

# The thermal and mechanical structure of a two-dimensional plume in the Earth's mantle

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On the condition that the distribution of velocity and temperature at the mid-plane of a mantle plume has been obtained (pages 213–218, this issue), the problem of determining the lateral structure of the plume at a given depth is reduced to solving an eigenvalue problem of a set of ordinary differential equations with five unknown functions, with an eigenvalue being related to the thermal thickness of the plume at this depth. The lateral profiles of upward velocity, temperature and viscosity in the plume and the thickness of the plume at various depths are calculated for two sets of Newtonian rheological parameters. The calculations show that the precondition for the existence of the plume,  $\delta_T/L \ll 1$  ( $L$  = the height of the plume,  $\delta_T$  = lateral distance from the mid-plane), can be satisfied, except for the starting region of the plume or near the base of the lithosphere. At the lateral distance,  $\delta_T$ , the upward velocity decreases to 0.1–50% of its maximum value at different depths. It is believed that this model may provide an approach for a quantitative description of the detailed structure of a mantle plume.

## 1. Introduction

A mantle plume is understood to be a hot, narrow, upwelling flow in the Earth's mantle, accompanied by an efficient transfer of mass and energy from depth to the upper layers of the Earth. Mantle plumes may play an important role in mantle dynamics and plate tectonics (Morgan, 1971, 1972). It has been suggested that the origin of sea-floor spreading and plate motion is due to special two-dimensional plumes beneath mid-ocean ridges. The origin of the surface hot spots and linear island chains may be due to cylindrical plumes. R. Meissner (1981) suggested that the

development of continental margins is a consequence of two-dimensional plumes intruding a variable viscosity tectonosphere. The thermal and mechanical structure of mantle plumes must be known for an understanding of the dynamical processes of the Earth.

Essentially, no fluid–dynamical description of a mantle plume has been attempted so far. Only the numerical experiments of Parmentier et al. (1975) have yielded plume-like structures in cylindrical, base-heated, variable-viscosity Newtonian flows. Although an analytic similarity solution was given for a two-dimensional mantle plume by Yuen and Schubert (1976), a plume-like structure of upward

velocity in a Newtonian plume could not be found, because the preconditions under which the similarity solution exists were not in existence (Li et al., 1983).

Newtonian (or Nabarro–Herring) creep with a linear constitutive relation seems to be the dominant process in regions of very high temperatures and very low stresses (Vetter and Meissner, 1979). An upwelling flow of mantle plumes certainly belongs to regions of high temperature in the asthenosphere. We formulate an analytic theory for the structure of a two-dimensional plume in a medium with Newtonian rheology. Based on this theory, the parameters of mantle plumes, such as temperature, velocity, viscosity, plume-thickness, and their distributions, are calculated.

## 2. Mathematical analysis

The general hydrodynamical equations are used to describe the motion of mantle material. For flows in a Newtonian two-dimensional mantle plume, the hydrodynamical equations can be simplified according to Yuen and Schubert (1976), and Li and Guan (1979) as

$$(\partial U/\partial x) + (\partial V/\partial y) = 0 \quad (1)$$

$$(\partial/\partial y)[\mu(\partial U/\partial y)] + \rho g \alpha (T - T_\infty) = 0 \quad (2)$$

$$U(\partial T/\partial x) + V(\partial T/\partial y) = k(\partial^2 T/\partial y^2) \quad (3)$$

where the  $x$ -axis is vertically upward, the  $y$ -axis is horizontally to the right,  $U$  and  $V$  are velocity components in  $x$ - and  $y$ -directions, respectively.  $T$  is temperature,  $T_\infty$  is the ambient mantle temperature,  $\rho$  is the density,  $\mu$  is the viscosity,  $g$  is the gravitational acceleration,  $\alpha$  is the coefficient of expansion, and  $k$  is the thermal conductivity.  $y = 0$  is the mid-plane of the plume or the axis of symmetry (i.e., the boundary of the upward flows of two neighbouring cells). The zero point of the coordinate is the point where the axis of symmetry intersects the base of the lithosphere. Let us consider only the flows in the region  $y \geq 0$ ,  $x \leq 0$ .

