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RESPONSE: Discussion of "MIXED MODE DUCTILE FRACTURE USING THE STRAIN ENERGY DENSITY CRITERION," by E.E. Gdoutos*

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We wish to express our appreciation for the discussor's continuous interest in the subject of mixed mode fracture and for providing the opportunity to further clarify the fracture criterion of maximum dilatational energy, or $(dW/dV)_v^{max}$. The discussor is right to state that the strain energy density theory advocated by the Institute of Fracture and Solid Mechanics of Lehigh University provides a methodology with which solutions for brittle and ductile fracture problems may be achieved by means of a number of hypotheses. The strain energy criterion based on the work theorem of statics itself is, however, not novel and can be readily found in a number of classical mechanics textbooks. The main question raised by the discussor is therefore not the validity of the theory of strain energy density rather than the hypothesis proposed using the fracture criteria of $(dW/dV)_{min}^{max}$ or $(dW/dV)_v^{max}$ for the application of the classical theory to determine the angle of crack initiation.

Any hypothesis can only be considered acceptable after it has been consistently tested to be valid by physical observations. This is a sound exercise of scientific investigation rather than simple mindedly branded as "fitting test data". The latter is true if the analysis performed is erroneous. The discussor's analysis is erroneous and has indeed led itself to "absurd" (the discussor's choice of word) conclusions and predictions. In the first instances, Fig. 1 in the Discussion is wrong and resulted from the incorrect expressions of (3b), (6), and (8) of the Discussion. The correct expressions should be

$$12\mu\, b_{12} = -(1 - 2\nu)(1 + \nu)\sin \tag{3b}$$

$$-\sin\theta k_1^2 - 2\cos\theta\, k_1 k_2 + \sin\theta\, k_2^2 = 0 \tag{6}$$

$$\theta_c = 2\beta - \pi \tag{8}$$

Equations (6) and (8) are deduced by satisfying the conditions

$$\frac{\partial}{\partial\theta}\left[\left(\frac{dW}{dV}\right)_v\right] = 0$$

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$$\frac{\partial^2}{\partial\theta^2}\left[\left(\frac{dW}{dV}\right)_v\right] < 0$$

and can be graphically represented in Fig. 1 of this Response. It is evident from both (8) and the figure that negative crack extension angles θ_c can be predicted and agree with the experimental observation rather than the "absurd" prediction of positive crack extension angles shown in the discussor's graph.

It should be pointed out that the proposed fracture criterion of $(dW/dV)_v^{max}$ is based on the physical consideration that crack initiation is governed by the change of dilatational energy density, a hypothesis in the strain energy density criterion the discussor has accepted. Unfortunately, the discussor failed to recognize the fact that should the prediction be proved to be drastically different such as the change of sign of predicted angle from negative to positive, this would lead to violation of the above stated hypothesis. Failing to recognize this apparent logic indeed "stems from the discussor's lack of a knowledge of the strain energy density criterion" (which again are the discussor's choice of words).

The question of the $(dW/dV)_v^{max}$ criterion in plasticity or in the presence of permanent deformation was raised as it may lead to some conceptual difficulties and mathematical inconsistencies. These problems have apparently not been shared by prominent scholars in plasticity including Kachanov [1] and Hill [2]. While we do appreciate the discussor's concern if superposition of two arbitrary stress states is performed in nonlinear analysis, the superposition involved in (A1) of the authors' paper* should not lead to any mathematical inconsistencies as the hydrostatic pressure - the second stress state superposed in the equation - will cause no permanent deformation in the Prandtl-Reuss materials chosen in the analysis. In other words, the same material response is obtained if arbitrary hydrostatic stresses are to be superposed on a specific stress state such as

$$\sigma_{ij} = (\sigma_{ij} - \sigma\delta_{ij}) + \sigma\delta_{ij} - \sigma_1\delta_{ij} + \sigma_1\delta_{ij} - \dots$$

We do however agree with the discussor's assertion that we made a rather "careless" remark that "For both the inclined angles (β = 30 deg and 60 deg), the maximum $(dW/dV)_v$ is located at θ = -30 degrees where the crack initiation is predicted to take place". Contrary to what the discussor asserts, this was not due to any lack of understanding on our part, and we appreciate the opportunity to clarify that the measured angles for β = 30 deg and 60 deg lay in the range of -25 deg to -45 deg. The above statement was intended to express the point that the predicted fracture angles for both β = 30 deg and 60 deg were approximately -30 deg. By careful examination of Figs. 9 and 10 in the authors' paper, the discussor should be able to identify the difference in the fracture angles θ_c for the thin plates embedded with an inclined angle crack of β = 30 deg and 60 deg at the maximum $(dW/dV)_v^{max}$. The main objective of the authors' paper was to provide a fracture criterion which can be used to predict fracture angles which are in general agreement with the experimental observation. Due to limitations imposed by the available computer memory,

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Last but not least, it is our view that the use of such harsh and abrasive remarks as those made by the discussor is contrary to the spirit of a normal healthy and constructive exchange of ideas and information about a challenging scientific problem. However, it is apparent to us that the discussor has chosen to resort to the use of discrediting remarks not only in this Discussion but also in the series of discussions he has cited himself in the subject of mixed mode fracture. We were consequently convinced that this is the type of communication the discussor is accustomed to and we have responded in kind. However, it is our wish that we could in future engage in scientific discussions and dialogue with mutual respect and understanding.

