LETTER TO THE EDITOR

A Hall Dynamo Model for the Reversed Field Pinch

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Abstract—A mechanism for the Reversed Field Pinch (RFP) dynamo is proposed, based on the non-linear Hall effect of a saturated helical MHD instability. The sign and magnitude of the effect are shown to be those required for the RFP dynamo. Predictions of the model are in accord with RFP fluctuation measurements.

1. INTRODUCTION

It is well known that the sustainment and creation of a reversed field configuration in the RFP cannot be explained by an Ohm's law of the form

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta\mathbf{J}. \tag{1}$$

This is because the applied electric field $\mathbf{E}$ is in the toroidal direction and yet, in a steady-stage configuration where $d\mathbf{B}/dt = 0$, it is observed that poloidal current is driven at the reversal surface. Many models have been put forward to address this problem. For instance the mean-field effect of non-axisymmetric perturbations (with velocity $\delta\mathbf{v}$ and field $\delta\mathbf{B}$) due to the non-linear Lorentz term may be included to give an Ohm's law of the form (Elasser, 1950; Parker, 1955; Steenbeck et al., 1966; Gimblett and Watkins, 1975; Gimblett and Allan, 1976; Moffat, 1978; Krause and Radler, 1980; Keinigs, 1983)

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} + \langle\delta\mathbf{v} \times \delta\mathbf{B}\rangle = \eta\mathbf{J}, \tag{2}$$

where triangular brackets indicate an ensemble average. Under particular assumptions concerning the nature of the assumed-turbulent perturbation spectrum this model can describe a steady-state reversed field configuration (Gimblett and Watkins, 1975). A specialised case of this model, perhaps more applicable to experimental observations which do not see such a turbulence as the dominant magnetic activity, is where the above perturbations are taken as the linearised eigensolutions of a single helical MHD mode and ensemble averaging is replaced by a helical and temporal averaging. In the ideal MHD approximation this gives rise to a mean field effect of the correct order of magnitude but that is only present for modes with real growth rate. Unfortunately it is hard to relate this to experimental observations in modern RFP's that do not show purely growing modes (e.g. Antoni and Ortolani, 1983; Hutchinson et al., 1984). When the resistive MHD eigensolutions are used the constraint that modes must be purely growing is relaxed but the mean field effect appears too small.

Other "dynamo" models have been proposed which rely on current drive by non-local electric fields (Rusbridge, 1977; Rusbridge, 1982; Jacobson and Moses, 1984). This mechanism is applicable when RFP fields are stochastic, a situation which has some experimental backing (Hutchinson et al., 1984) although important ramifications concerning transport.

In this letter we propose a new dynamo model which does not require the magnetic field to be stochastic but is based on a generalised Ohm's law which specifically includes the Hall term:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta\mathbf{J} + \frac{1}{n_e e}(\mathbf{J} \times \mathbf{B}). \tag{3}$$

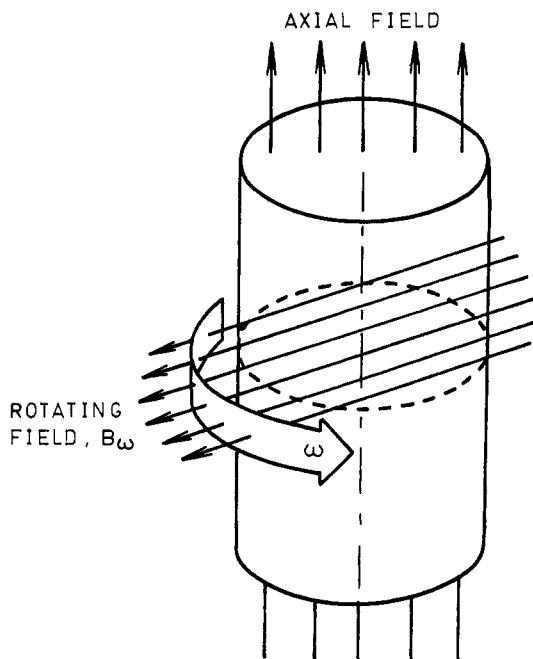(Here n_e denotes the electron density and e, the electron charge).

FIG. 1.—Diagram showing how current is driven in the Rotamak. A rotating magnetic field is applied such that it sweeps through a low density (pre-ionised) gas contained within the cylinder shown. Under the appropriate conditions (inequalities 4 and 5) the electrons are convected with the field and a Hall electron current is driven in the direction of the arrow shown. The resulting current loop is held in toroidal equilibrium by the applied axial field. The final configuration is thus a compact torus.

