

# Width of adiabatic shear bands

An equation for the half-width of an adiabatic shear band is derived. With reference to the equation, the importance of thermal conductivity, temperature, and viscosity in determining the bandwidth are illustrated. It is shown that calculations of shear band width are fairly close approximations to those obtained experimentally. The apparent insensitivity of shear band width to the loading conditions is in agreement with experimental findings of other researchers.

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## List of symbols

- $c_V$  specific heat at constant volume
- $k$  wave number
- $K$  fraction of plastic work converted into heat
- $P = - \left. \frac{\partial \tau}{\partial T} \right|_0$
- $Q = \left. \frac{\partial \tau}{\partial \gamma} \right|_0$
- $R = \left. \frac{\partial \tau}{\partial \dot{\gamma}} \right|_0$
- $t$  time
- $T$  temperature
- $T_M$  melting point
- $y$  displacement along axis perpendicular to shear band length
- $\alpha$  disturbance, *see* equations (2)
- $\gamma$  shear strain
- $\dot{\gamma}$  shear strain rate
- $\delta$  shear band half-width
- $\eta$  coefficient of viscosity
- $\lambda$  thermal conductivity
- $\rho$  density
- $\tau$  shear stress
- $\tau_f$  flow stress

## Subscripts

- a amplitude of disturbance
- c medium surrounding shear band
- 0 basic state
- \* within the shear band

Unlike tension or compression tests, there is no change in cross-sectional area during a torsion test on a thin-walled tube. In torsion, the shear-stress-shear-strain curve eventually reaches a maximum corresponding to  $d\tau = 0$ . During deformation at strains above this maximum, plastic flow is localized (*see*, for example, Refs. 1 and 2).

In tension tests, decreases in cross-sectional area limit the ductility of the specimen. Because there is no necking or similar geometrical effect in torsion, the shear strains to fracture are large, and the test has, therefore, become a means of measuring the mechanical properties of materials at high strains and strain rates.<sup>3,4</sup>

Bands of intense shear have been observed during punching and machining tests,<sup>5-7</sup> as well as during ballistic impact experiments.<sup>8,9</sup> In the hot forging of steels, bands of intense shear have also been observed; these have often been referred to as thermal crosses.<sup>10,11</sup> Two valuable reviews on the topic of shear banding have been written by Bedford *et al.*<sup>12</sup> and Rogers,<sup>13</sup> and these describe the

different processes in connection with which shear bands are found.

The applied strain rate and the thermal properties of the material, such as the specific heat and the thermal conductivity, are of fundamental importance to the appearance and the geometry of shear bands. Indeed, if it is accepted that increasing the strain rate normally leads to an increase in flow stress, then the thermal effects of the plastic work lead to shear localization. Such terms as thermoplastic shear instability are now in current use to describe these phenomena.<sup>14</sup>

Conceptually, if the effects of microstructure (i.e. inclusion count, presence of micro-cracks, and so forth) are ignored, the effects of strain rate and temperature in the torsion test are straightforward. For a given material, testpiece geometry, and applied strain rate, the rate of strain hardening will eventually be balanced by the rate of thermal softening; this corresponds to  $d\tau = 0$ . After this critical point, plastic flow will continue in a localized manner, often along a small number of shear bands.<sup>13</sup> If the strain rate is decreased, the shear band width will increase. In the limit, for very slow tests, the width of the shear band will approach the gauge length of the specimen, i.e. shear-band localization will be suppressed until much higher strains. A theory of thermal shear banding has recently been developed,<sup>14</sup> and it is the purpose of this paper to discuss this theory in relation to observations of shear bands in torsion tests.

