Vol. 33 No. 8 SCIENCE IN CHINA (Series A) August 1990

EXCITATION MECHANISM OF THERMOCAPILLARY OSCILLATORY CONVECTION

HU WEN-RUI (胡文瑞) AND TANG ZE-MEI (唐泽眉)
(*Institute of Mechanics, Chinese Academy of Sciences, Beijing* 100080, *PRC*)

Received April 6, 1989; revised July 24, 1989.

ABSTRACT

It is suggested that the oscillation of thermocapillary convection may be excited by the buoyancy instability. By means of numerical simulation of the finite-element method, the temperature distributions in the liquid bridge are qualitatively analyzed. The temperature gradient in a certain flow region of liquid bridge may turn to be parallel to the direction of gravity when the temperature difference ΔT between two boundary rods of liquid bridge is larger than the critical value. The buoyancy instability may be excited, and then the thermocapillary oscillatory convection appears, as the temperature difference increases further. The distribution of the critical Marangoni number in the micro-gravity environment is derived from the data on the ground experiments. The results show that the onset of thermocapillary oscillatory convection is delayed in the case of smaller typical scale of liquid bridge and lower gravity environment.

Keywords: thermocapillary convection, micro-gravity fluid mechanics, oscillatory flow.

I. INTRODUCTION

In the low- or micro-gravity environment the convection driven by the buoyancy almost disappears, consequently the oscillation in the convection no longer exists. This provides an ideal condition for space materials processing. The convection driven by surface tension becomes an important process in the model of liquid bridge which is used to simulate the crystal growth in the floating zone of melts[1]. The temperature oscillation may be excited in the convection driven by the surface tension under certain conditions[2]. The onset of oscillation will certainly influence the formation processes of space materials. Therefore, to study and reveal the reason for oscillation in thermocapillary convection becomes an important topic.

When estimating the relative relation between buoyancy and surface tension, the dynamical Bond number is usually introduced as

$$B_d = \frac{\rho g \beta l^2}{|\partial\sigma/\partial T|}, \tag{1.1}$$

where ρ, T and σ are, respectively, the density, temperature and surface tension of the fluid, β the thermal expansion coefficient, g the gravity, and l the typical scale of the fluid. The buoyancy may be neglected in comparison with the surface tension in the case of a smaller Bond number. Therefore, the ground experiments of smaller typical scale (smaller l and $g = g_0$) satisfying $B_d \ll 1$ may be designed to simulate the micro-gravity experiments in space (large l and $g \ll g_0$), where g_0 is the gravitational acceleration on the earth surface. Experiments of ground and the space thermocapillary convection are alike in many aspects, however, the onset mechanisms of their oscillatory convection are obviously different.

The thermocapillary oscillatory convection with a small Bond number has been studied through ground experiments. In the experiments the oscillatory temperature was measured by thermocouple[2-10]. The Marangoni number is introduced as

$$Ma = -\frac{\left(\frac{d\sigma}{dT}\right)\Delta T l}{\rho \nu \kappa}, \tag{1.2}$$

where ν and κ are, respectively, the kinematic viscous coefficient and thermal diffusion coefficient, ΔT is the temperature difference between the higher temperature at the upper rod and the lower temperature at the lower rod. The oscillation will appear in the convection region when Ma is larger than the critical value $(Ma)_c$. Relatively large-scale experiments have been performed with the micro-gravity sounding rocket. The critical value $(Ma)_c$ of the rocket experiment on the silicon oil is one order of magnitude larger than that of the ground experiment. However, the former is a bit smaller than the latter when using the $NaNO_3$ medium[11,12]. The different results may be due to the micro-gravity environment during the experiments. The critical value of $(Ma)_c$ in space shuttle is larger than that in the sounding rocket[13]. Some Italian scientists showed the relation of $(Ma)_c$ value to the typical scale, and argued that the Marangoni number does not seem the best parameter for measuring the onset of oscillation[14]. Obviously, it is far from perfect to describe the oscillatory convection by using only one parameter Ma. But what are the parameters that play the essential role among so many dynamical and geometrical similarity parameters? That is the question to be studied.

