Structures on Sliding Base Subject to Horizontal and Vertical Motions

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Abstract: In this paper particular investigation is directed towards the combined effects of horizontal and vertical motions of real earthquakes to structures resting on sliding base. A simplified method is presented to treat the nonlinear effects of time dependent frictional force of the sliding base as a function of the vertical reaction produced by the foundation. As an example, the El Centro 1940 earthquake record is used on a structural model to show the structural responses due to a sliding base with different frictional and stiffness characteristics. The study shows that vertical ground motion does affect both the superstructure response and the base sliding displacement. Nevertheless, the sliding base isolator is shown to be effective for the reduction of seismic response of a superstructure.

Introduction

The aseismic capacity of a building can be increased by using the infilled frame structure. However, as a result of the increased stiffness of the infilled frame, the fundamental frequency of the building will be shifted towards the high energy frequency domain of an earthquake excitation, and this may lead to the amplification of the dynamic response.

Liauw and Kwan (1985) showed that the collapse strength of an infilled frame can be considerably increased with the associated increase in the stiffness of the structure. Therefore, in the region of a high seismicity, the incorporation of base isolators under the infilled frame may be required.

The fundamental frequency of a low-rise to medium-rise building normally is in the range of frequencies where earthquake energy is strong. The frequency of the building can be shifted out of this range by using a base isolation system. More recent earthquake records showed that the spectral accelerations in a near fault region may not diminish until building periods greater than two seconds are reached (Anderson and Mohasub 1986). In order to withstand the wind forces on such building, additional devices such as lead plugs, mechanical fuses or mild steel energy absorbing devices are often needed. An alternative device is to use sliding joints as an isolation system which has been studied (Ahmadi 1983; Arya 1984; Liauw, Tian and Cheung 1986; Lin and Tadjbakhsh 1986; Mostaghel, Hejazi and Tanbakuchi 1983; Tadjabakhsh and Younis 1984; Tadjabakhsh and Constaninou 1984; Westermo and Udwadia 1984) and shown to be very effective.

A simple rigid-block model and a model of a single-degree-of-freedom oscillator representing the structure which is resting on sliding base

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subjected to harmonic and random motions have been widely studied. However, very little attention has been given to the effects of vertical and rocking motions of the sliding system.

The break-away strength of the friction element of the sliding system can be designed to be large enough to withstand the wind force, but not enough to prevent sliding under strong earthquake action. This condition is provided by the equilibrium between the inertia force of the superstructure and the frictional force. When the inertia force of the superstructure overcomes the frictional force, the superstructure will slide along the interface of the base and the foundation. Otherwise the superstructure will remain as a conventionally fixed base structure.

The frictional force is a function of the vertical reaction which is produced by the supporting element on the foundation mat. Hence, both these vertical and frictional forces are varied when there is vertical or rocking motion. It is, therefore, necessary to study the effects of vertical and rocking motions on the sliding system which is subjected to a real earthquake. Since the frictional force consists of harmonics, its influence on superstructure is dependent on the characteristic of the superstructure. In designing such a device, a nonlinear dynamic analysis is essential.

A rigorous treatment of the nonlinear effects of time dependent frictional force is very complicated. In this paper, a simplified method for earthquake analysis of structures with foundation uplift (Chopra and Yim 1985) and base isolation (Sarrazui and Morales 1986) is extended to sliding base system.

Since only the fundamental mode of a shear-type building is significant, a simple oscillator model can be used to study the effects of vertical and rocking motions on base sliding. The frictional element acts as a mechanical filter through which the earthquake excitation is transmitted but altered to the superstructure. Having thus simplified, the response of structure, which is subjected to the altered earthquake excitation, can be calculated assuming that there is no base isolation. Thus the problem becomes one that can be analysed by a usual finite element program.

Motion Equations

The superstructure with equivalent mass m, stiffness k, and damping coefficient c is represented in Fig. 1 as a single-degree-of-freedom model in the fundamental mode.

The foundation mat is idealized as a rigid plane and resting on two spring-damper systems, which characterize the soil-foundation interaction. The foundation mat rests on the spring-damper element by gravity or by bolts.

