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Development of Thermoplastic Shear Bands in Simple Shear

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ABSTRACT

The development of thermoplastic shear bands in coupling-rate- and temperature-dependent material under simple shear is examined, using a simplified model together with Fourier transformation and a matching technique at the moving boundary of a shear band. The analysis reveals that the development of a shear band is dominated by the coupling rate and temperature effect of material. The strength of the material acts as a destabilizer, whilst heat diffusion makes the band expand. In addition, shear bands are susceptible to long-wave disturbances and this coincides with instability analysis. A few computational examples are presented.

INTRODUCTION

Thermoplastic shear bands are closely related to failure and cracking in structural materials. (This subject, particularly its phenomenological aspects, has been reviewed by Rogers.[1]) Theoretical investigations are based on either the criterion of maximum shear stress[2] or perturbation analysis.[3,4] In both approaches it is assumed that there is a critical state, beyond which instability may develop, and this is usually taken to be the condition for the emergence of thermoplastic shear bands. However, the two approaches give only a condition for instability, and no indication of the emergence of narrow shear bands.

In an analysis of quasi-steady thermoplastic shear bands,[5] it was found that the observed width of shear bands is governed by the balance of plastic work rate and thermal conduction and an approximation is obtained as $\delta \sim \sqrt{(\lambda\theta_*/\tau_*\dot{\gamma}_*)}$, where δ is the half-width of the band, λ is the thermal conductivity, and τ_*, θ_* and $\dot{\gamma}_*$ are the characteristic stress, temperature and strain rate within the band. A fair agreement between this estimate and observed values was shown in Ref. 6; in addition, some computational work[7,8] has been carried out to show the process of shear banding.

It is desirable to know the dynamics of shear banding and the factors governing the process. In this paper an analytical model is developed to achieve this aim.

APPROXIMATION AND GOVERNING EQUATIONS

It is assumed that the deformation mode is simple shear. The equations of motion and energy are[9]

$$\rho \frac{\partial^2 \gamma}{\partial t^2} = \frac{\partial^2 \tau}{\partial y^2} \tag{1}$$

$$\tau \frac{\partial \gamma}{\partial t} = \rho c_v \frac{\partial \theta}{\partial t} - \lambda \frac{\partial^2 \theta}{\partial y^2} \tag{2}$$

where τ and γ are the shear stress and strain respectively, ρ is the density, c_v is the specific heat at constant volume, λ is the thermal conductivity, t and y are the time and space coordinates respectively. The y-axis is normal to the direction of shear, and the origin is located on the centre-line of the testpiece.

For a developed shear band ($\delta \lesssim 10^{-1}$ mm) the characteristic times for mechanical equilibrium, and heat diffusion are, from eqns. (1) and (2), respectively

$$t_d \sim \rho\dot{\gamma}_*\delta_*^2/\tau_*, \qquad t_h \sim \rho c_v \delta_*^2/\lambda \tag{3}$$

where the asterisk subscript denotes a value characteristic of the shear band. Hence one can obtain the ratio

$$t_d/t_h \sim \lambda\dot{\gamma}_*/c_v\tau_* \tag{4}$$

which is independent of the shear zone width. Since λ and c_v are material constants and τ_* should be of the same order as the flow stress, then the higher the strain rate, the greater the value of t_d. However, even under a severe shear, say $\dot{\gamma}_* \sim 10^7\,\mathrm{s}^{-1}$, the ratio is not greater than 10^{-2} for a typical steel, for which $\lambda \sim 10^2\,\mathrm{W\,m^{-1}\,K^{-1}}$, $c_v \sim 10^3\,\mathrm{J\,kg^{-1}\,K^{-1}}$, and the flow stress is about 10^8 Pa. However, for a transient process the ratio of the characteristic length scales t_h/t_d is the reciprocal of eqn. (4), and it would be appropriate to assume that the process was approximately adiabatic. Therefore, in order to discuss developed thermoplastic shear bands, and to compare the theory with observations of shear bands in failed testpieces one can reasonably take the former estimation. Hence eqn. (1) reduces to

$$\frac{\partial^2 \tau}{\partial y^2} = 0 \tag{5}$$

Since the deformation is bilaterally symmetric with respect to the y-axis, eqn. (5) has the solution

$$\tau = \tau(t) \tag{6}$$

The foregoing argument becomes even more clear from the following dimensionless equations:

$$\varepsilon \frac{\partial \bar{\dot{\gamma}}}{\partial \bar{t}} = \frac{\partial^2 \bar{\tau}}{\partial \bar{y}^2} \tag{7}$$

$$\frac{\bar{\tau}\bar{\dot{\gamma}}}{2} = \frac{\partial \bar{\theta}}{\partial \bar{t}} - \frac{\partial^2 \bar{\theta}}{\partial \bar{y}^2} \tag{8}$$

where $\bar{\dot{\gamma}} = \dot{\gamma}/\dot{\gamma}_k$, $\bar{\tau} = \tau/\tau_k$, $\bar{\theta} = \theta/\theta_m$, $\bar{y} = y/\delta_k$ and $\bar{t} = t/t_k$; θ_m is the melting temperature, $\dot{\gamma}_k$ and τ_k are the characteristic strain rate and stress, both material constants, and δ_k and t_k are taken to be $\lambda\theta_m/2\tau_k\dot{\gamma}_k$ and $\rho c_v \delta_k^2/\lambda$, the latter quantity being related to t_h. The smallness of $\varepsilon = \lambda\dot{\gamma}_k/c_v\tau_k$ reduces eqn. (7) to eqn. (5). From now on the over-bar used to indicate a dimensionless quantity will be omitted.

Therefore the approximate model for shear bands is as follows:

$$\tau = \tau(t) \tag{9a}$$

$$\frac{\tau\dot{\gamma}}{2} = \frac{\partial \theta}{\partial t} - \frac{\partial^2 \theta}{\partial y^2} \tag{9b}$$

$$y = 0, \qquad \frac{\partial \theta}{\partial y} = 0 \tag{9c}$$

$$y = \delta(t), \qquad \theta_{\delta_-} = \theta_{\delta_+} \tag{9d}$$

$$\left.\frac{\partial \theta}{\partial y}\right|_{\delta_-} = \left.\frac{\partial \theta}{\partial y}\right|_{\delta_+} \tag{9e}$$

$$v|_{\delta_-} = RV(t) \tag{9f}$$

$$t = 0, \qquad \theta = \theta_0(y) \tag{9g}$$

where $R = v_0/\delta_k\dot{\gamma}_k$, and $\theta_0(y)$ and v_0 are the initial disturbances of temperature and velocity respectively and $V(t)$ is dimensionless boundary velocity at $V = 1$ and $t = 0$. As instability is characterized by a very large plastic strain (>1%) and a high strain rate (>$10^3\,\mathrm{s}^{-1}$), the constitutive equation is assumed to be the one for temperature-dependent viscoplastic conditions:[10]

$$\tau = \tau(\dot{\gamma}, \theta) \tag{10}$$

Outside the band, the material is assumed to remain rigid, no matter how high the temperature is. The governing equation here is the one for homogeneous diffusion with

$$\theta(0, y) = B \tag{11}$$

where B is the assumed uniform initial temperature.

ANALYTICAL SOLUTION

Since the aim of the paper is to understand the mechanics of shear band formation, and its relation to instability analysis and the factors controlling the process, an analytical solution is more preferable, although in order to obtain one some approximations need to be made.

One of the simplifications is a linear version of the constitutive equation (10):

$$\tau = \dot{\gamma} + 1 - \theta \tag{12}$$

The linear relation of τ and $\dot{\gamma}$ is consistent with observations made by Campbell[10] of specimens under high strain rate loading (>$10^3\,\mathrm{s}^{-1}$). Moreover, linear softening approximates the behaviour of a variety of metals between room temperature and their melting points.[11]

Substitution of eqn. (12) into eqn. (9) leads to an inhomogeneous equation in θ. The solution to it then can be expressed as

$$\theta(t, y) = \left\{\theta_1(t, y) - \int_0^t \frac{1-\tau(\eta)}{2}\,\tau(\eta)\exp\left(-H(\eta)\right)\mathrm{d}\eta\right\}\exp\left(H(t)\right) \tag{13}$$

where

$$H(t) = \int_0^t \frac{\tau(\eta)}{2}\,\mathrm{d}\eta$$

and η is a variable in integration, in which

$$\tau(t) = 1 - \theta_\delta(t) \tag{14}$$

The temperature θ_1 satisfies the equation

$$\frac{\partial \theta_1}{\partial t} - \frac{\partial^2 \theta_1}{\partial y^2} = \frac{\tau(t)}{2}\,\theta_1 \tag{15}$$