For lack of sufficient experimental data, especially, the data about the variation features of oscillatory flow field, little has been revealed theoretically on oscillatory convection. One idea closely related to the ground experiments holds that the oscillation of thermocapillary convection results from the azimuthal instability at the free surface[3-6]. This mechanism needs to be analyzed theoretically. Another argument suggests that the oscillation is produced by the non-equilibrium of the heat-transfer processes[15], and the related experiment is expected to be completed in 1992 on a space shuttle. Our model suggests that the buoyancy instability may play an important role on the onset of oscillation[16,17].

The present paper analyzes in detail the mechanism of buoyancy instability. The numerical simulation is carried out by using our program of the finite-element method; the reverse temperature gradient in the flow field and the influence of buoyancy instability are discussed qualitatively.

II. Liquid Bridge Model

The experimental liquid medium is assumed to be bounded by two coaxial rods with the same diameter $2r_0$. The temperatures ($T_0 + \Delta T$ and T_0) at the upper and lower rods keep constant. The convection is driven by the gradient of the surface tension at the free surface of the liquid bridge due to the non-uniformity of temperature distribution. Usually, the Boussniesq approximation is used to describe the liquid behavior. For the axisymmetric liquid bridge, the equations at the cylindrical coordinate may be expressed as

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{2.1}$$

$$u\frac{\partial u}{\partial r} - w\frac{\partial u}{\partial z} = -\frac{\partial}{\partial r}\left(\frac{p}{\rho}\right) + \nu\left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2}\right), \tag{2.2}$$

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{\partial}{\partial z}\left(\frac{p}{\rho} + gz\right) + \nu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right) + g\beta(T - T_0), \tag{2.3}$$

$$u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \kappa\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right), \tag{2.4}$$

where p is the pressure of fluid, $(u, 0, w)$ is velocity, $\nu = \mu/\rho$ and $\kappa = k/\rho c_p$ are, respectively, the kinematic viscous and thermal diffusion coefficients. The boundary conditions for the floating zone of the liquid bridge may be written as

$$u(r, 0) = w(r, 0) = 0, \tag{2.5}$$

$$u(r, l) = w(r, l) = 0, \tag{2.6}$$

$$u(0, z) = 0, \quad \frac{\partial w(0, z)}{\partial r} = 0, \tag{2.7}$$

$$2\mu\left(\frac{\partial u}{\partial r} - \frac{\partial w}{\partial z}\right)r_b' + \mu\left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}\right)(1 - r_b'^2) = N^{1/2}\sigma_T'\left(\frac{\partial T}{\partial r}r_b' + \frac{\partial T}{\partial z}\right), \quad r = r_b(z), \tag{2.8}$$

$$p_1 - p_2 - \sigma\left(\frac{1}{r_b N^{1/2}} - \frac{r_b''}{N^{3/2}}\right) = \frac{2u}{N}\left[\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z}r_b'^2 - \left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z}\right)r_b'\right], \quad r = r_b(z). \tag{2.9}$$

$$T(r, 0) = T_0 \text{ (const.)}, \quad T(r, l) = T_0 + \Delta T \text{ (const.)}, \tag{2.10}$$

$$\frac{\partial T(0, z)}{\partial r} = 0, \tag{2.11}$$

$$k\frac{\partial T(r, z)}{\partial n} = -\varepsilon c(T^4 - T_e^4), \quad \text{when } r = r_b(z), \tag{2.12}$$

where $r = r_b(z)$ is the free surface equation of the liquid bridge, l the height of the liquid bride, $\sigma(T)$, ε and c are, respectively, the surface tension, radiation coefficient and the Stefen-Boltzmann constant, the superscript "′" denotes the differential, the subscript e denotes the external field, and $N = 1 + r_b'^2$. It can be shown that the convection in the liquid bridge is a nonlinear problem with undetermined free surface boundary.