REFERENCES

[1] L.M. Kachanov, *Fundamentals of the Theory of Plasticity*, North-Holland Publishing Co., London (1971).

[2] R. Hill, *The Mathematical Theory of Plasticity*, Oxfodd University Press, London (1950).

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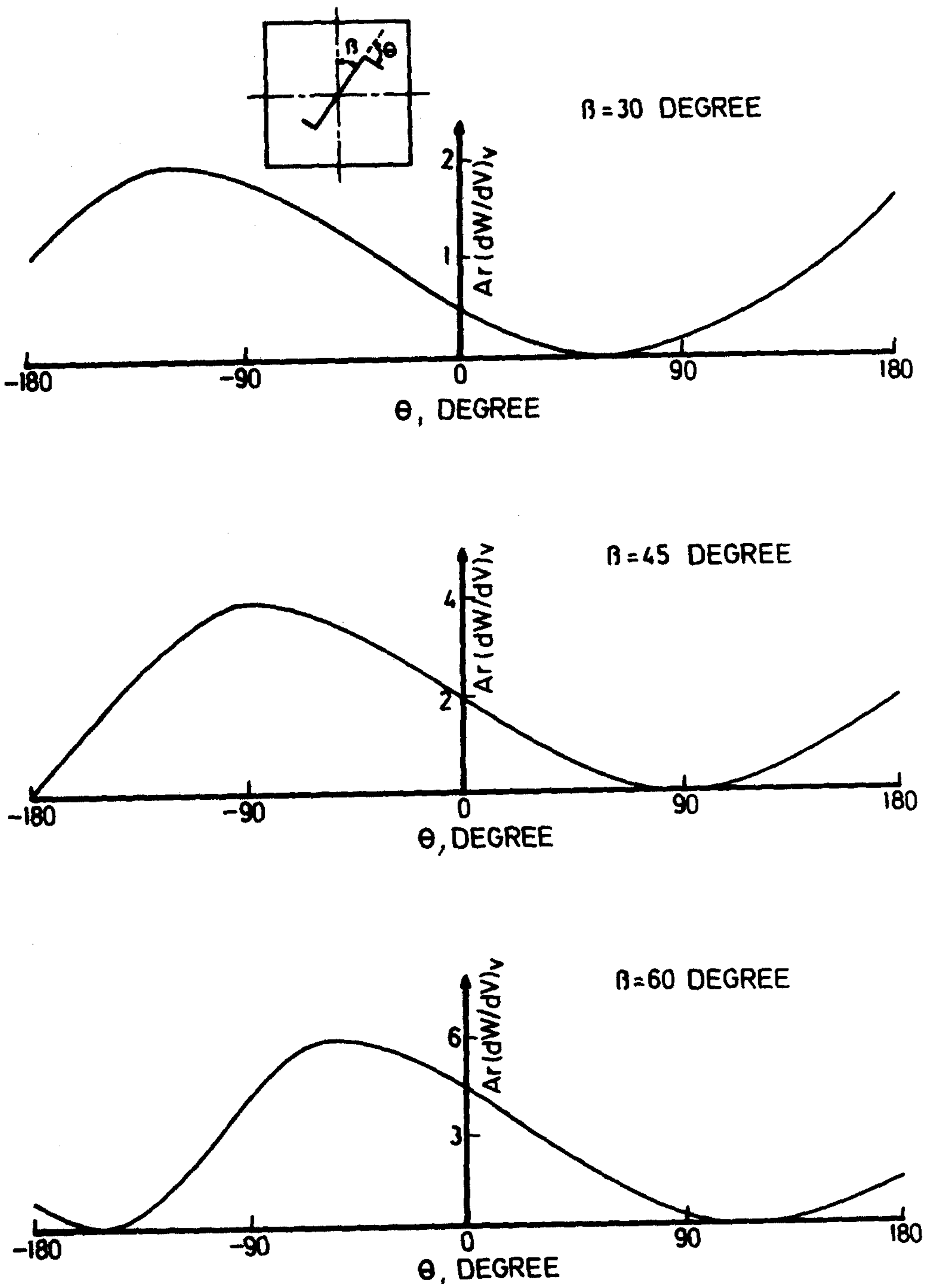

Figure 1. Fracture angle θ versus maximum dilatational energy density for ß=30, 45 and 60 degrees.