Dynamic Response of a Laminated Plate With Friction Damping

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A sandwich-type plate with metal facings and felt core, fastened by bolts, was studied using both test and finite-element analysis. This type of plate is cheap, light, damping-effective and without pollution; therefore, it is widely used in astronautical engineering. The tests were conducted for different felt thicknesses, bolt numbers, and fastening forces. The results show that the damping depends on friction between the plates and the felt. As compared with an identical stiffness solid plate, the damping of laminated plates can be increased up to 30 times. A mesh with rectangular elements was adopted in the finite-element analysis. In accordance with the slipping mechanism, a rectangular plate clamped on one edge was analyzed with the foregoing elements to determine the resonant frequency and the damping. The difference between the calculated and tested results was within 5 percent for the resonant frequency.

Introduction

In recent years, viscoelastic materials have been frequently used for decreasing structural vibration. Considerable damping can be achieved by using them, but the damping capacity of these materials is both frequency and temperature sensitive to a much greater extent than that found with friction. Therefore, frictional damping has also been studied and design criteria for the useful application of friction damping in vibrating structures have been established. As described in this paper, the damping in plate-type structures can be significantly increased by using laminated plates correctly fastened to allow controlled interfacial slip during vibration, and the damping and stiffness performance of such structures have been studied theoretically and experimentally.

Structure of the Laminated Plate and its Mechanical Mode

The sandwich-type plate consists of aluminum facings and felt core fastened by some bolts, as shown in Fig. 1. The thickness of the facing is 2 mm; the thickness of the core is 3 mm or 6 mm; the length and width of the plate are 420 mm and 220 mm.

The bolts were fitted with calibrated springs to control the clamping force, so that the actual clamping load could be found by measuring their deflection. The clamping force offered by each bolt is 24 kg. To simplify the actual state, let us suppose the clamping force per unit area of plate is a uniform load Q and we have $F=f\,Q$. The frictional factor $f=0.4$, was obtained by measurement. F is the frictional force in the interacting surface between the facing and core. Obviously, there was a shearing stress τ in the aforementioned surface when the plate was bent by the excited force. As shown in Fig. 2, F and τ are periodic forces. The facings and core were connected together for $\tau_0 < F$; the bolted laminated plate with felting core was considered as an ordinary laminated plate. The slip occurred in the interface for $\tau_0 \geq F$; thus the first function of the core is separating for upfacing and downfacing. Its antishearing function was replaced by an

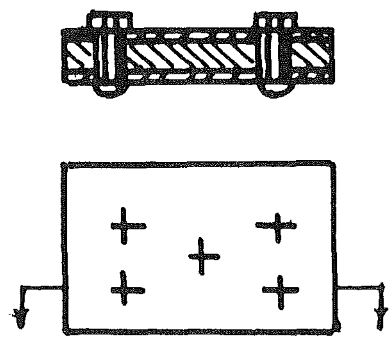

Fig. 1

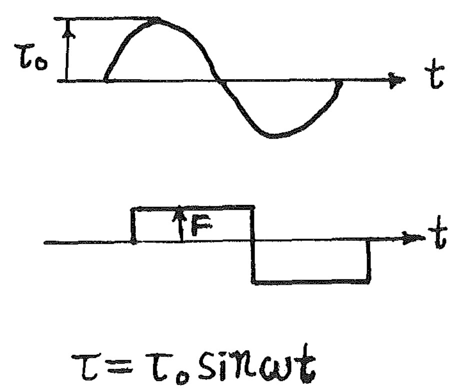

Fig. 2

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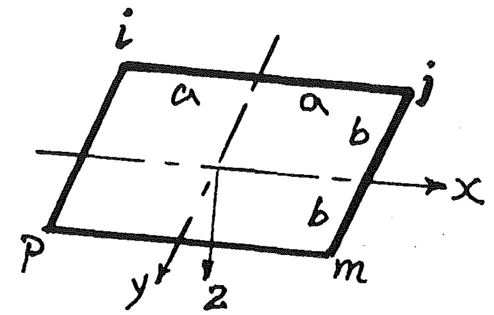

Fig. 3

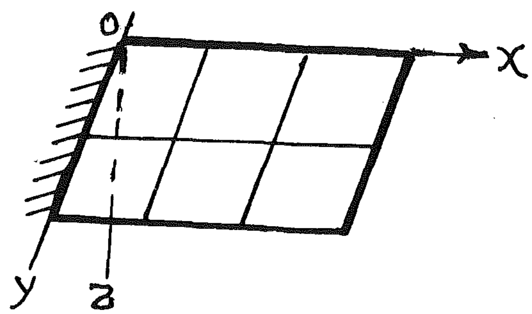

Fig. 4

idealized core. The shearing mode of the ideal core was determined by test. The damping of the laminated plate depends on frictional dissipation.