To manifest the physical process of convection, we have to introduce the non-dimensional quantities as follows:

$$\xi = \frac{r}{l}, \ \zeta = \frac{z}{l}, \ \bar{U} = \frac{u}{w_0}, \ \bar{w} = \frac{w}{w_0}, \tag{2.13}$$

$$P = \frac{p}{\rho w_0^2}, \ \Theta = \frac{T - T_0}{\Delta T}, \ \lambda = \frac{2r_0}{l},$$

$$B_0 = \frac{\rho g l^2 \beta \Delta T}{\sigma}, \quad C_a = \frac{\sigma' T \Delta T}{\sigma}, \ G_r = \frac{g\beta\Delta T l^3}{\nu^2}, \ P_r = \nu / x \tag{2.14}$$

$$Re = \frac{w_0 l}{\nu}, \quad Ma = P_r \cdot Re = \frac{w_0 l}{x},$$

where the typical velocity $w_0 = \lambda \sigma'_T \Delta T / \mu$ is the character velocity driven by the gradient of surface tension in the convective liquid. The non-dimensional quantity B_0 defined in (2.14) is the static Bond number; Ca, Gr, Re, Ma and Pr are, respectively, the thermocapillary number, the Grashof number, and the Reynold number of the surface tension typical velocity, the Marangoni number and the Prandtl number. The convection driven by surface tension is influenced by many factors, such as, the dynamical influence of gravity, surface tension, viscous force and pressure gradient. Heat transfer and radiation are also essential for the thermodynamical processes, in addition to the complex boundary conditions. All these factors, when combined together, cause ambiguity in the key point.

There are four variables u, w, p, and T in Eqs.(2.1)—(2.4). Using continuity equation (2.1), we introduce the streaming function

$$\bar{w} = \frac{1}{\xi} \frac{\partial(\xi\phi)}{\partial\xi}, \quad \bar{U} = -\frac{\partial\phi}{\partial\zeta}. \tag{2.15}$$

Eliminating pressure p from Eqs. (2.2) and (2.3) and introducing vorticity

$$\omega = \frac{\partial \bar{U}}{\partial\zeta} - \frac{\partial\bar{w}}{\partial\xi}, \tag{2.16}$$

we obtain the non-dimensional form of Eqs. (2.1)—(2.4)

$$\frac{\partial^2\phi}{\partial\xi^2} + \frac{1}{\xi}\frac{\partial\phi}{\partial\xi} - \frac{\phi}{\xi^2} + \frac{\partial^2\phi}{\partial\zeta^2} = -\omega, \tag{2.17}$$

$$R_e\left[\frac{1}{\xi}\frac{\partial(\xi\phi)}{\partial\xi}\frac{\partial\omega}{\partial\zeta} - \frac{\partial\phi}{\partial\zeta}\frac{\partial\omega}{\partial\xi} + \frac{1}{\xi}\frac{\partial\phi}{\partial\zeta}\omega\right]$$

$$= \frac{\partial^2\omega}{\partial\zeta^2} + \frac{\partial^2\omega}{\partial\xi^2} + \frac{1}{\xi}\frac{\partial\omega}{\partial\xi} - \frac{\omega}{\xi^2} - \frac{G_r}{R_e}\frac{\partial\Theta}{\partial\xi}, \tag{2.18}$$

$$M_a\left[-\frac{\partial\phi}{\partial\zeta}\frac{\partial\Theta}{\partial\xi} + \frac{1}{\xi}\frac{\partial(\xi\phi)}{\partial\xi}\frac{\partial\Theta}{\partial\zeta}\right] = \frac{\partial^2\Theta}{\partial\xi^2} + \frac{1}{\xi}\frac{\partial\Theta}{\partial\xi} + \frac{\partial^2\Theta}{\partial\zeta^2}, \tag{2.19}$$

where $\lambda = 1$ is assumed. We can obtain correspondingly the non-dimensional forms of the boundary conditions from(2.5)—(2.12), where the non-dimensional radiation number is taken as $Rd = \varepsilon c l \Delta T^3 / k$.