2. THEORY

(i) Rotating field current drive

It has been shown very clearly, both theoretically and experimentally, that the Hall term can significantly modify plasma behaviour from that predicted under the usual MHD description (BLEVIN and THONEMAN, 1965; JONES and HUGRASS, 1981; HUGRASS and GRIMM, 1981; HUGRASS, 1982; HUGRASS et al., 1981; HUGRASS et al., 1979). In particular in the Rotamak (HUGRASS et al., 1980; DURANCE et al., 1982; JONES, 1984) and other devices (e.g. DUTCH and MCCARTHY, 1986) it has proved possible to drive large Hall currents by the application of rotating magnetic fields (Fig. 1) which satisfy the following two criteria:

$$\omega_{ce}/v_{ei} \gg 1 \tag{4}$$

and
$$\omega_{ci} < \omega < \omega_{ce}. \tag{5}$$

Here ω_{ce} and ω_{ci} are the electron and ion cyclotron frequencies calculated with respect to the rotating field, v_{ei} is the electron–ion collision frequency and ω is the rotating field frequency. These conditions respectively ensure that the Hall term dominates the resistive dissipation term in the Ohm's law and that the rotating magnetic field will drive the electrons but not the ions. The explanation for the current drive effect is simple. In the above limits the relevant Ohm's law may be written as

$$\mathbf{E} + \mathbf{v}_e \times \mathbf{B} = 0, \tag{6}$$

where $\mathbf{v}_e$ is the electron fluid velocity. In analogy with Alfvén's theorem this equation dictates that the magnetic field is "frozen" into the *electron* fluid (LIGHTHILL, 1960). Thus an applied rotating or travelling magnetic field will convect electrons in the direction of its propagation, driving a Hall current unless the geometry is such that a counteracting electrostatic field is generated.
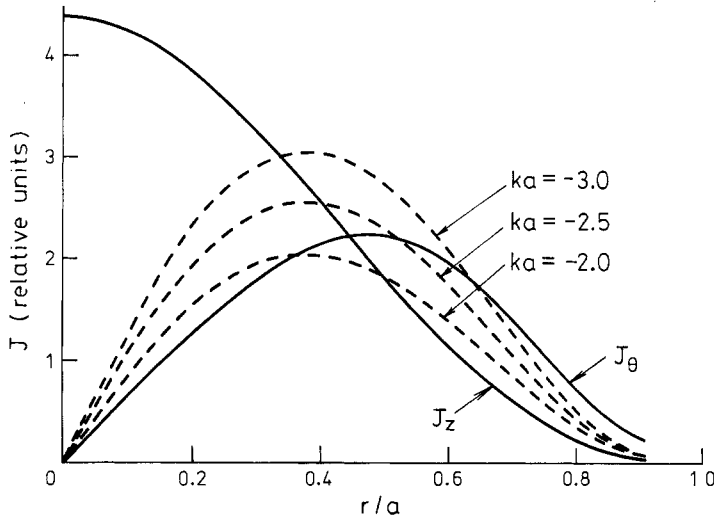
FIG. 2.—Comparison of the predicted J_θ profile calculated from the experimentally observed J_z profile (of BROTHERTON-RATCLIFFE and HUTCHINSON, 1984) using equation (7). The full lines show the experimentally observed J_θ and J_z. The three dotted lines show the calculated J_θ for the three indicated values of ka.

(ii) A Hall dynamo model for the RFP

There exists a strong analogy between the observed magnetic fluctuations in the RFP and the rotating magnetic field used to drive current in the Rotamak device. In the RFP the principal fluctuations are low frequency helical perturbations that have phase velocities in the laboratory frame much smaller than the electron fluid velocity (HUTCHINSON et al., 1984). In general, a direct consequence of this is that a high frequency helical travelling field will exist in the electron fluid frame. If this travelling field satisfies the inequalities 4 and 5 then the strong tendency towards electron "flux freezing" will convect the electron fluid in such a way as to reduce the rate at which the flux varies through a given electron fluid element. In the limit that this convection results in the flux through a given electron fluid element remaining invariant in time, the electron fluid trajectory will actually be aligned with the ignorable coordinate of the helical perturbation. Of course, realistically such a state would not be expected to occur due to finite plasma resistivity. But rather there would exist a tendency for the average plasma current to follow approximately such a trajectory, thus generating the required poloidal current.

