Technical Notes

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Second-Order Thickness Terms in Unsteady Wing Theory

Weikai Gu*
Academia Sinica, Beijing, China

RECENTLY, there has been some speculation as to which of the unsteady flow terms arising in the calculation of second-order theory on a three-dimensional body should be retained. A part of the necessary conditions for the second-order theory of irrotational compressible flow past a thin three-dimensional body have been set forth by Qian.[1] An extension of his analysis to flow past three-dimensional wings of thickness ratio δ shows that the necessary conditions for the second-order theory in the neighborhood of the wing are that all of the following conditions be satisfied:

$$(M\delta)^2 \ll 1 \tag{1}$$

$$\delta^2 \ll 1 \tag{2}$$

$$K(M\delta)^2 \ll 1 \tag{3}$$

$$K\delta^2 \ll 1 \tag{4}$$

$$|M-1| \gg (M\delta)^2 \tag{5}$$

$$K = \text{or} \ < \mathcal{O}(1) \tag{6}$$

where M is the freestream Mach number and K the reduced frequency (or, more generally, a dimensionless measure of the time ratio of change). In this case, we have

$$\phi_{0yy} + \phi_{0zz} - (M^2-1)\phi_{0xx} - 2M^2\phi_{0xt} - M^2\phi_{0tt} = 0 \tag{7}$$

$$\begin{aligned}\phi_{1yy} + \phi_{1zz} &- (M^2-1)\phi_{1xx} - 2M^2\phi_{1xt} - M^2\phi_{1tt} \\ &= M^2[(\gamma-1)(\phi_{0x}+\phi_{0t})(\phi_{0xx}+\phi_{0yy}+\phi_{0zz}) \\ &+ 2(\phi_{0x}\phi_{0xx}+\phi_{0y}\phi_{0yy}+\phi_{0z}\phi_{0zz}) + 2(\phi_{0x}\phi_{0xt} \\ &+ \phi_{0y}\phi_{0yt}+\phi_{0z}\phi_{0zt})]\end{aligned} \tag{8}$$

$$\phi_{0z} = H_t + H_x \qquad \text{at } z=0 \tag{9}$$

$$\phi_{1z} = \phi_{0x}H_x + \phi_{0y}H_y - \phi_{0zz}H \qquad \text{at } z=0 \tag{10}$$

$$C_{p0} = -2(\phi_{0x}+\phi_{0t}) \qquad \text{at } z=0 \tag{11}$$

$$\begin{aligned}C_{p1} &= -2(\phi_{1x}+\phi_{1t}) + (M^2-1)\phi_{0x}^2 - \phi_{0y}^2 - \phi_{0z}^2 \\ &+ 2M^2\phi_{0x}\phi_{0t} + M^2\phi_{0t}^2 - 2(\phi_{0xz}+\phi_{0tz})H \qquad \text{at } z=0\end{aligned} \tag{12}$$

where the equation of the wing surface is assumed to be $z = H(x,y,t) = \mathcal{O}(\delta)$, Φ is the nondimensional velocity perturbation potential, ϕ_0 the linear part of Φ, and ϕ_1 the second-order part of Φ. Note that δ is the expansion parameter.

For high Mach numbers,

$$M^2 \gg 1 \tag{13}$$

we have

$$\phi_{0yy} + \phi_{0zz} - M^2\phi_{0xx} - 2M^2\phi_{0xt} - M^2\phi_{0tt} = 0 \tag{14}$$

$$\begin{aligned}\phi_{1yy} + \phi_{1zz} &- M^2\phi_{1xx} - 2M^2\phi_{1xt} - M^2\phi_{1tt} \\ &= M^2[(\gamma-1)(\phi_{0x}+\phi_{0t})(\phi_{0yy}+\phi_{0zz}) + 2(\phi_{0y}\phi_{0yy} \\ &+ \phi_{0z}\phi_{0zz}) + 2(\phi_{0y}\phi_{0yt}+\phi_{0z}\phi_{0zt})]\end{aligned} \tag{15}$$