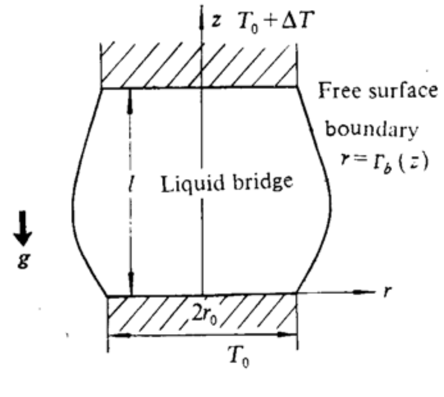

Fig. 1. Schematic model of liquid bridge, gravity is in the same direction as $-z$.

III. Numerical Simulation

The finite-element method was used to simulate numerically the flow field and temperature distribution induced by the temperature difference between two boundary rods in the liquid bridge. The fluid motion in the liquid satisfied the equations of vorticity, stream function and energy, and also the boundary conditions including the stress equilibrium at the free surface. The Prandtl number determined by the liquid property was taken as 20. The upper and lower boundaries were considered as the plane of solid. Two boundary conditions for streaming function and vorticity were that there should be neither penetration nor slip at the solid rods. The free surface was dealt with as a cylindrical surface, and the wetting angle was 90°. It could be seen that the boundary of liquid and solid, the free surface and the symmetric axis are at streaming line. The static flow field, the linear temperature distribution in the z-direction in the liquid bridge and temperature difference ΔT were adopted as the initial condi-

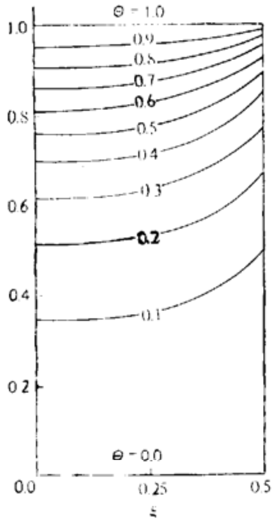

Fig. 2. Temperature distribution in the liquid bridge for $Ma = 100$.

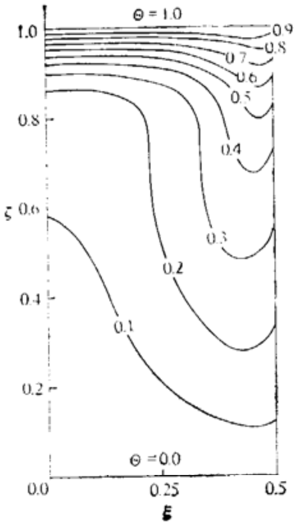

Fig. 3. Temperature distribution in the liquid bridge for $Ma = 1000$.

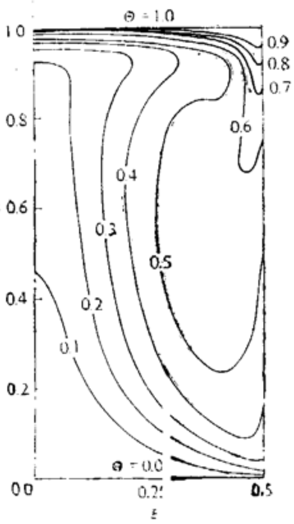

Fig. 4. Temperature distribution in the liquid bridge for $Ma = 5000$.

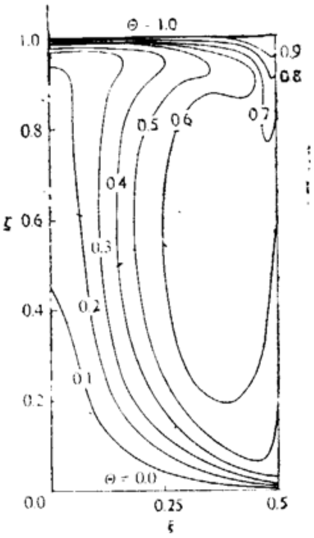

Fig. 5. Temperature distribution in the liquid bridge for $Ma = 10000$.

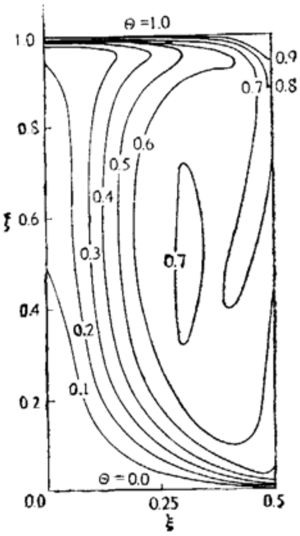

Fig. 6. Temperature distribution in the liquid bridge for $Ma = 15000$.