To deal with the moving boundary $\delta(t)$, we consider the initial boundary conditions

$$
\begin{aligned}
y &= 0, & \frac{\partial \theta_1}{\partial y} &= 0 \\
y &= \delta_s, & \frac{\partial \theta_1}{\partial y} &= S(t)\exp(-H(t)) \\
t &= 0, & \theta_1 &= \theta_0(y)
\end{aligned}
\tag{16}
$$

where δ_s is an imaginary fixed boundary chosen to be greater than $\delta(t)$, and $S(t)$ is an arbitrary function determined by matching the condition at $y = \delta(t)$, i.e. eqns. (9d) and (9e).

The solution θ to the problem, but with initial boundary values (16), can be expressed as Fourier cosine series:

$$
\begin{aligned}
\theta(t, y) = {} & \exp(H(t))\frac{1}{\delta_s}\left\{c_0 + \int_0^t S(\eta)\exp(-H(\eta))\,\mathrm{d}\eta \right. \\
& \left. - \delta_s \int_0^t \frac{1-\tau(\eta)}{2}\tau(\eta)\exp(-H(\eta))\,\mathrm{d}\eta\right\} + \frac{2}{\delta_s}\sum_{n=1}^{\infty}\exp(H(t) - \alpha_n^2 t) \\
& \times\left\{c_n + (-1)^n\int_0^t S(\eta)\exp(\alpha_n^2\eta - H(\eta))\,\mathrm{d}\eta\right\}\cos\frac{n\pi y}{\delta_s}
\end{aligned}
\tag{17}
$$

where $\alpha_n = (n\pi/\delta_s)$, $0 < y < \delta_s$; c_n are constants determined by the initial condition (9g) as

$$
c_n = \int_0^{\delta_s} \theta_0(\xi)\cos\frac{n\pi\xi}{\delta_s}\,\mathrm{d}\xi \tag{18}
$$

where ξ is a variable in integration; eqn. (9f) requires that

$$
\begin{aligned}
RV(t) &= v|_{\delta_-} = \int_0^{\delta}\dot{\gamma}\,\mathrm{d}y \\
&= \frac{2}{\delta_s}\sum_{n=1}^{\infty}\exp(H(t) - \alpha_n^2 t)\frac{\delta_s}{n\pi}\left\{c_n + (-1)^n\int_0^t S(\eta)\exp(\alpha_n^2\eta - H(\eta))\,\mathrm{d}\eta\right\} \\
&\qquad \times\left\{\sin\frac{n\pi\delta(t)}{\delta_s} - \frac{n\pi\delta(t)}{\delta_s}\cos\frac{n\pi\delta(t)}{\delta_s}\right\}
\end{aligned}
\tag{19}
$$

The solution for the rigid material outside the band can be easily obtained as

$$
\theta(t, y) = \int_0^t (\theta_*(t-\eta) - B)\frac{y}{2\sqrt{(\pi\eta^3)}}\exp(-y^2/4\eta)\,\mathrm{d}\eta + B \tag{20}
$$

if it is assumed that $B=$ constant and the edge effect of testpiece is neglected; $\theta_*(t)$ here is the temperature at an imaginary boundary $y=0$.

In all there are four unknown functions: $\theta_*(t)$, $S(t)$, $\delta(t)$ and $\tau(t)$. They can be determined by solving eqns. (9d), (9e), (14) and (19) simultaneously.

MECHANICS OF SHEAR BANDING

Equation (17) shows that three factors control a non-uniform shear field, namely $S(t)$, $H(t)$ and $\alpha_n^2 t$. The first one is related to the heat flux flowing out of the shear band, the second represents a cumulative effect of the strength of the material, and the third concerns the decaying mode of heat diffusion within the band. The second and third factors are both exponential, and so are much more important than the first. Even at the early stage of shear band development, the heat flow (accounted for by $S(t)$) to surrounding material appears to be negligibly small, because $\partial\theta/\partial y|_{\delta_0}=0$, where $\delta_0=\delta(0)$. Therefore $H(t)$ and $\alpha_n^2 t$ are bound to be the governing factors in shear band formation.