Formulation of the Rectangular Laminated Element

The state of deformation of the laminated plates can be fully described by five quantities: the uniform transverse displacement W and the longitudinal displacements of the midplanes of the facings U_1, V_1, U_3 and V_3. Let us use x, y, z to denote Cartesian coordinates. U, V are components of displacement parallel to x, y-axes. Symbols 1, 2, 3 denote upfacing, core, and downfacing, respectively. The symbol h_i ($i=1, 2, 3$) denotes layer thicknesses of laminated plate. Thus, on the basis of the linear elastic theory of the laminated plate, we have the following geometric relationships:

$$U_2 = \frac{1}{2}\left[(U_1+U_3) - \frac{1}{2}(h_1-h_3)\frac{\partial W}{\partial x}\right] - \frac{1}{h_2}\left[(U_1-U_3) - \frac{1}{2}(h_1+h_3)\frac{\partial W}{\partial x}\right]Z$$
$$V_2 = \frac{1}{2}\left[(V_1+V_3) - \frac{1}{2}(h_1-h_3)\frac{\partial W}{\partial y}\right] - \frac{1}{h_2}\left[(V_1-V_3) - \frac{1}{2}(h_1+h_3)\frac{\partial W}{\partial y}\right]Z \qquad (1)$$

From the formula (1) we can further obtain the strain components of the facings and the shearing strain components of the core. The finite element of the rectangular laminated plate is defined by nodes i, j, m and p, as shown in Fig. 3. The number of degrees of freedom per node is seven (W, $\partial W/\partial x$, $\partial W/\partial y$, U_1, V_1, U_3 and V_3) and the element degrees of freedom become 28. This, in turn, allows for 28 unknown coefficients $\alpha_i (i=1, \ldots, 28)$ in the polynomial expressions representing the displacement functions. These polynomials are chosen as follows:

$$\begin{aligned} W &= a_1 + a_2x + a_3y + a_4x^2 + a_5xy + a_6y^2 + a_7x^3 \\ &\quad + a_8x^2y + a_9xy^2 + a_{10}y^3 + a_{11}x^3y + a_{12}xy^3 \\ U_1 &= a_{13} + a_{14}x + a_{15}y + a_{16}xy \end{aligned}$$

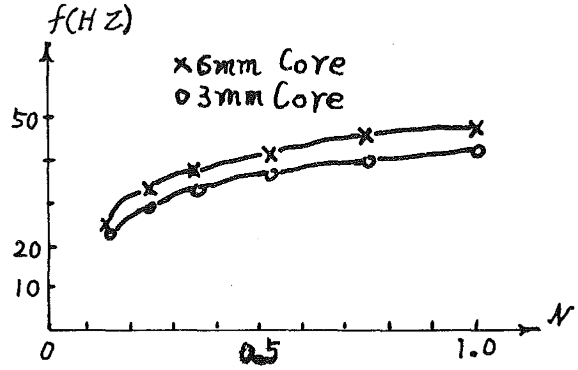

Fig. 5

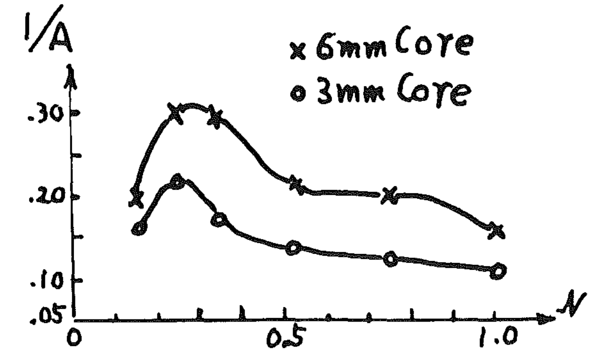

Fig. 6

$$\begin{aligned} V_1 &= a_{17} + a_{18}x + a_{19}y + a_{20}xy \\ U_3 &= a_{21} + a_{22}x + a_{23}y + a_{24}xy \\ V_2 &= a_{25} + a_{26}x + a_{27}y + a_{28}xy \end{aligned} \qquad (2)$$