$$\phi_{0z} = H_t + H_x \qquad \text{at } z=0 \tag{16}$$

$$\phi_{1z} = \phi_{0y}H_y - \phi_{0zz}H \qquad \text{at } z=0 \tag{17}$$

In this case and at the usual reduced frequencies, the problems can be reduced to a sequence of familiar steady lifting surface problems that can be solved by existing methods.[1] For a transonic flow, Eq. (5) will fail and the first-order equation (7) will include three more nonlinear terms in the right side,

$$M^2[(\gamma+1)\phi_{0x}\phi_{0xx} + (\gamma-1)\phi_{0t}\phi_{0xx} + 2\phi_{0x}\phi_{0xt}]$$

where γ is the ratio of specific heats. The second-order equation (8) will include seven more terms in the right side

$$\begin{aligned}M^2[2\phi_{0x}(\phi_{0y}\phi_{0xy}+\phi_{0z}\phi_{0xz}) + N\phi_{0x}^2\phi_{0xx} \\ + (N-1)(\phi_{0z}^2\phi_{0xx}+\phi_{0z}^2\phi_{0yy}+\phi_{0x}^2\phi_{0zz}+\phi_{0y}^2\phi_{0zz})]\end{aligned}$$

where $N=(\gamma+1)/2$. In addition, the pressure coefficient will include four more terms

$$\begin{aligned}M^2\left(1-\frac{2-\gamma}{3}M^2\right)\phi_{0x}^3 + M^2[1-(2-\gamma)M^2]\phi_{0t}\phi_{0x}^2 \\ -\frac{2-\gamma}{3}M^4\phi_{0t}^3 - (2-\gamma)M^4\phi_{0x}\phi_{0t}^2\end{aligned}$$

When Eq. (1) fails and

$$(M\delta)^2 = \mathcal{O}(1) \tag{18}$$

the first-order equation will still be linear, but the second-order equation will include five more terms in the right side,

$$\begin{aligned}2M^2\phi_{0y}\phi_{0z}\phi_{0yz} + NM^2(\phi_{0y}^2\phi_{0yy}+\phi_{0z}^2\phi_{0zz}) \\ + (N-1)M^2(\phi_{0z}^2\phi_{0yy}+\phi_{0y}^2\phi_{0zz})\end{aligned}$$

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*Research Associate, Aerodynamics Division, Institute of Mechanics.

In this case, the boundary conditions [Eqs. (9) and (10)] will also fail and they must be calculated on the surface of the wing. The expansion of the pressure coefficient will be divergent and Eqs. (11) and (12) will fail; hence, its original expression must be used.

When we deal with a rolling wing,[2] we can choose a set of axes fixed to and rolling with the wing. The nondimensional variable x axis coincides with the axis of roll. If the conditions

$$K\sigma/\mu = \mathcal{O}(1) \text{ or } \mathcal{O}(\delta) \tag{19}$$

$$\mu^2 < \sigma^2 \tag{20}$$

are satisfied, the additional term due to the Coriolis acceleration in the second-order equation is only

$$-2KM^2 y\phi_{0xy}$$

where the dimensionless parameter σ is a measure of the lateral extent of the boundaries and μ is a measure of the vertical extent of this neighborhood. The pressure coefficients include two more linear terms

$$-2(Kz\phi_{0y} - Ky\phi_{0z}) \qquad \text{for Eq. (11)}$$

$$-2(Kz\phi_{1y} - Ky\phi_{1z}) \qquad \text{for Eq. (12)}$$

If the condition

$$\mu^2 \ll \sigma^2 \tag{21}$$

is satisfied, besides Eqs. (1-6), the second term on the left side of Eq. (7) can be neglected. This flow is a local two-dimensional one.

In addition to the above, there are many simpler cases, for example, when $K \ll \mathcal{O}(1)$, which are explained in detail in Ref. 3. In general, the second-order approximation will not be uniformly valid in the neighborhood of the wing edges and the Mach cone emanating from the vortex of the wing. But this difficulty can be circumvented by the PLK method.[1] The validity of the second-order theory beyond the variable range described here is very interesting for engineering calculations. Hence, this theory should be used with caution outside of this range until sufficient test data are available to indicate what range of parameters mentioned above should be used for any particular body.