A pair of friction elements is used to represent the frictional mechanism of slippage interface. The equations expressing the dynamic equilibrium of the system are given as follows:

$$m(\ddot{x} + h\ddot{\theta}) + c\dot{x} + kx = -m(\ddot{x}_o + \ddot{y}) \quad (1)$$

$$hm\ddot{x} + I_t\ddot{\theta} + 2C_f b^2\dot{\theta} + 2K_f b^2\theta = -hm(\ddot{y} + \ddot{x}_o) \quad (2)$$

$$m_t\ddot{v} + k_f v + c_f\dot{v} = -m_t\ddot{v}_g \quad (3)$$

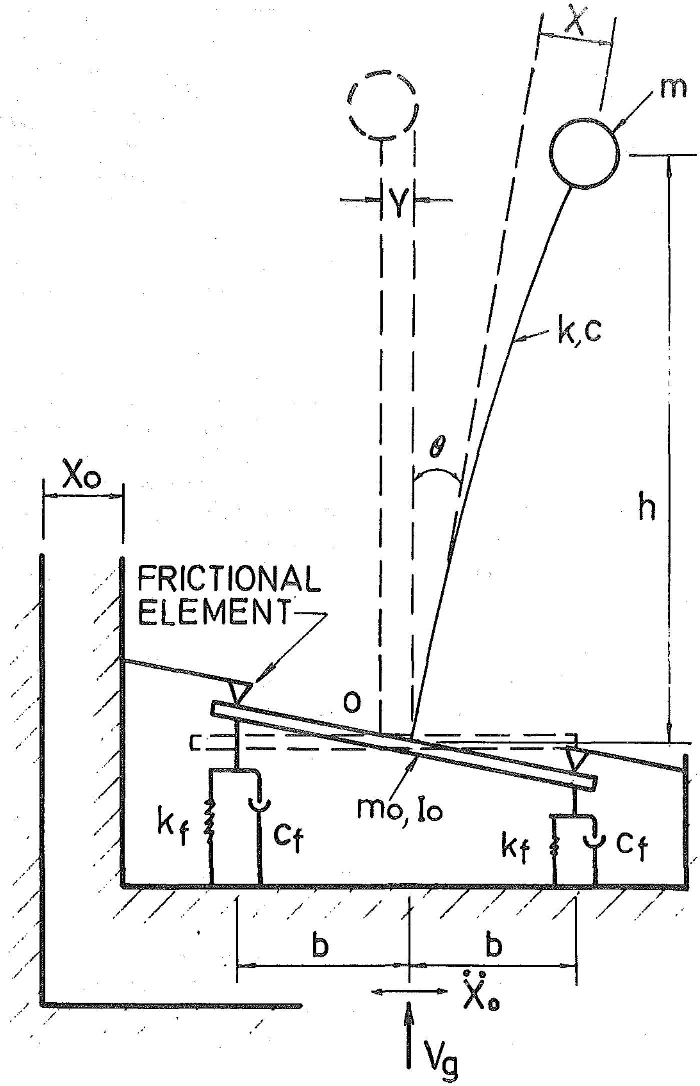

FIG. 1. Model of Superstructure with Sliding Base Foundation

$$m_t(\ddot{y} + \ddot{x}_o) + m(\ddot{x} + h\ddot{\theta}) = +\mu N\zeta \quad (4)$$

where $\quad m_t = m_o + m \quad (5)$

$$I_t = mh^2 + I_o \quad (6)$$

$$\zeta = \text{sgn}(\dot{y}) = -\frac{\dot{y}}{|\dot{y}|} \quad (7)$$

Eq. 1 is the horizontal motion equation of the fixed base superstructure, which is subjected to acceleration $(\ddot{x}_o + \ddot{y})$ at the base. Eq. 2 is the moment equilibrium equation of the fixed base superstructure, where $hm(\ddot{x} + \ddot{y} + \ddot{x}_o + h\ddot{\theta})$ is the over-turning moment provided by the inertia force of the superstructure, $I_o\ddot{\theta}$ is the inertia moment of the base mat, and $(2k_f b^2 \theta + 2C_f b^2 \dot{\theta})$ is the moment provided by a pair of vertical reactive forces of the supporting spring-damper system. Eq. 3 is the vertical equilibrium equation of the whole system. Eq. 4 is the horizontal equilibrium equation between the inertia force of the superstructure and the friction force of the frictional element. In a sliding mode, the magnitude of the friction force is dependent on the frictional coefficient and the vertical force between the frictional element and the supporting spring.