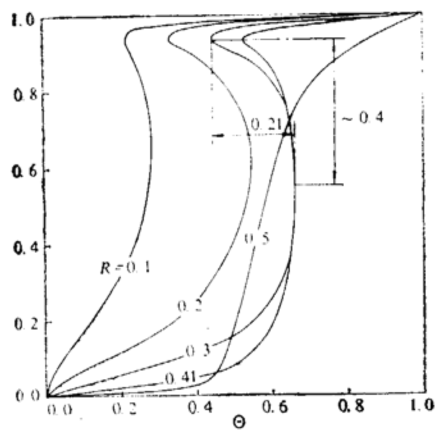

Fig. 7. Temperature profiles in the liquid bridge at different radii for $Ma = 10000$.

tions. Therefore, the vorticity and streaming function were zeros at the beginning. Solving the equations of streaming function, vorticity and energy with iterative processes, we obtained the temperature distributions (Fig. 2—6).

The whole fluid region was divided into 360 triangular elements of different scales, the larger the variation in temperature and flow, the denser the element nets. The linear interpolation functions were used in calculations.

As seen from Figs. 2—6, the temperature difference between both rods increases gradually. The gradient of surface tension, the dynamical viscous coefficient and the

Prandtl number keep invariable in one experiment. Therefore, the Reynolds number based on the typical velocity of surface tension $R_e = |\partial\sigma/\partial T|\Delta Tl/\mu\nu$ increased linearly with temperature difference ΔT. The Marangoni number $Ma=Re \cdot Pr=\left|\frac{\partial\sigma}{\partial T}\right|$ $\Delta Tl/\mu\kappa$ is the ratio of thermal convective effect to the diffusive effect induced by the surface tension-driven convection;it also increases linearly with the increasing ΔT. In Fig.2 the distribution of iso-thermal counters is almost parallel to the horizontal direction which shows the dominant influence of diffusion effect, because the ΔT is smaller and the convection effect is weaker. The surface tension decreases with increasing temperature. The fluid is driven to flow from the upper to the lower region at the free surface due to the high temperature at the upper rod, and then returns from the lower to the upper region near the symmetric axis. The heat is transferred by the fluid convection, and the isotherms turn upward near the symmetric axis, and downward near the free surface. The effect of convection increased with the increasing temperature difference. Therefore, the temperature gradients became larger both in the upper region near the symmetric axis and in the lower region near the free surface, and the thermal boundary layer became thinner as shown in Figs. 5 and 6. There is an internal region in the center with relatively uniform temperature, the temperature gradient in the region being parallel to the direction of gravity. Fig. 7 indicates the temperature distributions at different heights and radii for $Ma = 1 \times 10^4$, showing quantitatively the values and ranges of the reversed temperature difference in the fluid region. Our calculation results about the distributions of temperature and flow field agree qualitatively with experimental[1,6].

IV. Oscillatory Convection

The variation in temperature and velocity distributions increase in the liquid bridge as the applied temperature difference ΔT increases gradually, making the Marangoni boundary layer thinner near the free surface and the temperature gradient higher near the solid boundaries. The temperature oscillation appears at the liquid bridge as ΔT approaches to a critical value. The oscillatory convection as a part of transition process from laminar to turbulence has aroused much theoretical interest. In material processing, the temperature oscillation will influence the growth process at the front surface. The study on how to avoid oscillation is meaningful. So the thermocapillary oscillatory convection is an important subject of micro-gravity fluid physics.

Generally, in the research of thermocapillary oscillatory convection attentions are focused on the relation between the critical Marangoni number and the height l or the relation between the ratio of height to radius of the liquid bridge. As pointed out by many authors, $(Ma)_c$ seems not to be the ideal parameter to describe the onset of oscillation. The basic equations suggest the Marangoni number, with the typical velocity w_0 adopted as the thermocapillary flow velocity of the viscous type, in the energy equation (2.19) should be important for measuring the relation between convection and conduction. There are three typical flow regions for thermocapillary convection: the Marangoni boundary layer near the free surface, the boundary layer near the upper and lower rods; and the internal flow region. The flow and thermal processes in these regions are coupled with one another. The onset of oscillation depends on the coupling processes.