References

[1]Qian, F. X. and Gu, W. K., "Accuracy and Application of a Second-Order Theory for Three-Dimensional Supersonic and Low Hypersonic Unsteady Flow Around a Thin Wing," *Acta Aeronautica et Astronautica Sinica*, Vol. 2, No. 1, 1981, pp. 1-9.

[2]Gu, W. K., "The Second-Order Roll Damping of Rolling Wings at Supersonic Speeds," to be published *Journal of Spacecraft and Rockets*.

[3]Gu, W. K., "The Analysis of Second-Order Theory," Institute of Mechanics, Academia Sinica, Beijing, China, Res. Rept. D.V. 15, 1981.

Wake Periodicity in Subsonic Bluff-Body Flows

J.L.F. Porteiro
University of South Florida, Tampa, Florida

Introduction

A SIGNIFICANT part of the total drag experienced by bluff bodies is generated by the low pressures existing in the near wake. This drag component is known as base drag and is strongly dependent on the fore- and afterbody geometry. This note reports an experimental study of the influence of the boundary-layer thickness and base mass transfer on the time-dependent, near-wake pressure and velocity fields of an axisymmetric blunt-based cylinder in subsonic flow. Both of these parameters are known to have a significant influence on the base pressure.[1-3]

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*Associate Professor, Chemical and Mechanical Engineering Department.

Experimental Apparatus and Technique

This investigation was conducted in the Rutgers axisymmetric near-wake tunnel (RANT II). This is the same facility used by Porteiro et al.[4] in their studies of the influence of mass transfer on the wake of a blunt-based body. The present work is a continuation of that study. The tunnel is an open-jet facility designed and constructed for interference-free studies of turbulent, subsonic, axisymmetric near-wakes behind a 1.9 cm diam cylindrical model. The model support sting is hollow, allowing the transfer of mass to and from the boundary layer and base region. Boundary-layer blowing and suction is carried out through a porous metal sleeve of 1.9 cm o.d. extending from the model support sting to a distance 3 diam upstream of the model base. Base mass transfer takes place through a porous metal plate 1.9 cm in diameter and 0.159 cm thick. Pressure regulators were used to stabilize the bleed air pressure and flow meters were used for metering the airflow.

Boundary-layer velocity measurements were made at a location 3 diam upstream of the base with a miniature total pressure probe and a Statham ± 3.45 kPa·g transducer. The probe location was zeroed by electrical continuity and its position could be determined within 0.025 mm in 152.4 mm of total travel. Total pressure measurements were taken at 27 radial locations chosen to provide detailed information on the velocity profile. Reference static pressure measurements were also taken at an axial location 3 diam upstream of the base. Base pressure was measured with a static pressure tap located at the center of the base of the model. An alcohol micromanometer providing readings to 0.05 mm of water was used for these measurements.

The fluctuating component of the base pressure was measured with a Kulite XCS-093-5 pressure transducer with a natural frequency of 70 kHz. The transducer dc output was analyzed with a Hewlett-Packard 3490A wave analyzer up to a frequency of 62 kHz.

Near-wake velocity studies were made with a constant-temperature hot-wire anemometer. An analysis of the frequency spectrum of the hot-wire signal was carried out at the stagnation point, in the far wake, and in the shear layer at points located in the vicinity of the base and at a halfway point in the near wake.

This investigation was carried out with a nominal Mach number of 0.11. The corresponding Reynolds number was 2.57×10^6/m. The nominal stagnation pressure was 102 kPa abs and the value of the stagnation temperature 280 ± 10 K. The approaching boundary layers were turbulent.

Experimental Results and Discussion

The fluctuating component of the near-wake velocity was studied by analyzing the frequency spectrum of the hot-wire signal up to a frequency of 62 kHz.

When the hot wire was placed in the shear layer very close to the base, analysis of the frequency showed no peaks in the signal amplitude for any combination of boundary-layer thickness and base mass transfer, except those involving moderate suction rates. Moderate suction rates produced a slight but noticeable increase in the signal amplitude at a frequency corresponding to a Strouhal number of 0.19. Altering the boundary-layer thickness brought about changes in the frequency of the amplitude maximum.

Analyses were also carried out at the location in the shear layer having the maximum level of turbulence ($X \cong 0.5D$) at the stagnation point and on the far-wake centerline at an ax-