In eq. 2 the viscosity is expressed as follows (Yuen and Schubert, 1976)

$$\mu = (T/2B) \exp[(E^* + P_0(x)V^*)/(RT)] \quad (4)$$

where  $E^*$  is the activation energy of the thermally activated deformation process,  $V^*$  is its activation volume,  $R$  is the gas constant.  $P_0$  is the static pressure and  $B$  is a constant. In the present paper the calculated results are given for two sets of Newtonian rheological parameters, and their values are given as follows (Yuen and Schubert, 1976):

Newtonian I

$$B = 2.4 \times 10^{-3} \text{ cm s K g}^{-1},$$

$$V^* = 11 \text{ cm}^3 \text{ mol}^{-1}, E^* = 95 \text{ kcal mol}^{-1};$$

Newtonian II

$$B = 1.8 \times 10^{-4} \text{ cm s K g}^{-1},$$

$$V^* = 9 \text{ cm}^3 \text{ mol}^{-1}, E^* = 104 \text{ kcal mol}^{-1}.$$

On the condition that the velocity  $U_w$  and temperature  $T_w$  at the mid-plane of a plume have been found, let us introduce two-dimensionless functions

$$U/U_w = g(\eta)(T - T_\infty)/(T_w - T_\infty) = p(\eta) \quad (5)$$

where  $\eta = y/\delta_T(x)$  is a new dimensionless independent variable, and  $\delta_T$  is the thermal thickness of the plume at depth  $x$ . It is evident that

$$g(0) = 1, p(0) = 1, p(1) = 0$$

Substituting eq. 5 into eq. 2, we obtain

$$g'(\eta) + \frac{n[T_w(x) - T_\infty(x)]\delta_T^2(x)}{\mu[p(\eta)x]U_w(x)} \int_0^\eta p(\eta)d\eta \quad (6)$$

where  $n = \rho g \alpha$ .

Let

$$Q(\eta) = \int_0^\eta p(\eta)d\eta$$

Then

$$Q'(\eta) = p(\eta) \quad (7)$$

From eq. 1 the following equation can be obtained

$$V = - \left[ \frac{dU_w}{dx} \delta_T(x) + U_w(x) \frac{d\delta_T(x)}{dx} \right] \int_0^\eta g(\eta)d\eta + U_w \frac{d\delta_T}{dx} g(\eta) \cdot \eta \quad (8)$$

Let

$$f(\eta) = \int_0^\eta g(\eta)d\eta \quad (9)$$

then

$$f'(\eta) = g(\eta) \quad (10)$$

From eq. 1 we obtain

$$\begin{aligned} p''(\eta) = & \frac{U_w \delta_T^2}{k(T_w - T_\infty)} \left( \frac{dT_w}{dx} - \frac{dT_\infty}{dx} \right) p(\eta) g(\eta) \\ & + \frac{U_w \delta_T^2}{k(T_w - T_\infty)} \frac{dT_\infty}{dx} g(\eta) \\ & - \frac{1}{k} \left( \frac{dU_w}{dx} \delta_T^2 + U_w \delta_T \frac{d\delta_T}{dx} \right) p'(\eta) f(\eta) \end{aligned} \quad (11)$$

Let

$$p'(\eta) = R(\eta) \quad (12)$$

$$p''(\eta) = R'(\eta) \quad (13)$$

Then eqs. 10, 6, 7, 12 and 11 can be rewritten as follows

$$df/d\eta = g \quad (14)$$

$$dg/d\eta = - \left\{ [n(T_w - T_\infty) \delta_T] / (\mu U_w) \right\} Q \quad (15)$$

$$dQ/d\eta = p \quad (16)$$

$$dp/d\eta = R \quad (17)$$

$$\begin{aligned} \frac{dR}{d\eta} = & \frac{U_w \delta_T^2}{k(T_w - T_\infty)} \left( \frac{dT_w}{dx} - \frac{dT_\infty}{dx} \right) gp \\ & + \frac{U_w \delta_T^2}{k(T_w - T_\infty)} \frac{dT_\infty}{dx} g \\ & - \frac{1}{k} \left( \frac{dU_w}{dx} \delta_T^2 + U_w \delta_T \frac{d\delta_T}{dx} \right) fR \end{aligned} \quad (18)$$