With the two assumptions that of $S(t)=0$ and $c_n=0$ $(n\neq 1)$, which represent the most influential part of heat diffusion and the simplest case, eqn. (19) becomes

$$\begin{aligned} \sin(\alpha_1\delta(t)) - Z &= \alpha_1\delta(t)\cos(\alpha_1\delta(t)) \\ Z &= \pi\exp(\alpha_1^2 t - H(t)) \end{aligned} \tag{21}$$

provided $\delta_0=\delta_s$ and a constant-velocity boundary condition is introduced. It is clear from eqn. (21) that shrinkage of the shear deformation field requires a decreasing value of Z, i.e.

$$\frac{d}{dt}(H(t)-\alpha_1^2 t) > 0 \tag{22}$$

and

$$\tau(t)/2 > (\pi/\delta(0))^2 \tag{23}$$

Therefore this shrinkage is due to the material strength τ, whereas heat diffusion tends to smooth shearing.

For a material governed by the temperature-independent, viscous constitutive relation $\tau=\tau(\dot{\gamma})$, the solution to eqn. (9) is

$$\begin{aligned} \theta &= \theta_3(t, y) + \int_0^t \frac{\tau\dot{\gamma}}{2}\,d\eta \\ \frac{\partial\theta_3}{\partial t} &- \frac{\partial^2\theta_3}{\partial y^2} = 0 \end{aligned} \tag{24}$$

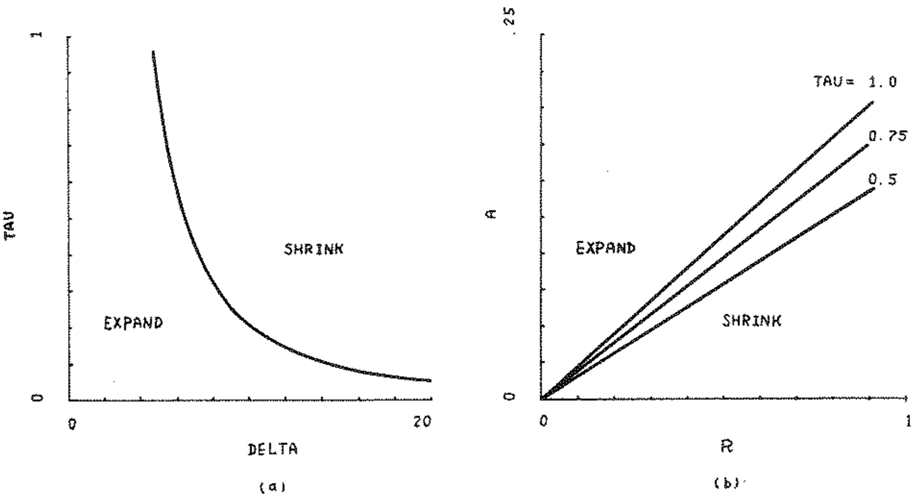

Fig. 1. The expansion and contraction of shear band fields, shown as diagrams of (a) stress τ/τ_k versus initial band width δ_0/δ_k, and (b) amplitude of disturbance A versus velocity R, with τ/τ_k as a parameter.

Unlike the solution (13), this solution indicates that the strength τ will not be incorporated into a non-uniform shear field. This case corresponds to simple heat diffusion, whereas $\tau = \tau(\theta)$ would lead to a trivial solution. Therefore, one can conclude that only in a material governed by a coupling-rate- and temperature-dependent constitutive relation can the shearing zone shrink, with the strength of the material acting as a destabilizer.

With decreasing $\tau(t)$, which usually happens in thermal shearing in accord with thermal softening, a narrowing shear band will be transformed into an expanding one at a certain moment, because the right-hand side of the inequality (23) is constant. This implies that there is a stable phase of deformation dominated by heat diffusion.

Figure 1(a), based on the inequality (23), shows that long-wave disturbances are more liable to cause shrinkage than short-wave ones. The effect of the amplitude A of the disturbance and boundary velocity R ($A = R/\delta_0$) on shear band formation is shown in Fig. 1b). Obviously the higher the velocity R, the narrower the shear band that will form.

It is interesting to compare the inequality (23) with the instability criterion based on perturbation analysis,[9]

$$\tau_0 P_0 > \lambda R_0 k^2 \qquad (25)$$

where $P_0 = -(\partial\tau/\partial\theta)$ and $R_0 = (\partial\tau/\partial\dot{\gamma})$ are the thermal softening and the strain rate hardening respectively, and k is the wavenumber. After transforming it into dimensionless form and using eqn. (12), (25) becomes

$$(\delta_{\mathrm{k}} k)^2 \equiv (\bar{k})^2 < \bar{\tau}/2 \tag{26}$$

Comparison of the disturbances predicted by the two approaches leads to $\bar{k} = \pi/\bar{\delta}_0$; therefore (26) is identical to (23). This illustrates that instability criterion (25) does define the localization of the shearing field in this particular case.