Under statical condition the supporting spring-damper elements are subjected to the initial vertical reactive forces resulting from the gravity force of the building or the initial clamping forces of the bolts. The horizontal resistance at the base against wind on the building is provided by the base frictional force, which is a function of the vertical reactive force of the supporting spring-damper element and the frictional coefficient on the sliding base interface. During vibration of the whole structure, the frictional force will vary as the vertical reactive force of the supporting system changes. When the upward reactive force of one spring is greater than the initial force, this spring support will provide no reactive force at this uplifted edge.

The base excitation is specified by the horizontal and vertical ground motions with acceleration $\ddot{x}_o(t)$ and $\ddot{v}_g(t)$ respectively. Under the influence of this excitation, the foundation mat will rotate through an angle $\theta(t)$ and undergo a vertical movement $v(t)$ defined at its centre of gravity in the unstressed position.

Method of Analysis

Eqs. 1–3 are satisfied in sticked mode and sliding mode except that y must be equal to zero for the former case.

Eq. 4 is the equilibrium condition between the base shearing force produced by the inertia of the superstructure and frictional force of the frictional element. When the former is less than the latter, the system will be in the sticked mode. Hence the starting time of a sliding mode can be determined from Eq. 4. The vertical force between the base mat and the frictional element is

$$N = N_R + N_L \quad \text{(8)}$$

where sub-index R and L represent the right side and the left side respectively.

The upward reactive forces in the supporting spring-damper elements acting against the foundation mat are:

$$N_R = N_o - k_f(v + b\theta) \quad \text{(9)}$$

$$N_L = N_o - k_f(v - b\theta) \quad \text{(10)}$$

where N_o = the initial normal force produced by the gravity force of the building or the clamping forces of the bolts. Since the foundation mat is assumed to be in contact with the frictional element in compression, a downward reactive force cannot develop at this interface, and the limiting conditions must be imposed on Eqs. 9 and 10 as follows:

$$N_R = 0 \quad \text{when} \quad [N_o - k_f(v + b\theta)] < 0 \qquad (11a)$$

$$N_L = O \quad \text{when} \quad [N_o - k_f(v - b\theta)] < 0 \qquad (11b)$$

From Eq. 4

$$\ddot{y} = -\frac{1}{m_t}[m(\ddot{x} + h\ddot{\theta}) - \mu N\zeta] - \ddot{x}_o \qquad (12)$$

Substituting Eq. 12 into Eqs. 1 and 2, then combining with Eq. 3, the matrix form of equations of motion in sliding mode can be obtained as:

$$\begin{bmatrix} m_1 & hm_1 & O \\ hm_1 & I_t - \dfrac{(mh)^2}{m_t} & O \\ O & O & m_t \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \\ \ddot{v} \end{Bmatrix} + \begin{bmatrix} k & & \\ & 2b^2k_f & \\ & & k_f \end{bmatrix} \begin{Bmatrix} x \\ \theta \\ v \end{Bmatrix}$$

$$+ \begin{bmatrix} C & & \\ & 2b^2C_f & \\ & & C_f \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{\theta} \\ \dot{v} \end{Bmatrix} = -\begin{Bmatrix} \left(\dfrac{m}{m_t}\right)\mu N\zeta \\ \left(\dfrac{hm}{m_t}\right)\mu N\zeta \\ m_t\ddot{v}_g \end{Bmatrix} \qquad (13)$$

