method suggested by this paper is correct but the computer time required is less than that with the finite-element method.

These frequencies, which are not those of sloshing of the fluid, are those of the plate of the container. The mode of the plate along its diameter shown in Fig. 2 corresponds to a frequency of 48.33 Hz. It can be predicted that its peak will appear in the lower half of the plate because of the presence of the fluid.

NOMENCLATURE

C_i – the circumference of the plate, cm; $D_i = E_i h_i^3/12(1 - \nu_i^2)$ – bending stiffness of the plate, kg·cm; E_i – the modulus of elasticity, kg/cm²; h_i – the plate thickness, cm; h_3 – the shell thickness, cm; R_0 – the radius of the middle surface of the shell, cm; Q' – the force term of the forced vibration of the solution without fluid; Q" – the coupled force term of the forced vibration of the solution with fluid; U_i – displacement of the plate in x coordinate, cm; U, V, W and $\bar{U}$, $\bar{V}$, $\bar{W}$ – the axial, circumferential, and radial components of the displacement of a point on the middle surface of the shell and their values at the boundary, cm; ν_i – the Poisson's ratio; ρ_i – the density of material of the plate, kg/cm³; ρ_3 – the density of material of the shell, kg/cm³; ρ_i' – the density of liquid, kg/cm³; σ – the free surface of fluid, cm²; σ_i – the area of the plate, cm²; σ_i' – the interface of the plate with liquid, cm²; σ_3' – the interface of the shell with liquid, cm²; $\Phi = \partial\psi/\partial t$ – the velocity potential of the fluid, 1/sec; ψ – the displacement potential of liquid; Ω – the volume of the fluid, cm³; $\delta_0 = h_3^2/12R_0^2$.

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