The temperature exhibits a linear distribution when the applied temperature difference ΔT is small. More heat will be transferred to the liquid bridge as ΔT is increased, which makes the boundary layer near the solid rods grow thinner and the temperature gradient larger. The temperature profile at a certain radius in the flow field exhibits a turning point when ΔT is increased to a certain value. And it comes to be S-shaped when ΔT further increases. The temperature distributions at different radii for $Ma=1\times10^4$ are given in Fig. 7. Reversed temperature difference is roughly $\delta\Theta \approx 0.2$ within the typical scale $\Delta \approx 0.4$ at radius $r = 0.3$. Both the values of Δ and $\delta\Theta$ become larger in the case of a larger Ma. These results implies that a region, with the temperature gradient parallel to the gravity, appeared in the liquid bridge when ΔT was larger than a certain value, and the reversed temperature difference and the scale of the region would be correspondingly increased when ΔT further increased. In this case, the buoyancy gets larger and larger and finally induces the buoyancy instability. The thermocapillary oscillatory convection might be caused by the buoyancy instability in the liquid bridge. But it is the driven force of the convection that mainly causes the gradient of the surface tension.

According to definition of non-dimensional parameter (2.14), the Marangoni number may be rewritten as

$$Ma = \frac{Ra}{B_0C_a}\frac{1}{\Delta^3\delta\Theta}, \tag{4.1}$$

where the Rayleigh number is

$$Ra = \frac{g\beta\delta T(\delta l)^3}{\kappa\nu}, \tag{4.2}$$

and δT and δl are, respectively, the dimensional quantities of $\delta\Theta$ and Δ. The quantity $Bo/Ca = \rho g\beta l/|\sigma_T'|$ is also named the Bond number. Hence, the critical Marangoni number $(Ma)_c$ is propotional to $(Ra)/\Delta^3\delta\Theta$, which is a complex function of ΔT. It could be seen that $\delta\Theta$ and Δ have an order of magnitude $O(1)$ for relatively large ΔT. Therefore, qualitatively speaking, the buoyancy instability may be induced if the Rayleigh number is larger than the critical value, from which, the corresponding critical Marangoni number could be derived. From ground experiments, the critical Marangoni number is

$$(Ma)_{c0} = (1-3)\times10^4, \tag{4.3}$$

and the corresponding dynamic Bond number is

$$BoCa = O(1). \tag{4.4}$$

Therefore, the critical Rayleigh number can be estimated as

$$(Ra)_c \approx \Delta^3\delta\Theta\times(1-3)\times10^4.$$

We have, then,

$$(Ra)_{c0} = (1-3)\times10^3, \tag{4.5}$$

using $\Delta^3\delta\Theta = O(10^{-1})$. This value of the critical Rayleigh number is reasonable. The critical Marangoni number in the micro-gravity environment may be expressed as

$$\frac{(Ma)_c}{(Ma)_{c0}} = \frac{g_0 l_0^2}{g l^2}, \tag{4.6}$$

where the subscript 0 denotes the typical value of the ground experiment. The distributions of critical Marangoni number under different conditions are given in Fig. 8. The smaller typical scale and lower gravitational environment with a larger critical Marangoni number postpone the appearance of oscillatory convection.

These results imply that, the material processes in the space environment enjoy large-ranged stable convection in the liquid bridge and might produce ideal large-sized crystals. However, in the case of reducing gravity, the phase change process at the interface of solid and liquid, and the variations in the value and direction of acceleration in the space environment influence the flow field and heat transfer in the liquid bridge, as well as the critical value of the instability convection.