The boundary conditions are

$$\begin{aligned} f(0) = 0, \quad g(0) = 1 \\ Q(0) = 0, \quad p(0) = 1 \\ R(0) = 0, \quad p(1) = 0 \end{aligned} \quad (19)$$

This set of eqs. (14–19) provides an eigenvalue problem for a set of ordinary differential equations with five unknown functions, with an eigenvalue being relative to the thermal thickness of the plume,  $\delta_T(x)$ , and its derivative about  $x$ . Because there is a term  $d\delta_T/dx$  in eq. 18, we must first find  $\delta_T(x_0)$  as an initial value of  $\delta_T(x)$ , in order to solve eqs. 14–19.

Integrating eq. 18 for  $\eta$  over the region (0, 1),

we have

$$\begin{aligned} p'(1) - p'(0) = & \frac{U_w \delta_T^2}{k(T_w - T_\infty)} \left( \frac{dT_w}{dx} - \frac{dT_\infty}{dx} \right) \int_0^1 pgd\eta \\ & + \frac{U_w \delta_T^2}{k(T_w - T_\infty)} \frac{dT_\infty}{dx} \int_0^1 gd\eta \\ & - \frac{1}{k} \left( \frac{dU_w}{dx} \delta_T^2 + U_w \delta_T \frac{d\delta_T}{dx} \right) \\ & \times \int_0^1 p'fd\eta \end{aligned} \quad (20)$$

Taking note of  $\int_0^1 p'fd\eta = -\int_0^1 pgd\eta$ , from eq. 20, we can obtain

$$\begin{aligned} \frac{d\delta_T^2}{dx} = & \left[ -\frac{2}{T_w - T_\infty} \left( \frac{dT_w}{dx} - \frac{dT_\infty}{dx} \right) + \frac{2}{U_w} \left( \frac{dU_w}{dx} \right) \right. \\ & \left. - \frac{2}{T_w - T_\infty} \frac{dT_w}{dx} C_1 \right] \delta_T^2 + C_2 \end{aligned} \quad (21)$$

where

$$C_1 = f(1) / \int_0^1 pgd\eta \quad (22)$$

$$C_2 = R(1) / \int_0^1 pgd\eta \quad (23)$$

The coefficient  $C_1$  and  $C_2$  can first be estimated from the solution of a constant viscosity plume (Li and Guan, 1979). Substituting eq. 21 into eq. 18, a solution of eqs. 14–19 can be found using a Newtonian iteration method and an “optimum search method” for the eigenvalue. From the solution obtained, new coefficients  $C_1$  and  $C_2$  can be estimated again. This process is repeated until a stable value for the coefficients  $C_1$  and  $C_2$  is obtained. On the condition that  $\delta_T(x_0)$  has been found,  $[\delta_T(x_0 + \Delta x) - \delta_T(x_0)]/\Delta x$  can be used to take the place of  $d\delta_T/dx$  in eq. 18. Then the solution of eqs. 14–19 at any depth can be obtained using the above-mentioned method.

### 3. Numerical results for the structure of the plume

Numerical solutions of eqs. 14–19 have been obtained for the Newtonian I and II plumes. The values for the physical parameters of the Earth's mantle were adopted as follows:  $\rho = 3.3 \text{ g cm}^{-3}$ ,  $g = 10^3 \text{ cm s}^{-2}$ ,  $\alpha = 3.5 \times 10^{-5} \text{ K}^{-1}$ ,  $k = 1.0 \times$

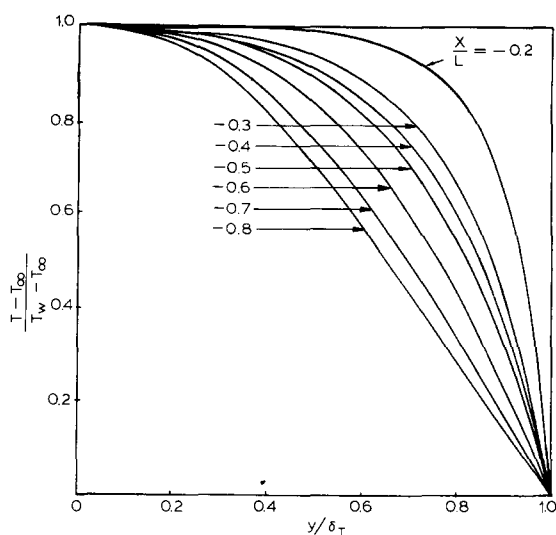


Fig. 1. Lateral profiles of the dimensionless temperature in a Newtonian I plume for different values of  $x/L$ .