In the sticked mode, the sliding motion y = zero and the equations of motion becomes

$$\begin{bmatrix} m & hm & O \\ hm & I_t & O \\ O & O & m_t \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \\ \ddot{v} \end{Bmatrix} + \begin{bmatrix} k & & \\ & 2b^2k_f & \\ & & k_f \end{bmatrix} \begin{Bmatrix} x \\ \theta \\ v \end{Bmatrix}$$

$$+ \begin{bmatrix} C & & \\ & 2C_fb^2 & \\ & & C_f \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{\theta} \\ \dot{v} \end{Bmatrix} = -\begin{Bmatrix} m\ddot{x}_o \\ hm\ddot{x}_o \\ \ddot{v}_g \end{Bmatrix} \qquad (14)$$

Both Eqs. 13 and 14 are linear, but the whole system is nonlinear. At any time instant, the numerical analysis consists of the following three steps: (1) Calculate the normal force by using Eqs. 8–10, and note that N_R or N_L must be greater than zero; (2) determine the motion mode (whether it is in sliding or sticked mode) by comparing the base shearing force induced by the inertia force of the superstructure with the friction force of the frictional element; and (3) calculate the response of the whole system by solving the equations of motion corresponding to the motion mode obtained in the previous step. Since the inertia force and the frictional force are different at every time instant, the calculation must be executed by a recurrence method, the end time and end condition of one mode being the starting time and initial condition of another mode respectively. The

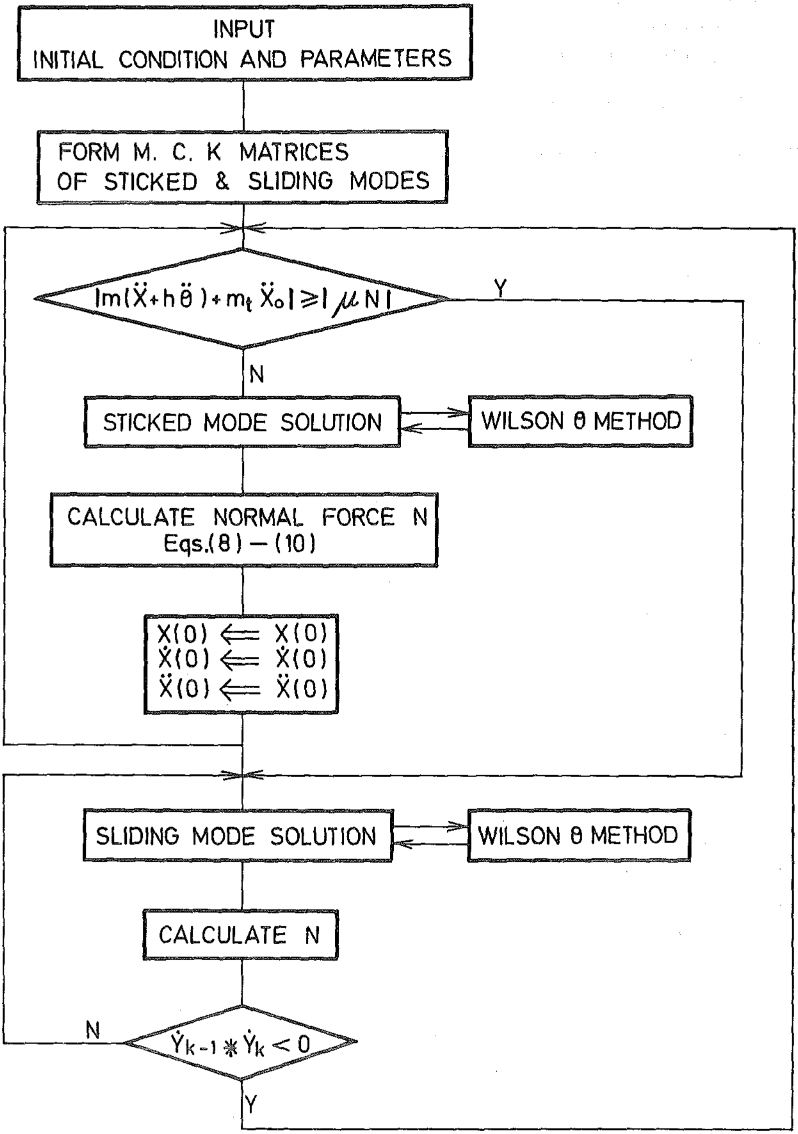

FIG. 2. Calculation Flow Charts

calculation process can be illustrated in the flow chart in Fig. 2. The validity of this program has been checked by harmonic analysis.