## Theory

For simple shear, by taking strain rate, the heat resulting from plastic work, and thermal conductivity into account, Bai's approximate model<sup>14</sup> gives

$$\left. \begin{aligned} \rho \frac{\partial^2 \gamma}{\partial t^2} &= \frac{\partial^2 \tau}{\partial y^2} \\ K\tau \frac{\partial \gamma}{\partial t} &= \rho c_V \frac{\partial T}{\partial t} - \lambda \frac{\partial^2 T}{\partial y^2} \end{aligned} \right\} \dots \dots \dots (1)$$

where the nomenclature is as given in the 'List of symbols'. The constant  $K$ , denoting that portion of the plastic work converted into heat, is  $\sim 0.9$  (Ref. 15). The  $y$ -axis is taken to be normal to the shear band length. Considering disturbances of the form

$$\gamma_a \exp(\alpha t + iky)$$

it is possible to obtain the homogeneous equations

$$\left. \begin{aligned} \rho \alpha^2 \gamma_a + k^2 \tau_a &= 0 \\ K\tau_0 \alpha \gamma_a + k \dot{\gamma}_0 \tau_a - (\rho c_V \alpha + \lambda k^2) T_a &= 0 \end{aligned} \right\} \dots \dots \dots (2)$$

where the subscripts 0 and a represent the basic state and the amplitude of the disturbance, respectively.

Assuming the constitutive equation has the form  $\tau = \tau(\gamma, \dot{\gamma}, T)$ , and using the identities

$$Q = \frac{\partial \tau}{\partial \gamma} \Big|_0, \quad P = - \frac{\partial \tau}{\partial T} \Big|_0, \quad R = \frac{\partial \tau}{\partial \dot{\gamma}} \Big|_0,$$

the spectral equation of  $\alpha$  (taken by equating the determinant of equations (2) to zero) is

$$\rho^2 c_v \alpha^3 + [KP\dot{\gamma}_0 + (\lambda + c_v R)k^2] \alpha^2 + (\lambda Rk^2 + \rho c_v Q - k\tau_0 P)k^2 \alpha + \lambda Qk^4 = 0 \quad (3)$$

For all possible disturbances ( $0 < k < \infty$ ),  $\alpha > 0$  if  $Q > 0$ , as long as

$$\frac{K\tau_0 P}{\rho c_v Q} > 1 + 2 \left( \frac{K\lambda P\dot{\gamma}_0}{\rho c_v^2 Q} \right)^{1/2} \quad (4)$$

This is the condition under which the disturbances would develop with increasing time.

Generally, shear bands are examined in post-mortem testpieces. For example, they can be identified as white-etching lines of martensite in steels, and narrow bands in other materials. The shear bands observed in post-mortem samples are obviously representative of a late stage in their evolution and, as such, can be compared with the corresponding theory. Referring to the second of equations (1): as  $t \rightarrow \infty$ , this reduces to

$$K\tau\dot{\gamma} \approx \lambda \frac{\partial^2 T}{\partial y^2} \quad (5)$$

An estimate of the term  $\lambda(\partial^2 T/\partial y^2)$  within a shear band is

$$\lambda \frac{\partial^2 T}{\partial y^2} \approx \lambda \left( \frac{T_* - T_c}{\delta^2} \right) \quad (6)$$

Usually,  $T_c$  is significantly lower than  $T_*$ ; therefore, it is possible to make the approximation  $\Delta T \approx T_*$ .

Substituting these simplifications into equation (5) gives

$$\delta \approx \left( \frac{\lambda T_*}{\tau_* \dot{\gamma}_*} \right)^{1/2} \quad (7)$$

where the subscript \* denotes characteristic quantities within the band. Equation (5) appears to be physically reasonable, because an increase in thermal conductivity  $\lambda$  and a decrease in the plastic-work rate would both tend to increase the band width.

### Comparison of theory with experiments

Although the torsion test is of fundamental significance and is often used, it is surprising that very few researchers concerned with shear bands have actually measured the widths of bands obtained experimentally.

Lindholm *et al.*<sup>16</sup> carried out torsion tests at strain rates between 0.009 and 330 s<sup>-1</sup> on OFHC copper. At strain rates  $\leq 9.6$  s<sup>-1</sup>, no maxima were observed in the stress-strain curves, whereas for tests at strain rates between 174

and 330 s<sup>-1</sup>, maxima were observed at shear strains of  $\sim 5$ . Metallographic sectioning showed that for tests at the lower strain rates the specimen gauge length remains uniformly deformed. For tests at the higher rates, however, shear bands 0.34 mm in width were observed, irrespective of strain rate.