$10^{-2} \text{ cm}^2 \text{ s}^{-1}$ , and  $n = 0.1155 \text{ g cm}^{-2} \text{ s}^{-2} \text{ K}^{-1}$ .

Figures 1 and 2 show the thermal structure of plumes for Newtonian I and II rheological parameters, respectively. At any depth the temperature decreases monotonically from its maximum at the centre of the plume to the ambient mantle temperature. We can see that for the same value of  $y/\delta_T$ ,

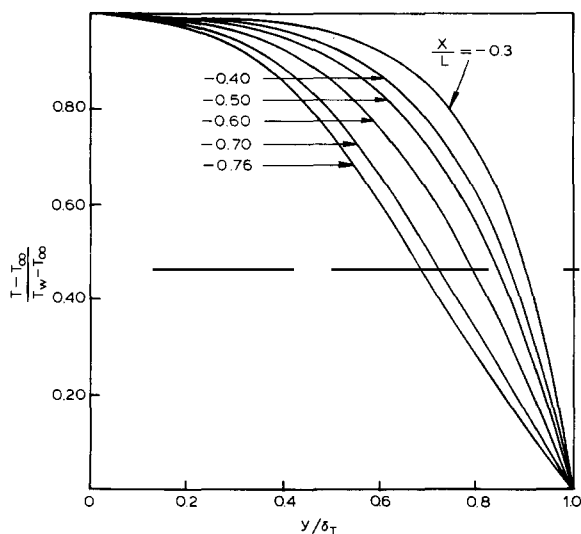


Fig. 2. Lateral profiles of the dimensionless temperature in a Newtonian II plume for different values of  $x/L$ .

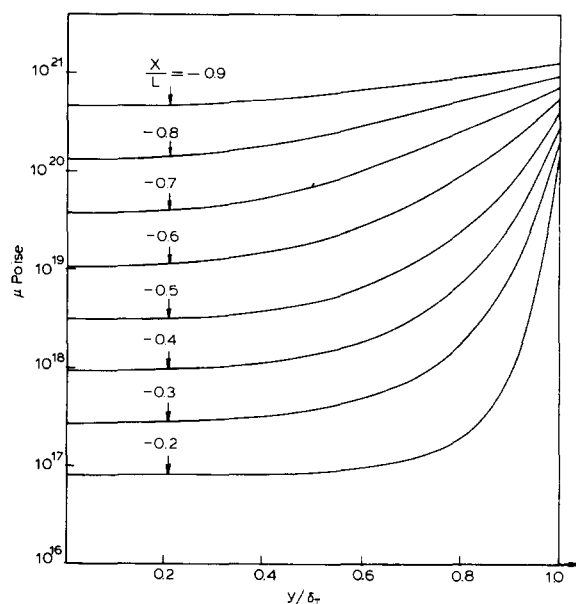


Fig. 3. Lateral profiles of the viscosity in a Newtonian I plume for different values of  $x/L$ .

the greater the depth, the lower is the dimensionless temperature.

The lateral viscosity profiles with parameters of  $x/L$  for the Newtonian I and II plumes are shown in Figs. 3 and 4, respectively. For all lateral viscosity profiles the minimum is at the mid-plane and the maximum is at the boundary of the plume. For profiles at greater depth, the maximum of the viscosity is only slightly larger than the minimum.

