THE FATIGUE LIFE ESTIMATION OF NOTCHED SPECIMENS UNDER RANDOM LOADING

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The LY12-cz aluminium alloy sheet specimens with a central hole were tested under constant amplitude loading, Rayleigh narrow band random loading and a typical fighter broad band random loading. The fatigue life was estimated by means of the nominal stress and the Miner's rule. The stress cycles were distinguished by the rainflow count, range count and peak value count, respectively. The comparison between the estimated results and the test results was made. The effects of random loading sequence and small load cycles on fatigue life were also studied.

1. Specimens and test machine

The central hole specimens with a concentration factor of $K_t = 2.60$ were made from LY12-cz aluminium alloy thin sheet. The chemical compositions of the alloy are shown in Table 1. The ultimate strength of the LY12-cz aluminium alloy is 450.8 N/mm^2 . At 0.2% strain, the stress is 352.8 N/mm^2 .

The shape and size of the specimens are shown in Fig. 1. The fatigue test under constant amplitude loading, Rayleigh narrow band random loading and the typical fighter broad band random loading were carried out in MTS 810 (10 t) fatigue test machine. The specimens were subjected to repeated axial loads.

Table 1
The chemical compositions of the alloy

Compositions	Mg	Cu	Mn	Al
Contents %	1.62	4.56	0.49	Remainder

2. Constant amplitude fatigue test

The mean stress was 142.10 N/mm^2 in the constant amplitude test. The test frequency was 5

Table 2
The results of constant amplitude fatigue test

No.	Stress amplitude (N/mm^2)	Number of cycles ($\times 10^4$)	Mean ($\times 10^4$)
1		1.46	
2	142.10	1.05	1.12
3		1.16	
4		0.88	
5		1.67	
6	132.30	1.44	1.64
7		1.83	
8		1.82	
9	122.50	2.04	1.90
10		3.57	
11	93.10	3.11	3.33
12		7.34	
13	73.50	6.91	7.12
14	58.80	13.3	15.1
15		17.2	
16	49.00	30.6	30.6

Fig. 1. LY12-cz central hole specimen 1 mm thick.

Fig. 2. $S-N$ curve.

Hz. The test results are shown in Table 2. The $S-N$ curve can be described as

$$S = AN^B \quad \text{or} \quad \log S = \log A + B \log N. \quad (1)$$

The stress amplitude is S and number of cycle to failure is N . In eq. (1), A and B are two parameters determined by the least square method. Their values are $A = 3402.76 \text{ N/mm}^2$ and $B = -0.3396$ with a correlative factor of $r = 0.9889$. The corresponding $S-N$ curve is shown in Fig. 2.

3. Rayleigh narrow band random fatigue test

It is well known that the stress amplitude with a Rayleigh distribution in case of Gaussian narrow band random loading is associated with the expression [1]

$$P(S) = S/\sigma^2 \exp(-S^2/2\sigma^2), \quad (2)$$

in which P is the probability density function and σ is the root-mean-square (RMS) of the stress.

Fig. 3. Rayleigh distribution.

Fig. 4. A sample of the Rayleigh narrow band random loading.

Refer to Fig. 3 for the Rayleigh distribution. The minimum stress amplitude is $S_{\min}$ and maximum is $S_{\max}$. The area under the curve is divided into

Table 3
The test results for Rayleigh narrow band random loading

No.	RMS stress (N/mm ²)	Number of blocks to failure	Number of cycles to failure ($\times 10^4$)	Mean ($\times 10^4$)	Loading sequences
17		419	5.36		
18		435	5.44		
19		384	4.92		
20		363	4.65		
21	87.22	347	4.44	4.81	Random
22		403	5.16		
23		421	5.39		
24		359	4.60		
25		341	4.36		
26		287	3.67		
27		396	5.07		
28		484	6.20	5.76	
29	87.22	476	6.09		Low-High
30		449	5.75		
31		347	4.44		
32		501	6.41		
33	87.22	447	5.72	6.17	High-Low
34		641	8.20		
35		474	6.07		
36		458	5.86		
37		496	6.35		
38	87.22	504	6.45	6.03	Low-High-Low
39		427	5.47		
40	94.47	247	3.16	3.16	
41		383	4.90	4.70	
42	93.20	351	4.49		
43	91.24	353	4.52	4.52	Random
44	86.73	475	6.08	6.08	
45		492	6.30		
46	80.46	537	6.87	6.59	