We can arrive at the above conclusion in a different way. By introducing eqn. (12) and the condition $\dot{\gamma} \doteq 0$ at $y = \delta$ into eqn. (19), eqn. (11) becomes

$$RV(t) = \int_0^{\delta(t)} \theta(t, y)\,\mathrm{d}y - \theta_\delta(t)\delta(t) \tag{27}$$

Differentiation of (27) with respect to time t under the constant velocity boundary condition leads to

$$\left.\frac{\partial \theta}{\partial y}\right|_\delta \delta(t)\frac{\mathrm{d}\delta(t)}{\mathrm{d}t} = \frac{\tau(t)R}{2} + \left.\frac{\partial \theta}{\partial y}\right|_\delta - \delta(t)\left.\frac{\partial \theta}{\partial t}\right|_\delta \tag{28}$$

For $\partial\theta/\partial y \neq 0$ and $\delta(t) \neq 0$, eqn. (28) becomes an expression for shear band development:

$$\frac{\mathrm{d}\delta(t)}{\mathrm{d}t} = \left\{\frac{\tau(t)\bar{\dot{\gamma}}(t)}{2} + \overline{\frac{\partial^2\theta}{\partial y^2}} - \left.\frac{\partial\theta}{\partial t}\right|_\delta\right\} \Big/ \left.\frac{\partial\theta}{\partial y}\right|_\delta \tag{29}$$

When $\dot{\gamma} > 0$, within the shear band $\partial\theta/\partial y|_\delta$ must be negative. Moreover, $\partial\theta/\partial t|_\delta = \partial^2\theta/\partial y^2|_\delta$ at $\delta(t)$. Then eqn. (29) becomes

$$\frac{\mathrm{d}\delta(t)}{\mathrm{d}t} = \left\{\frac{\tau(t)\bar{\dot{\gamma}}(t)}{2} + \left(\overline{\frac{\partial^2\theta}{\partial y^2}} - \left.\frac{\partial^2\theta}{\partial y^2}\right|_\delta\right)\right\} \Big/ \left.\frac{\partial\theta}{\partial y}\right|_\delta \tag{30}$$

The most significant term is $\tau\bar{\dot{\gamma}}/2$, the plastic work rate within the shear band, which is always positive and therefore governs shear band contraction. However, there is usually a simple monotonic decreasing temperature distribution,

$$\overline{\frac{\partial^2\theta}{\partial y^2}} - \left.\frac{\partial^2\theta}{\partial y^2}\right|_\delta < 0$$

from which it is seen that heat diffusion in the shear band tends to expand the band.

For a cosine temperature distribution one can obtain a criterion, identical to (26) or (23), to the distinction of expansion and shrinkage of the thermoplastic shear band.

COMPUTATIONAL EXAMPLES

Calculations are carried out for several typical situations. The initial disturbance is supposed to be a thermal one. For simplicity, θ_0 is assumed to consist only of the basic mode

$$\theta_0(y) = A\left(\cos\frac{\pi y}{\delta_0} + 1\right) + B, \qquad y < \delta_0 \tag{31}$$

We have seen that the terms $H(t)$ and $\alpha_1^2 t$ play a more significant role than others; two additional assumptions are therefore introduced:

$$\theta_* \simeq \theta_\delta - \delta \frac{\partial \theta}{\partial y}\bigg|_{\delta_-} \tag{32}$$

$$S(t) \simeq (\theta_{\delta_0} - \theta_\delta)/(\delta_0 - \delta) \tag{33}$$

in their corresponding extension region, instead of accurate calculation of $\partial\theta/\partial y|_{\delta_-}$ and $S(t) = \partial\theta/\partial y|_{\delta_0}$.

With $B = 0{\cdot}167$ three cases, corresponding to boundary velocities of $R = 0{\cdot}0422$, $0{\cdot}211$ and $1{\cdot}054$, are considered. The initial disturbances are found to expand only if the amplitude of the disturbance exceeds some limit. The evolution of shear bands is shown in two ways; as the variation of temperature at $y = 0$ with shear band width (Fig. 2) and with time (Fig. 3). Although the evolution shows a marked time dependence with different disturbances, the variations with shear band width have less scatter in the same phase, say shrinkage. In both Fig.