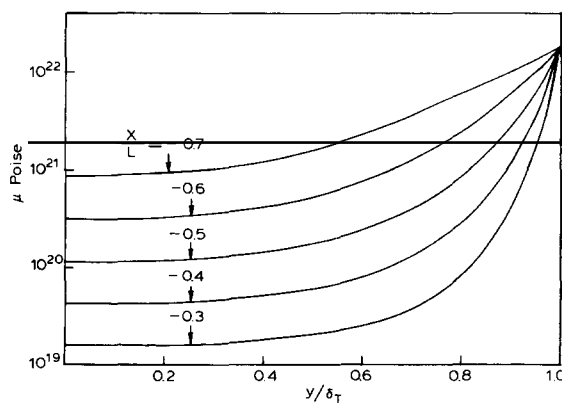


Fig. 4. Lateral viscosity profiles in a Newtonian II plume for different values of  $x/L$ .

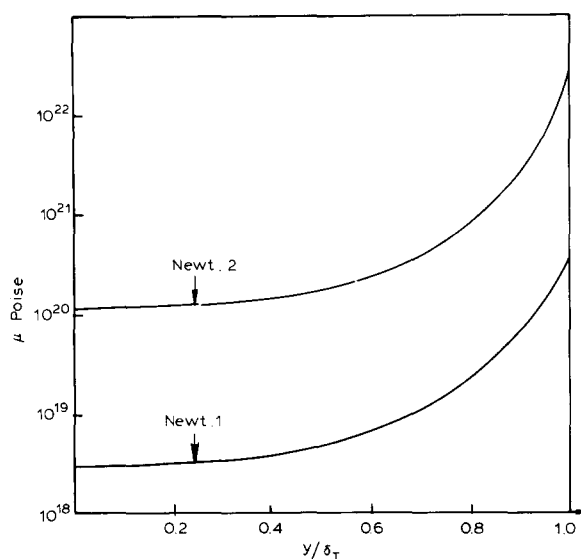


Fig. 5. Lateral viscosity profiles in Newtonian I and Newtonian II plumes at  $x/L = -0.5$ .

However, for profiles at a shallow depth, the maximum is 2–3 orders of magnitude larger than the minimum. Figure 5 shows a comparison between the viscosity profiles of Newtonian I and II plumes. For the same point of the plume, at  $x/L = -0.5$ , the viscosity of the material with a Newtonian II

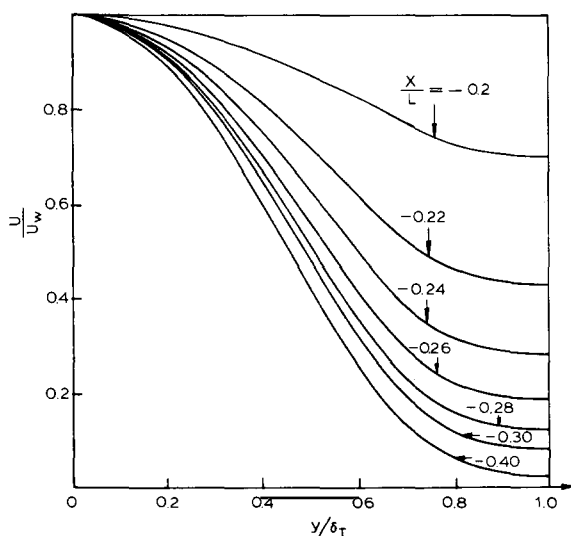


Fig. 6. Lateral profiles of the dimensionless upward velocity in a Newtonian I plume for different values of  $x/L$  in the region  $-0.4$  to  $-0.2$ .

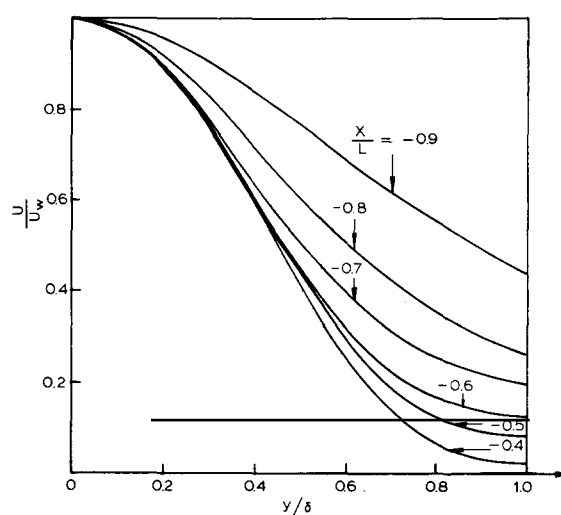


Fig. 7. Lateral profiles of the dimensionless upward velocity in a Newtonian I plume for different values of  $x/L$  in the region  $-0.9$  to  $-0.4$ .

creep law is at least one order of magnitude larger than that for Newtonian I.