128 intervals possessing equal possibility. The Rayleigh narrow band random loading was generated by the Monte-Carlo method. It contained 128 cycles. The minimum stress amplitude was taken as 39 N/mm² and the maximum was taken as 137.2 N/mm². The RMS stress amplitudes were varied from 80.46 to 94.47 N/mm². The ratio of maximum stress amplitude to RMS stress amplitude was less than 1.7. The mean stress was 142.10 N/mm². The ratio of the number of crossing mean stress to the number of peak above mean stress was about 1.0 which will be referred to as the "crossing" factor. The test frequency was 5 Hz. A sample of Rayleigh narrow band random loading is shown in Fig. 4. The fatigue test of low-high, high-low and low-high-low sequences were also made in order to study the effect of load sequences on fatigue life. The test results are given in Table 3. The relation between the RMS stress amplitude and the number of cycle to failure is also shown in Fig. 2.

4. The fighter broad band random fatigue test

A typical fighter aircraft load spectrum was used in the broad band random test [2]. There were 2601 peaks in a block. The minimum stress amplitude was 16 N/mm² and maximum stress amplitude was 184.64 N/mm². The RMS stress amplitude was 63.21 N/mm² and the mean stress was 142.10 N/mm². The fatigue test omitting small load cycles was also carried out in order to study the effect of omitting small load cycles on fatigue life. The small load cycles which stress amplitudes were below 37.24, 56.00, 74.68 and

Fig. 5. A sample of the fighter broad band random loading.

93.35 N/mm² were omitted. The corresponding peaks were 1601, 649, 355 and 163. The RMS stress amplitudes were from 63.21 to 131.61 N/mm² and $S_{\max}/\sigma$ was less than 2.95. The crossing fac-

tors were 0.72, 0.82, 0.90, 0.94 and 0.98 for random loading peaks of 2601, 1601, 649, 355 and 163, respectively. A sample of the fighter broad band random loading is shown in Fig. 5 and the test results are given in Table 4. The relation between the RMS stress amplitudes and the number of cycles to failure is also shown in Fig. 2.

Table 4
The test results of the fighter loading spectrum

No.	Num- ber of peaks	RMS stress (N/ mm ²)	Num- ber of blocks	Mean (N/ mm ²)	Number of cycles ($\times 10^4$)	Mean ($\times 10^4$)
47			78		10.1	
48			49		6.37	
49			77		10.0	
50			91		11.8	
51	2601	63.21	121	86.2	15.7	11.2
52			88		11.4	
53			114		14.8	
54			96		12.5	
55			83		10.8	
56			65		8.45	
57			113		9.04	
58	1601 ^a	77.91	97	105.0	7.76	8.40
59			105		8.40	
60			99		3.20	
61	649 ^b	102.70	143	122.7	4.63	3.98
62			126		4.08	
63	355 ^c	116.33	181	180.5	3.20	3.20
64			180		3.19	
65	163 ^d	131.61	223	211.5	1.81	1.72
66			200		1.62	

^a The stress amplitude below 37.34 N/mm² was omitted.

^b The stress amplitude below 56.01 N/mm² was omitted.

^c The stress amplitude below 74.68 N/mm² was omitted.

^d The stress amplitude below 93.35 N/mm² was omitted.

5. The fatigue life estimation

The fatigue life was estimated by the nominal stress and Miner's rule. The stress cycles under the random loads were distinguished by the rainflow count, range count and peak value count respectively. The damage per cycle was calculated by the $S-N$ curve obtained in the constant amplitude test as follows:

$$D_i = 1/N_i = (S_i/A)^{-1/B}, \quad (3)$$

$$D_b = \sum_i^K D_i, \quad (4)$$

$$\text{BTF} = 1/D_b. \quad (5)$$

The damage of the i th cycle is D_i and the corresponding number of cycles to failure is N_i with stress the amplitude S_i . The number of cycle in a block is K and BTF denotes the number of block to failure. The comparison between the estimated results and the test results are shown in Tables 5 and 6 and Figs. 6 and 7.