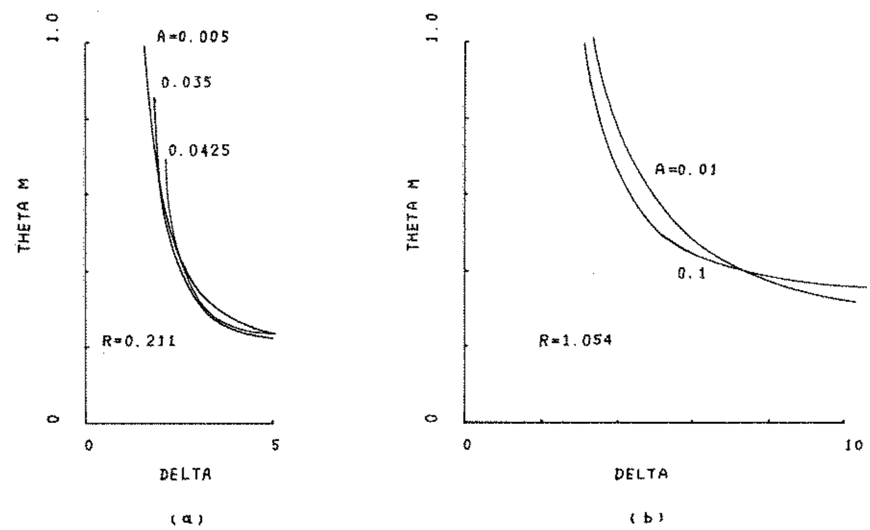

Fig. 2. Variation of temperature at centre of shear band with shear band width.

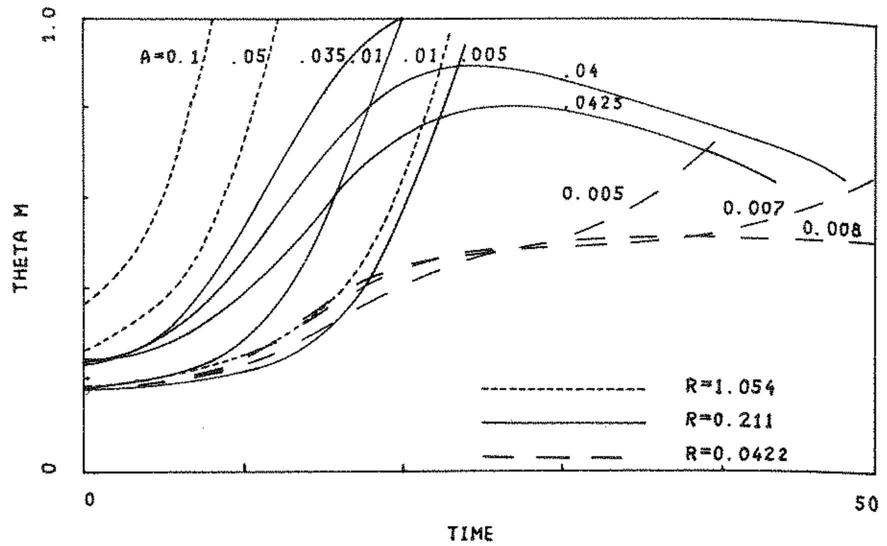

Fig. 3. Variation of temperature at centre of shear band with time for different velocities R and disturbance amplitudes A.

(2a) and (2b) the amplitudes span about one order, but the curves remain quite close. This indicates that the thermoplastic shear band has a strong intrinsic structure.

For the highest boundary velocity considered $R = 1{\cdot}054$, the temperature increases with narrowing band, and the higher the amplitude