Figures 6–9 show the upward velocity of a plume as a function of the dimensionless distance from its centre. The upward velocity also decreases monotonically from its maximum at the mid-plane

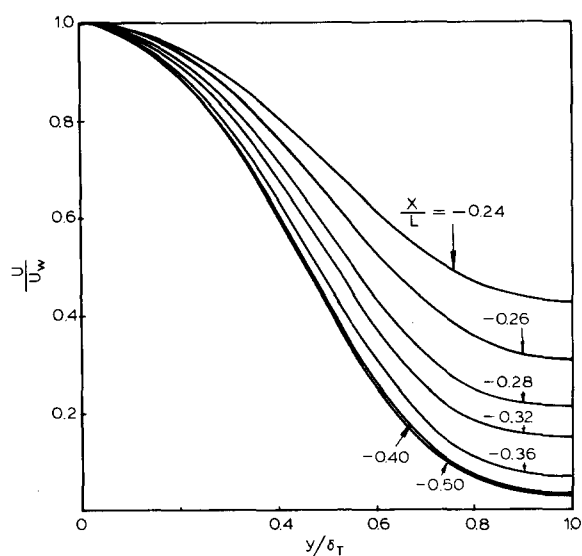


Fig. 8. Lateral profiles of the dimensionless upward velocity in a Newtonian II plume for different values of  $x/L$  in the region  $-0.5$  to  $-0.24$ .

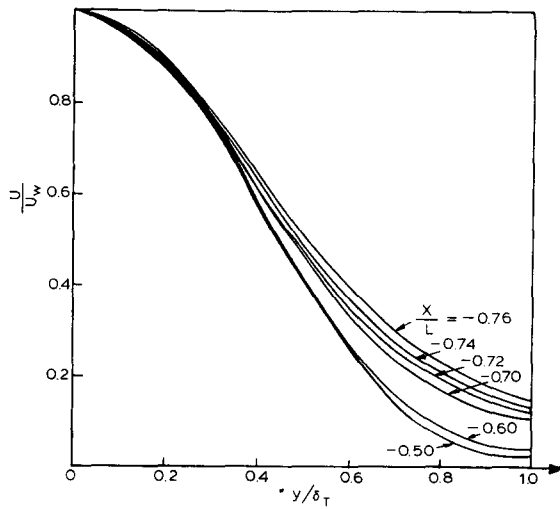


Fig. 9. Lateral profiles of the dimensionless upward velocity in a Newtonian II plume for different values of  $x/L$  in the region  $-0.76$  to  $-0.5$ .

to its boundary value  $U_b(x)$ , where  $U_b(x) = U[x, \delta_T(x)]$  is an important parameter of a plume. According to the definition suggested by Yuen and Schubert (1976), when  $U_b/U_w \leq 0.5$  then  $\delta_T$  can be considered as the thickness of the plume; when  $U_b/U_w > 0.5$  then there are no plume-like structures of the upward velocity. For example, in Fig. 6 for  $x/L = -0.2$ ,  $U_b/U_w = 705 > 0.5$ , then at this depth for Newtonian I rheological parameters there are no plume-like structures of the upward velocity. The results obtained in the present work show that for Newtonian I rheological parameters in the region of a dimensionless depth from about  $-0.22$  to  $-0.90$ , and for Newtonian II in the region from about  $-0.24$  to  $-0.76$ , the condition  $U_b/U_w \leq 0.5$  can be satisfied. In the regions outside those mentioned above, there are no plume-like structures of the upward velocity because the change in flow-direction takes place near there. Comparing Figs. 7 and 8 it is seen that  $U/U_w$  goes through a minimum at  $x/L = -0.4$  for Newtonian I viscosities. For a Newtonian II rheology the minimum is at  $\sim x/L = -0.5$ .