Example

To show the performance of the sliding isolation model subjected to a real earthquake excitation, the maximum acceleration of the superstructure and the relative sliding displacement at the base of the simple

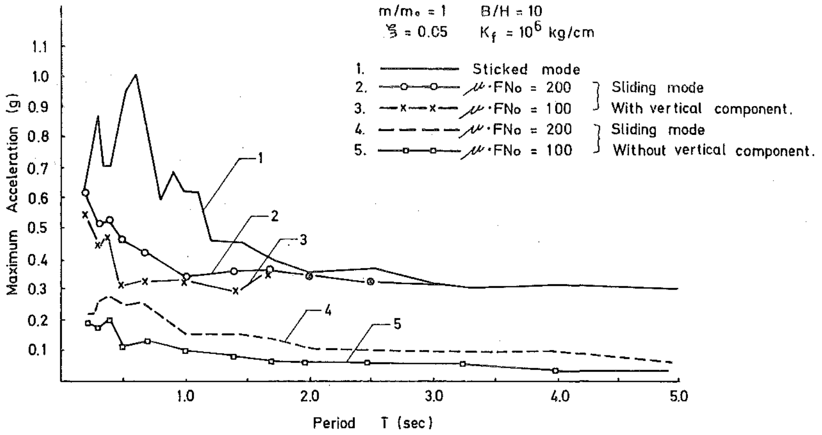

FIG. 3. Influence of Vertical Component of Motion on Acceleration Spectra. Curves 2 and 3 Coincide with Curve 1 Beyond $T = 3$ sec

oscillator model are plotted against its horizontal natural period T in Figs. 3–6. These results are calculated using the first 19 seconds of the El Centro 1940 earthquake acceleration record.

From the curves shown in Fig. 3, it can be seen that the acceleration responses are affected by the vertical ground motion. However, the earthquake responses of the sliding modes with and without vertical excitations are all reduced considerably from that of the sticked mode, especially in the region of high energy spectrum of the earthquake.

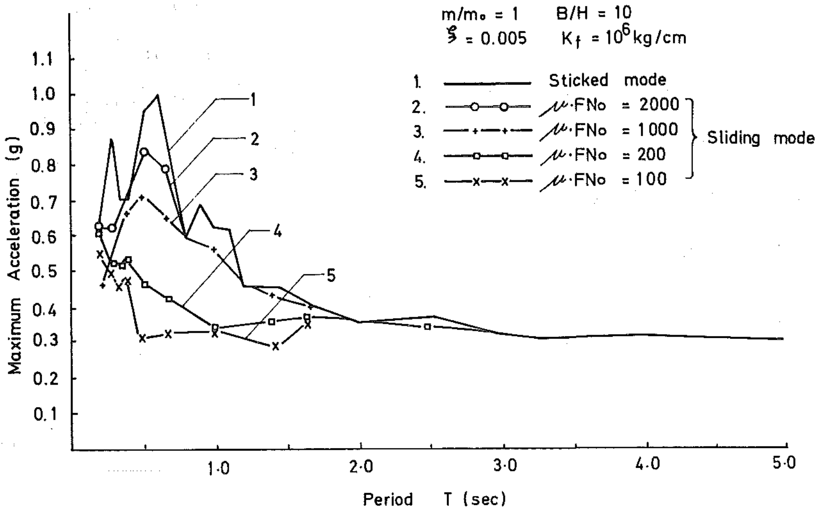

FIG. 4. Acceleration Spectra for Different Frictional Force (with Vertical Component of Motion). Curves 2 to 5 all Coincide with Curve 1 beyond $T = 3$ sec

