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EXTENDING TSAI-HILL AND NORRIS CRITERIA TO PREDICT CRACKING DIRECTION IN ORTHOTROPIC MATERIALS

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As is well known the Tsai-Hill criterion [1] and the Norris criterion [2] are failure criteria for predicting failure strength of orthotropic materials without discontinuity or stress raiser. They are often named as distortional strain energy criteria. In fact, for anisotropic materials, such as fibre composites, the total strain energy cannot be separated into pure volume strain energy and pure distortional strain energy. The two parts of energy are always coupled together. Now, we will extend the two criteria to predict crack growth direction in orthotropic materials with a discontinuity or a stress raiser.

To begin with, the Tsai-Hill distortional strain energy S_{TH} and the Norris distortional strain energy S_N are defined by the following two equations:

$$S_{TH} = \frac{1}{2}\left\{\left(\frac{\sigma_1}{X}\right)^2 + \left(\frac{\sigma_2}{Y}\right)^2 - \frac{\sigma_1\sigma_2}{X^2} + \left(\frac{\tau_{12}}{S}\right)^2\right\} \quad (1)$$

$$S_N = \frac{1}{2}\left\{\left(\frac{\sigma_1}{X}\right)^2 + \left(\frac{\sigma_2}{Y}\right)^2 - \frac{\sigma_1\sigma_2}{XY} + \left(\frac{\tau_{12}}{S}\right)^2\right\} \quad (2)$$

The extended Tsai-Hill and Norris criteria postulate that the crack initiates or propagates in the radial direction along which the Tsai-Hill and Norris distortional strain energies possess their minimum values, i.e.,

$$\partial S_{TH}/\partial\theta = 0 \quad \text{and} \quad \partial^2 S_{TH}/\partial\theta^2 > 0 \quad when\,\theta = \theta_c \quad (3)$$

$$\partial S_N/\partial\theta = 0 \quad \text{and} \quad \partial^2 S_N/\partial\theta^2 > 0 \quad when\,\theta = \theta_c \quad (4)$$

If S_{TH} and S_N have more than one minimum value around the crack tip or stress raiser, the maximum ones thereof predict the crack growth direction, namely,

$$S_{TH} = (S_{TH})_{\min}^{\max} \qquad when \qquad \theta = \theta_c \tag{5}$$

$$S_N = (S_N)_{\min}^{\max} \qquad when \qquad \theta = \theta_c \tag{6}$$

In a previous paper, [3] we have used the two extended criteria in predicting the cracking direction in the double grooved tension-shear specimen which consist of chopped strand mat glass fiber reinforced polyester laminate and give mixed-mode interlaminar fracture. FEM was used to calculate the stress distribution in the vicinity of the grooves. The analytical prediction was compared with experimental observation and good agreement was found.

The research described in the present short note is to ascertain if the extended criteria are also valid for other orthotropic materials.

In the following, a typical problem will be dealt with, which has been analysed using other criteria [4]. The computation model is as shown in Fig. 1. The material is a unidirectional graphite fiber reinforced epoxy, its material constants are $E_{11} = 151.9$ GPa, $E_{22} = 13.7$ GPa, $G_{12} = 5.83$ GPa, $\upsilon_{12} = 0.28$, X = 1.983 GPa, Y = 0.0703 GPa, S = 0.0998 GPa.

From anisotropic linear elastic fracture mechanics, the stresses in the near crack zone can be calculated by the following formulas:

$$\sigma_x = \frac{\sigma^\infty \sqrt{a}}{\sqrt{2r}} Re\left\{ \frac{\mu_1\mu_2}{\mu_1 - \mu_2}\left[\frac{\mu_2}{\psi_2} - \frac{\mu_1}{\psi_1}\right]\right\} \tag{7a}$$

$$\sigma_y = \frac{\sigma^\infty \sqrt{a}}{\sqrt{2r}} Re\left\{ \frac{1}{\mu_1 - \mu_2}\left[\frac{\mu_1}{\psi_2} - \frac{\mu_2}{\psi_1}\right]\right\} \tag{7b}$$

$$\tau_{xy} = \frac{\sigma^\infty \sqrt{a}}{\sqrt{2r}} Re\left\{ \frac{\mu_1\mu_2}{\mu_1 - \mu_2}\left[\frac{1}{\psi_1} - \frac{1}{\psi_2}\right]\right\} \tag{7c}$$

where

$$\psi_1 = (\cos\theta + \mu_1 \sin\theta)^{\frac{1}{2}} \tag{8a}$$

$$\psi_2 = (\cos\theta + \mu_2 \sin\theta)^{\frac{1}{2}} \tag{8b}$$

and μ_1 and μ_2 (and their conjugates $\bar{\mu}_1$ and $\bar{\mu}_2$) are two unequal roots of the following characteristic equation:

$$A_{11}\mu^4 - 2A_{16}\mu^3 + (2A_{12} + A_{66})\mu^2 - 2A_{26}\mu + A_{22} = 0 \tag{9}$$