Figures 10 and 11 show a comparison between the lateral profiles of temperature and upward velocity for the Newtonian I plume at  $x/L = -0.4$ , and for the Newtonian II plume at  $x/L = -0.5$ , respectively.

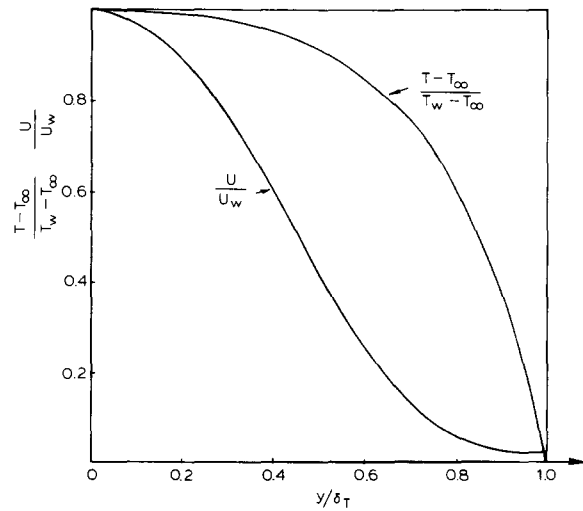


Fig. 10. Lateral profiles of the dimensionless upward velocity and temperature in a Newtonian I plume at  $x/L = -0.4$ .

Figure 12 shows the thickness of the plume. At the same depth, the thickness of a Newtonian II plume is larger than that of Newtonian I. For both Newtonian I and II plumes it can be seen that the larger the depth, the larger is the thickness of the plume.

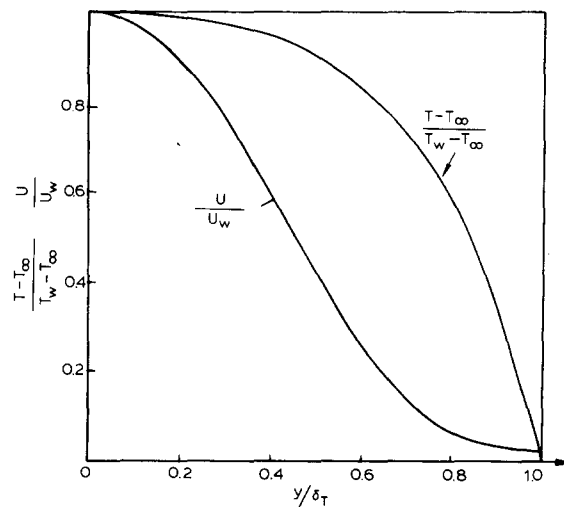


Fig. 11. Lateral profiles of the dimensionless upward velocity and temperature in a Newtonian II plume at  $x/L = -0.5$ .

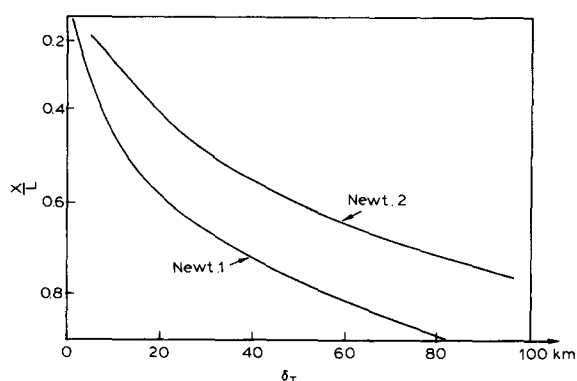


Fig. 12. Thickness of the plume versus dimensionless depth,  $x/L$ .

#### 4. Conclusion

The mantle plume is the rising part of a mantle convection cell. Its existence was found from investigating the structure of convection cells, but it is difficult to find its detailed structure in this way. The boundary-layer approach for a mantle plume on the other hand, as suggested by Yuen and Schubert (1976), makes it possible to estimate the structure of a plume independently. Based on this idea we have formulated an analytical solution by which the lateral profiles of temperature, upward

velocity, viscosity and thickness of the plume at various depths can be obtained for Newtonian temperature- and pressure-dependent rheologies. Hence, it is believed that the present model provides an approach for a quantitative description of the detailed structure of a mantle plume.

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