Costin *et al.*<sup>17</sup> conducted a number of torsion tests on two steels at different temperatures and strain rates. One steel was hot rolled, the other cold rolled. The cold-rolled steel showed shear banding at a strain rate of 500 s<sup>-1</sup>, where the shear bands were  $\sim 0.19$  mm in width. At quasistatic rates, the cold-rolled steel only exhibited uniform deformation. Irrespective of rate, the hot-rolled steel did not exhibit banding. These results were explained in terms of the differences in the strain-hardening capacity of the two steels.

Some other experimental observations of shear band widths, obtained under different loading conditions and for various materials, are listed in Table 1.

Using equation (7), it is possible to predict the shear band width of a given material if the appropriate material parameters, such as the coefficient of thermal conductivity and the stress state, are known. Moreover, besides the coefficient of thermal conductivity, the band temperature  $T_*$  and shear stress  $\tau_*$  may also correspond to material parameters; at least this will be so for a transformation band. In a transformation band, the temperature  $T_*$  is approximately the melting or transformation temperature, while  $\tau_*$  is of the order of the flow stress.<sup>23</sup> The problem is whether there is a certain characteristic strain rate corresponding to  $\dot{\gamma}_*$  for a given material. At high strain rates the viscosity of the material may become important. If this is so, then  $\dot{\gamma}_* \approx \tau_*/\eta$ , and equation (7) becomes

$$\delta = \tau_*^{-1} (\lambda \eta T_*)^{1/2} \quad (8)$$

From this equation, it can be concluded that the thermoplastic shear band width should be a material characteristic and that it should be fairly insensitive to details of the loading conditions. Materials data required for the calculation of bandwidths are given in Table 2, in which, for simplicity, the melting temperature  $T_M$  is used instead of the transformation temperature for estimating  $\delta$  from equation (8). Typical measurements of shear band widths in copper and steel are, respectively, 0.34 (Ref. 16) and 0.19 mm (Ref. 17). The corresponding predicted values are 0.44 and 0.1 mm, which are reasonably close approximations to the experimental widths, particularly since they are calculated using such a simple expression as equation (8).

It is interesting to note that the calculated bandwidths are larger than most of those given in Table 1. The reason for this is most probably that the observed widths are usually those of transformed bands, which are the central part of the shear band only. The apparent insensitivity of shear band width to loading conditions, as shown by equation (8), is in good agreement with the observations of Backman and Finnegan.<sup>18</sup> They found that the bandwidth depends more on the material than on the loading conditions (for example, impact velocity). Moreover, Backman and Finnegan found that there was little

Table 1 Reported observed shear band widths

Shear band width, mm	Materials	Loading condition(s)	Ref.
0.003 - 0.01	Various materials, including steel	High-velocity impact	18
0.0033 - 0.0114	Fe-Ni-C alloys		19
0.01 - 0.03	Low-carbon high-strength pipe steel	Normal and tangential impacts	20
0.01 - 0.03	Ti, steels, and uranium alloys	Dynamic expansion of shells	21
0.03	Ni-Cr steel	Punching with explosives	22
0.005 - 0.2	Rolled steel	Penetration, exploding cylinders, etc.	23

**Table 2** Materials parameters and predicted shear band widths  $2\delta$ 

	Al	Cu	Mild steel	Ref.
$\tau_0$ , MN m <sup>-2</sup>	~300	~200	~300	24
$T_M$ , K	775-877	1355	1800	25
$\dot{\gamma}_{273K}$ , W m <sup>-1</sup> K <sup>-1</sup>	236	403	50	...
$\eta$ , kPa s	2.1	3.6	2.8, 2.1	26
$2\delta$ , mm	0.136	0.444	0.106	...

variation in bandwidth for impact velocities between 0.5 and 3.2 km s<sup>-1</sup>.

## Conclusions

An equation for the half-width of an adiabatic shear band has been derived in terms of the coefficients of thermal conductivity and viscosity, the band temperature, and the flow stress. Despite the simplifications inherent in the derivation of this equation, it gives adequate predictions when compared with experiment. However, much more experimental work is required. Particularly necessary are measured shear band widths in various materials tests at different strain rates.

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