A_{ij} are the coefficients of compliance matrix of the material.

The calculation is straightforward. By using (7),(8) and (9), the stresses were obtained, and then the crack growth direction can be predicted by (5) and (6). The calculated results are shown in Table 1, which indicates the dependency of cracking direction, θ_c, on the fiber orientation β. For small β, θ_c is approximately equal to β; whereas, the deviation of θ_c from β grows larger with β. This tendency of prediction coincides largely with the observation of most kinds of composite materials. Figure 2 shows the two curves of $S_{TH} \sim \theta$ and $S_N \sim \theta$ for the case of β=45° to illustrate how to use (5) and (6) to determine the cracking direction, where a = 20mm, r = 0.1mm and σ^{∞}= 0.1 MPa.

To end with, the conclusion may be drawn that Tsai-Hill and Norris criteria, which are only used as failure criteria so far, also have the function of predicting crack growth direction in orthotropic materials.

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Table 1. Variation of predicted cracking direction with fiber orientation β

β	0°	5°	15°	30°	45°	60°	75°	85°
Tsai-Hill	0°	6°	15°	28°	39°	51°	64°	73°
Norris	0°	6°	16°	28°	40°	52°	64°	73°

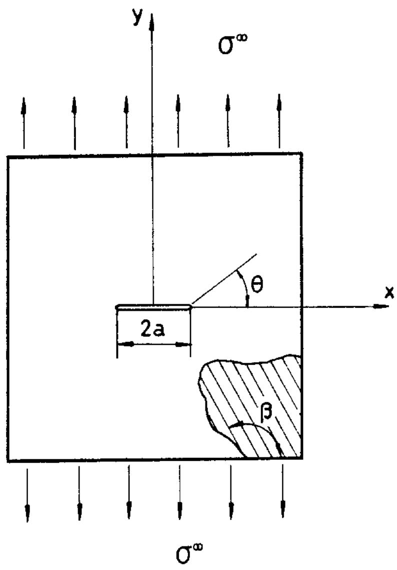

Figure 1. Computational model.

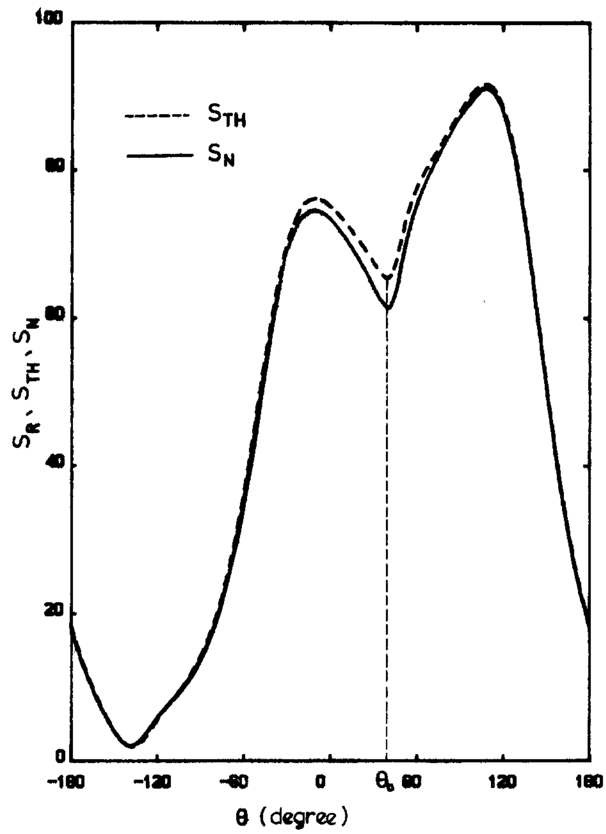

Figure 2. Curves of S_{TH} and S_N vs θ, β = 45°.