According to the approach of reference [1], we can use the foregoing polynomial expressions to obtain the stiffness matrix of the elements, the mass matrix of the elements and the damping matrix of the elements. They are $[K]^e$, $[M]^e$, and $[C]^e$, respectively.

$$[K]^e = \int_{V^e} [B]^T[D][B]dV \qquad (3)$$

$$[M]^e = \int_{V^e} [N]^T\rho[N]dV \qquad (4)$$

$$[C]^e = \int_{V^e} [N]^T\nu[N]dV \qquad (5)$$

In these formulas, the matrix $[B]$ describes the relationship between the element strain and nodal displacement, $[D]$ is the matrix of material constants, $[N]$ are shape functions, ρ is material density, V^e denotes the element volume, ν is the damping factor, derived according to the equivalent damping principle. Thus we have

$$\nu = \frac{4F(h_1+h_3+2h_2)\int_S\left(\frac{\partial W}{\partial x} + \frac{\partial W}{\partial y}\right)dS}{\omega\pi(h_1+h_2+h_3)\int_S w^2 dS} \qquad (6)$$

where ν can be evaluated by Gaussian quadrature formula, S denotes the area of the rectangular element plane, W, $\partial W/\partial x$, $\partial W/\partial y$ are unknown quantities.

Calculated Example and Test

A cantilever rectangular laminated plate, as an assembly of

Table 1

Bolt numbers	5	8	12	18	26	35
N	0.14	0.23	0.34	0.51	0.74	1.00
f test(Hz)	26.8	31.6	35.8	38.7	43.0	44.6
f calcu.(Hz)	28.0	32.1	35.4	38.8	41.6	43.1
$1/A$ test	0.210	0.300	0.300	0.211	0.200	0.156
$1/A$ calu.		0.303	0.318			

rectangular elements is illustrated in Fig. 4. The total matrix $[K]$, $[M]$ and $[C]$ can be obtained by the sum of the corresponding element matrices $[K]^e$, $[M]^e$, and $[C]^e$, respectively. Thus, for sinusoidal vibration, we can write the vibration equation of the plate as follows:

$$(-\omega^2[M]+i\omega[c]+[K])\{g\}=\{F\} \tag{7}$$

and the eigenequation

$$[K]\{g\}=\omega^2[M]\{g\} \tag{8}$$

where $\{g\}$ and $\{F\}$ denote the displacement and the load, respectively. From equation (8), we can obtain the natural frequency ω. From equation (6), we can obtain the damping matrix $[C]$, which depends on displacements $\{g\}$. This shows that equation (7) is a group of forced vibration equations that include nonlinear damping, which can be solved by iteration.

The test was conducted for different size, felt thickness, bolt numbers and fastening forces. Each plate was excited on its foundation. At the first mode resonance of the plate, we measured the vibration frequency f and the ratio $1/A$ of the exciting amplitude to the transverse response amplitude on the free end of the plate.

Let us consider the middle plate as an example; its tested results are illustrated in Figs. 5 and 6. Figure 5 shows the relation of the first mode resonance frequency f to the relative clamping force N. Figure 6 shows the relation of the factor $1/A$ with N, where N is a function of the distributed density of the bolts when all bolts offer the same clamping force. Another test was made for the identical stiffness solid plate and the laminated plate.

The calculated and tested results are listed in Table 1.

Conclusions

1 Obvious slipping between facing and core was observed, and this is the main damping mechanism.

2 The damping and stiffness of the plate increased with the felt thickness.

3 The stiffness of the plate increased with the number of bolts and fastening loads.

4 The damping of the plate is dependent on the bolt numbers, and there is a optimum range for which maximum damping can be obtained.

5 The difference between calculated and measured results is within 5 percent for the resonant frequency.

6 As compared with the identical stiffness solid plate, the damping of laminated plates can be increased 30 times.

7 The mechanical mode was defined on the basis of bringing the effect of the structural stiffness and frictional damping into full play. Therefore, the mode is suited to the optimal damping range.

References

1 Zienkiewicz, O. C., *The Finite Element Method in Engineering Science*, published by McGraw-Hill Publishing Company Limited, 1971.