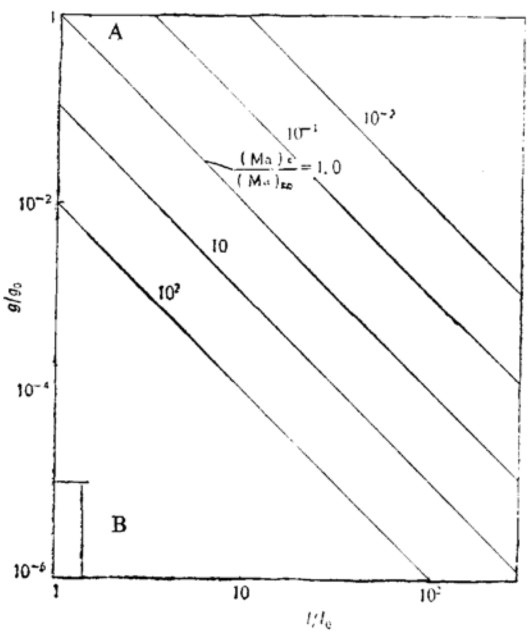

Fig. 8. The distribution of critical Marangoni number.
A, Ground experiment; B, space platform.

V. Discussions

The influence of convection on the space material processes is an important factor. The onset of oscillation is much concerned, because it changes the uniformity of solidification at the phase-change interface. In the present paper, the distributions of flow field and temperature field in the liquid bridge are calculated with the numerical simulation method, and the influence of buoyancy instability in the micro-gravity environment is analyzed[16,17]. The results show that, we have to reduce the level of micro-gravity and limit the scale of floating zone to avoid the oscillation even in the experiment of spacecraft. This conclusion has practical value for the preparation of space materials.

To verify the mechanism of buoyancy instability, it is necessary to compare the results in the small-scale ground experiments with those of the space experiments. In addition, the unsteady processes should be simulated numerically to study the transition process from laminar flow to turbulence. These studies are in progress.

Obviously, the mechanism of thermocapillary convection of oscillation is complex, for it depends on the range of typical non-dimensional parameters. Many studies show that different ranges of typical parameters may incur different mechanisms for the onset of oscillation. The buoyancy discussed in the present paper which was operated under certain conditions, may also induce oscillatory convection, thus showing that the oscillatory mechanism of thermocapillary convection awaits further studies.

References

[1] Ostrach, S., *Ann. Rev. Fluid Mech.*, **14**(1982), 313.
[2] Chun, Ch.-H. & Wuest, W., *Acta Astronautica*, **6**(1979), 1073.
[3] Chun, Ch.-H., *ibid.*, **7**(1980) 479.
[4] Chun, Ch.-H. & Wuest, W., *J. Crystal Growth.*, **48**(1980), 6.
[5] Schwabe, D. & Scharmann, A., *ibid.*, **46**(1979), 125.
[6] Preisser, F., Schwabe, D. & Scharmann, A., *J. Fluid Mech.*, **126**(1983), 545.
[7] Komotani, Y., Ostrach, S. & Vargas, M., *J. Crystal Growth.*, **66**(1984), 83.
[8] Monti, R., Napolitano, L. G. & Russo, G., *4th European Symp. on Material Science in Space*, ESA sp-101, 1983, 219.
[9] Monti, R., Napolitano. L. G. & Russo, G., *Acta Astronautica*, **11**(1984), 369.
[10] Monti, R., *5th European Symp. on Material Science under Microgravity*, ESA sp-222, 1985.
[11] Schwabe, D., Preisser, F. & Scharmann, A., *Acta Astronautica*, **9**(1982), 265.
[12] Schwabe, D. & Scharmann, A., *Advance Space Res.*, **4**(1984),43.
[13] Napolitano, L. G., *Naturwissen Schaften*, **73**(1986), 352.
[14] Monti, R., *Acta Astronautica*, **15**(1987), 557.
[15] Ostrach, S., Komotani, Y. & Lai, C. L., *Pch Physicochemical Hydrodynamics*, **6**(1985), 585.
[16] Hu, W. R., *39th Congress of IAF*, IAF-88-365, 1988.
[17] ————, *Proceedings of 16th Inter. Symp. on Space Tech.and Sci.*, Chapter 20, AGNE Publishing Inc., 1988.