Table 5

The comparison between the estimated results and test results for the Rayleigh narrow band random loading

RMS stress (N/mm ²)	Test results (Cycle number) ($\times 10^4$)	Estimated Results			Ratio ^a
		Rainflow ($\times 10^4$)	Range ($\times 10^4$)	Peak ($\times 10^4$)	
94.47	3.16	3.51	3.60	3.48	0.90
93.20	4.70	3.64	3.74	3.60	1.29
91.24	4.52	3.86	3.97	3.82	1.17
87.22	4.81	4.36	4.49	4.31	1.10
86.73	6.08	4.45	4.58	4.39	1.37
80.46	6.59	5.51	5.68	5.42	1.20

^a The ratio of test results to estimate results by rainflow count method.

Table 6

The comparison between the estimated results and test results for the fighter broad band random loading

Number of peaks	RMS (N/mm ²)	Test cycles ($\times 10^4$)	Estimated results			Ratio ^a
			Rainflow ($\times 10^4$)	Range ($\times 10^4$)	Peak ($\times 10^4$)	
2601	63.21	11.2	8.98	11.3	7.19	1.25
1601	77.91	8.40	5.65	6.46	4.89	1.49
649	102.70	3.99	2.82	2.96	2.61	1.41
355	116.33	3.21	2.01	2.07	1.91	1.59
163	131.61	1.43	1.43	1.45	1.37	1.00

^a The ratio of test results to estimate results by rainflow count method.

Fig. 6. The comparison between the estimated results and the test results for the Rayleigh narrow band loading.

In Table 5 and Fig. 6, it is seen that there are no obvious differences between the estimated results by rainflow count, range count and peak value count. In this case, the crossing factor is equal to about 1.0. In Table 6 and Fig. 7, there are some differences between the estimated results by the different methods. When the total number of peaks are 2601 and 1601 with crossing factors of 0.72 and 0.82, respectively, the damage estimated by peak value count is largest and that by range count method is smallest. The damage estimated by rainflow count method is moderate. No obvious differences are found when the total number of peaks are 649, 355 and 163 with crossing factors of 0.90, 0.94 and 0.98, respectively.

The ratio of the test results to the estimated results is from 0.90 to 1.37 in case of Rayleigh narrow band random loading. The mean value is 1.17. The ratio of the test results to the estimated results is from 1.0 to 1.59 for the fighter broad band random loading. The mean value is 1.35.

In Table 3, damage for the low-high sequence is

Fig. 7. The comparison between the estimated results and the test results for the fighter random loading.

largest while damage for the high-low sequence is smallest. The damage for low-high-low sequence is moderate while damage for random sequences is bigger than the damage for sequence loading. The test results for the random loading is about equal to 80% of that of sequence loading.

6. Conclusions

(1) No obvious differences are found for the results estimated by rainflow count method, range count method and peak value count method when applying the Rayleigh narrow band random loading. For the fighter broad band random loading, the damage estimated by the peak value count method is largest and the damage estimated by range count method is smallest. The damage estimated by rainflow count method is moderate. When small load cycles were omitted and crossing factors were extended to 1.0, consistent results

were obtained by the different methods.

(2) The fatigue life estimated by the nominal stress and Miner's rule are approximately the same as those from the tests and they are conservative.

(3) The relation between the RMS stress amplitudes and the number of cycles to failure N in Rayleigh random loading and the fighter random loading is nearly the same as that of the S - N relation for constant amplitude tests.

(4) The test results of fatigue life for random sequences are equal to about 80% of the test results for sequence loading in the Rayleigh narrow band random loading.

References

- [1] D.E. Newland, *An Introduction to Random Vibrations and Spectral Analysis*, Longman (1975).
- [2] J.B. Chang, "Round-robin crack growth predictions on center-cracked tension specimens under random spectrum loading", *ASTM STP 748*, 17-21 (1981).