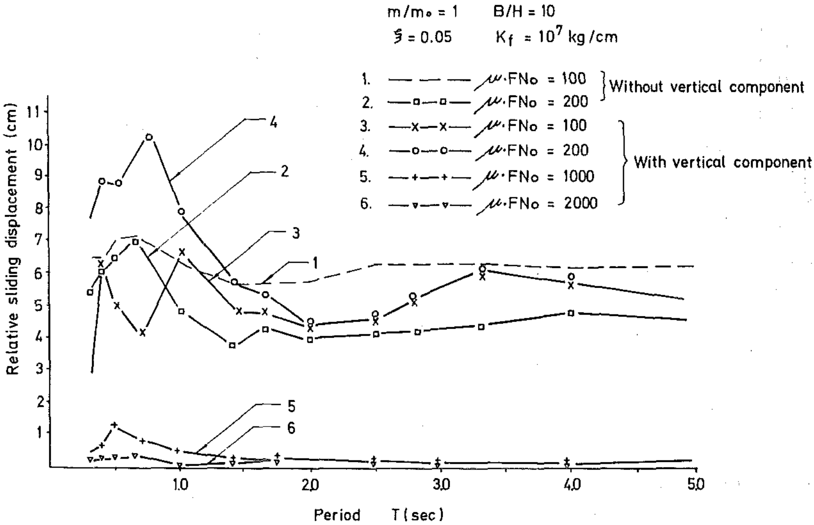

FIG. 5. Relative Displacement Spectra for Different Frictional Force

Fig. 4 shows the influence of different frictional forces on the acceleration response: the larger the frictional force, the higher the response. Fig. 5 shows the influence of different frictional forces on the relative sliding displacement between the base of the superstructure and the foundation.

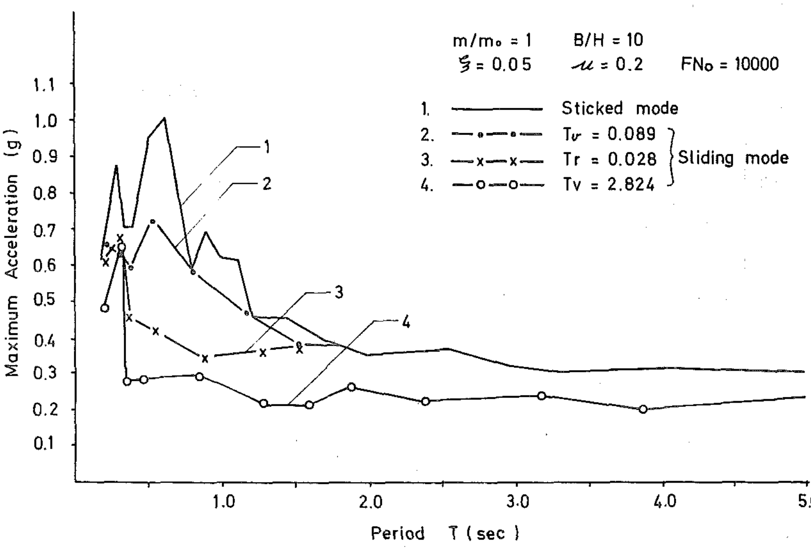

FIG. 6. Acceleration Spectra for Different Vertical Supporting Stiffness (with Vertical Component of Motion). $T_v = 2\sqrt{m_t/k_f}$. Curves 2 and 3 Coincide with Curve 1 beyond $T = 2$ sec