The qualitative picture of this RFP dynamo is therefore as follows: The applied toroidal electric field drives the electrons toroidally. The low frequency helical magnetic perturbations observed in the RFP then force the electron fluid into a (sustained) helical path such that in the final equilibrium state, which includes the small amplitude helical equilibrium of the observed perturbations, the flux through a given electron fluid element remains invariant.

Taking typical numbers (BROTHERTON-RATCLIFFE et al., 1985; BROTHERTON-RATCLIFFE and HUTCHINSON, 1984) for the HBTX1A device, where the mode amplitudes are roughly 2–3% of the equilibrium field we find that conditions 4 and 5 are indeed easily satisfied. In addition the helicity of the observed perturbations is such that they are resonant inside the reversal surface. The driven poloidal current is thus in the correct direction. Moreover, assuming that complete electron "flux freezing" occurs we may calculate that

$$J_\theta = -(ka/m)(r/a)J_z, \tag{7}$$

where ka is the product of the longitudinal wavenumber and the minor radial size of the plasma, m is the poloidal mode number and r/a is the non-dimensional minor radial coordinate. If we take the published J_z profile (BROTHERTON-RATCLIFFE and HUTCHINSON, 1984) we may calculate the J_θ profile by using the experimentally determined values of $m = 1$ and $ka = -2.5$ to -3.0. This is shown in Fig. 2 in comparison with the experimentally observed J_θ profile. Clearly the prediction is reasonable.

(iii) The Hall dynamo as a non-linear consequence of an instability

The ideas established above have utilised the "Hall" equivalent of Alfvén's theorem. In this section we

quantify these ideas by looking at the problem in the spirit of equation 2 but using the following Ohm's law:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = \eta\mathbf{J} + \frac{1}{n_e e}(\mathbf{J} \times \mathbf{B}) + \frac{1}{n_e e}\langle\delta\mathbf{J} \times \delta\mathbf{B}\rangle. \tag{8}$$

Here the last term represents a Hall electric field specifically due to the experimentally observed saturated helical perturbation. Perturbed quantities are prefixed by δ's and triangular brackets indicate a helical and temporal average.

In this model we imagine that resistive diffusion of the poloidal current density at the reversal surface is exactly balanced by the (poloidal) quadratic Hall effect of the saturated helical instability. For a single helical Fourier mode we may write using cylindrical coordinates (r, θ, z):

$$
\begin{aligned}
b_r(r, \theta, z, t) &= b_r(r, t)\cos(\chi) + b_r{}^+(r, t)\sin(\chi) \\
b_\theta(r, \theta, z, t) &= b_\theta(r, t)\sin(\chi) + b_\theta{}^+(r, t)\cos(\chi) \\
b_z(r, \theta, z, t) &= b_z(r, t)\sin(\chi) + b_z{}^-(r, t)\cos(\chi)
\end{aligned}
\tag{9}
$$

where $\chi = m\theta + kz$. This gives the poloidal quadratic Hall effect for a single mode as

$$\frac{1}{n_e e}\langle \mathbf{j} \times \mathbf{b}\rangle_\theta = \frac{1}{2Tn_e e\mu_0}\int_0^T \left\{\frac{1}{r}b_r{}^+ \frac{\partial}{\partial r}(rb_\theta) + \frac{1}{r}b_r\frac{\partial}{\partial r}(rb_\theta{}^+) - kb_\theta{}^+ b_z + kb_\theta b_z{}^+\right\}\mathrm{d}t. \tag{10}$$

If we estimate the size of any of the four terms in the equation above using the HBTX1A figures (BROTHERTON-RATCLIFFE *et al.*, 1985; BROTHERTON-RATCLIFFE and HUTCHINSON, 1984) of 2% fluctuation amplitude or 0.002 T, $|ka| = 2.5$, $a = 0.26$ m and $n_e \approx 2.7 \times 10^{19}$ m^{-3} we calculate the Hall electric field to be approximately 3.5 Vm^{-1} which with a measured conductivity of $\sigma \approx 2 \times 10^5$ mho m^{-1} gives a very rough estimate of the driven poloidal current density of 0.7 MA m^{-2}. This compares favourably with the experimentally observed poloidal current density of 0.45 MA m^{-2} at the reversal surface.

3. CONCLUSION

A mechanism for the RFP dynamo based on the non-linear Hall effect of a saturated MHD instability has been presented. The principle is very similar to that of "Rotating field current drive" (JONES, 1984) utilised in the Rotamak. Estimates indicate that the effect is of the correct magnitude and polarity to explain the sustainment of reversal in the presence of resistive diffusion. The model is in accord with experimental observations of fluctuations in modern RFP's. Quantitative results from a numerical investigation will be published shortly.

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