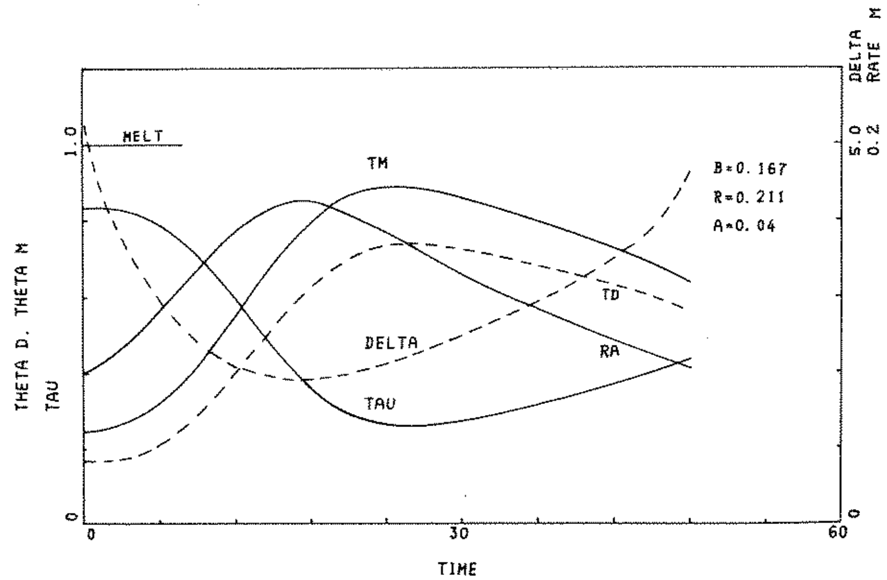

Fig. 4. Variation of some characteristic parameters of shear band formation with time, where TM and TD denote the maximum (at centre) and minimum (at boundary) temperature in the shear band. Also Rate M represents the highest rate of strain.

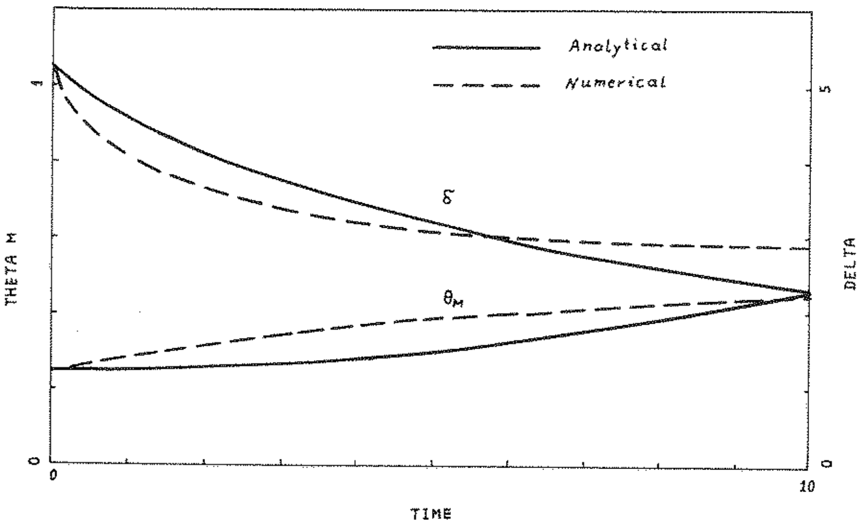

Fig. 5. Comparison of approximate solution and numerical computation.

of the disturbance, the shorter the time to reach melting point (Fig. 3). For the lowest velocity, $R = 0{\cdot}0422$, the reverse is true, although the band continues to narrow.

At the intermediate velocity ($R = 0{\cdot}211$) there are some transitional phenomena: from shrinkage to expansion, whether melting or not, and so on. The variations of temperature at $y = 0$ with time in the range $A = 0{\cdot}005$ to $0{\cdot}01$ are similar to those for $R = 1{\cdot}054$. This is the reflection of the remains of disturbance, since a shorter-wave (i.e. greater amplitude) disturbance should have less of a tendency to narrow the band. For a very short-wave disturbance ($A = 0{\cdot}035$ to $0{\cdot}0425$) the variations are reversed and appear to be similar to those for $R = 0{\cdot}0422$. In this phase the longer waves show stronger ability to shrink and then to reach higher temperature, whilst the shortest wave ($A = 0{\cdot}0425$) shows expansion and decreasing temperature after limited shrinkage, since its capacity for localization dies out.

Figure 4 shows the variations of several parameters of shear formation band with time. The general trend is of increasing strain rate and temperature, but decreasing stress with time for narrowing bands. This is obviously reasonable.

Finally, in Fig. 5 there is a comparison between the approximate solutions and the numerical computation, in which a routine difference scheme has been used to solve eqns. (9), (11) and (12). There is fair agreement between the two sets of results. This shows that the model assumptions (32) and (33), which greatly simplified the computation, are reasonable.

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