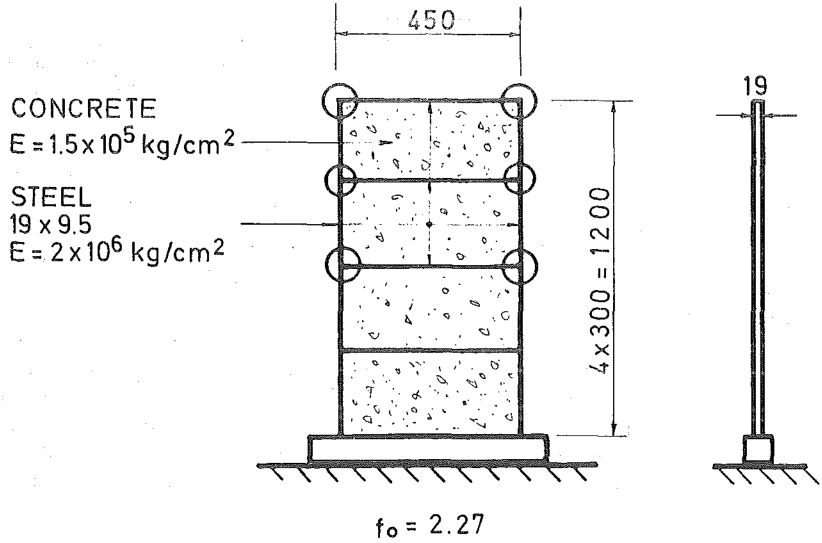

FIG. 7. Infilled Frame Model (Dimensions in *mm*) Subjected to El Centro Earthquake and Filtered Earthquake

In contrast to Fig. 4, the larger the frictional force, the smaller the displacement whatever the excitation with or without vertical component.

Fig. 6 shows the influence of the stiffness of vertical support on the acceleration response. When the natural period of the vertical system (T_v) is much higher or much lower than unity, the response is small. However, when the magnitude of T_v approaches unity, the vertical motion and the horizontal acceleration response of the system are amplified significantly.

As an example, an infilled frame model as shown in Fig. 7 is subjected to the horizontal earthquake excitation and a filtered earthquake excitation. The latter is obtained from the above-mentioned nonlinear analysis of the fundamental mode model of this infilled frame. The responses for maximum bending moment in the column and maximum plane stress in the infill are given in Table 1. It is obvious from the comparisons shown in Table 1 that the response of the infilled frame is considerably reduced through the filter action of the sliding isolator.

TABLE 1. Response of Infilled Frame

	Maximum Response		
Excitation (1)	Column bending moment (kg-cm) (2)	Panel stress (kg/cm²) (3)	Base shearing force (kg) (4)
EL Centro 1940 earthquake	14,315	1,715	53,505
Filtered earthquake	4,144	520	15,723

Conclusions

The sliding base isolator is an effective device for the reduction of seismic risk of a structure. Under vertical and rocking motions of a substructure, the frictional force at the base is time dependent and the frictional element has slip-stick characteristic. The simulation results show that both the acceleration response of the superstructure and the relative sliding displacement at the base are affected by the vertical ground motion. However, the earthquake response can be reduced considerably from the sticked mode in the region of high energy spectrum of the earthquake by means of sliding device.

Smaller initial frictional force in the sliding system subjected to seismic excitation can reduce the response of the superstructure, but at the expense of larger sliding displacement. Since the superstructure response and the base movement are a pair of contradictory factors, the proper selection of the initial frictional force must be treated with caution. As the acceleration responses can be significantly affected by the stiffness of the vertical support, the natural period of the system must also be considered in design.

Acknowledgment

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Appendix II. Notation

The following symbols are used in this paper:

b = width of base mat;
C = equivalent damping coefficient of superstructure;
C_f = equivalent damping coefficient of foundation;
h = equivalent height of superstructure;
I_o = moment inertia of base mat;
I_t = total moment inertia of whole system;
k = equivalent stiffness of superstructure;
k_f = equivalent stiffness of foundation;
m = equivalent mass of the superstructure;
m_o = mass of base mat;
m_t = total mass of the whole system;
N = vertical normal force;
N_R = vertical normal force at the right side of base;
N_L = vertical normal force at the left side of base;
v = vertical response acceleration of equivalent oscillator;
v_g = vertical acceleration of ground motion;
x = elastic displacement of equivalent oscillator;
x_o = displacement of foundation;
y = relative sliding movement between base mat and foundation;
ζ = sign function Eq. 7;
θ = rotation response of equivalent oscillator;
μ = coefficient of friction; and
ξ = c/c_c damping ratio.