# THE NON-AXISYMMETRICAL CONFIGURATION OF A 

# LARGE-SCALE <br> SOLAR AND STELLAR MAGNETIC FIELD 

WEN-RUI HU<br>Institute of Mechanics, Academia Sinica, Beijing, China

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#### Abstract

The non-axisymmetric and nonlinear solutions of the magnetostatic equations are given in three-dimensional space of spherical coordinates ( $r, \theta, \phi$ ). These solutions are applied to the large-scale solar magnetic field. Their basic features are similar to a dipole field near the polar regions and the polarity reverses near the equator. These features agree with observations for the large-scale solar magnetic field. The solutions can also be applied to investigating the connection between the structure of the magnetic field and the density distribution of the corona. It is shown that the tops of the closed magnetic field associate with density enhancements.

Similar results may apply to the large-scale configuration of the stellar field.


## 1. Introduction

It is believed that the solar corona and the large-scale solar magnetic field are threedimensional, that is, their configurations deviate obviously from the axisymmetric ones. The non-axisymmetric interplanetary magnetic field associates with the non-axisymmetric large-scale solar magnetic field. Observation at low resolution gives the largescale mean magnetic field of the Sun seen as a star. A magnetic neutral line runs generally north-south in low and middle latitudes and often east-west at high altitudes. Several large-scale regions of alternating polarity near the equator are separated by the neutral line (Wilcox and Howard, 1968; Svalgaard et al., 1974, 1975; Svalgaard and Wilcox, 1978; Levine, 1979).

Many theoretical models have suggested that, based on the approximation of potential field, the line-of-sight component of the photospheric magnetic field is used to determine the large-scale coronal field by the harmonic expansion of the Laplace equation (Altschuler and Newkirk, 1969; Newkirk and Altschuler, 1970; Altschuler et al., 1977; Riesebieter and Neubauer, 1979). On the other hand, if a magnetic dipole rotates obliquely in a vacuum, the polarity-reversal regions of the magnetic field will be formed near the plane perpendicular to the rotating axis. This idea is applied to the threedimensional structure of the heliospheric magnetic fields (Satio et al., 1978; Kaburaki and Yoshii, 1979). In the solar or stellar atmosphere, the magnetic field is coupled with the plasma. Therefore, the magnetostatic equilibrium should be studied, and the connection between the structure of the magnetic field and the density distribution may be obtained.

Recently, we have analyzed the non-axisymmetric magnetostatic equilibrium for a sunspot-like magnetic field (Hu et al., 1983a, b; Hu, 1983), and nonlinear models are
also suggested for a small-scale field in the cartesian coordinates (Low, 1983). In this paper, similar approaches are applied to the large-scale solar magnetic field in spherical coordinates. The basic nonlinear theory and the general solution in spherical coordinates are given in the next solution. A special solution is suggested in Section 3. In Section 4, an example is given. The configuration of this magnetic field has polarity reverses near the equator and is similar to a dipole field near the polar region. In the last section, we discuss the coupling relationship between the magnetic field and the plasma.

## 2. Nonlinear Theory

The magnetostatic equations are

$$
\begin{align*}
& \frac{1}{4 \pi}(\nabla \times \mathbf{B}) \times \mathbf{B}-\nabla p-\rho \mathbf{g}=0  \tag{2.1}\\
& \nabla \cdot \mathbf{B}=0  \tag{2.2}\\
& p=\rho \mathscr{R} T \tag{2.3}
\end{align*}
$$

where $\mathbf{B}, p, p, T$ denote the magnetic field, pressure, density, and temperature, respectively; $\mathscr{R}$ is the gaseous constant; and $g$ is the gravitational acceleration in the $\mathbf{e}_{r}$ direction in the spherical coordinates $(r, \theta, \phi)$.

According to Equation (2.2), the magnetic potential function $\psi$ may be introduced as

$$
\begin{equation*}
\mathbf{B}=\frac{1}{r \sin \theta}\left(\frac{1}{r} \frac{\partial \psi}{\partial \theta},-\frac{\partial \psi}{\partial r}, 0\right), \tag{2.4}
\end{equation*}
$$

where the azimuthal component of the magnetic field is not included. In this case, the $\phi$ component of Equation (2.1) reduces to

$$
\frac{\partial}{\partial \phi}\left(8 \pi p+B_{\theta}^{2}+B_{r}^{2}\right)=0
$$

Then we have

$$
\begin{equation*}
8 \pi p(r, \theta, \phi)+B_{\theta}^{2}(r, \theta, \phi)+B_{r}^{2}(r, \theta, \phi)=2 a(r, \theta) \tag{2.5}
\end{equation*}
$$

where $a$ is an arbitrary function of $r$ and $\theta$. Substituting (2.5) into the $\theta$ component of Equation (2.1) for canceling the pressure $p$, we obtain

$$
\begin{equation*}
B_{r} \frac{\partial r B_{\theta}}{\partial r}+B_{\theta} \frac{\partial B_{\theta}}{\partial \theta}=\frac{\partial a(r, \theta)}{\partial \theta} . \tag{2.6}
\end{equation*}
$$

According to definition (2.3), Equation (2.6) becomes

$$
\begin{equation*}
\frac{\partial \psi}{\partial \theta} \frac{\partial}{\partial r}\left(\frac{1}{\sin \theta} \frac{\partial \psi}{\partial r}\right)-\frac{\partial \psi}{\partial r} \frac{\partial}{\partial \theta}\left(\frac{1}{\sin \theta} \frac{\partial \psi}{\partial r}\right)=-r^{2} \sin \theta \frac{\partial a(r, \theta)}{\partial \theta} \tag{2.7}
\end{equation*}
$$

The solution $\psi(r, \theta, \phi)$ may be solved from Equation (2.7) when the distribution $a(r, \theta)$ is given. The pressure and density may then be obtained from relation (2.5) and the $r$-component of Equation (2.1), respectively,

The configuration of the magnetic field depends critically on the distribution $a(r \cdot \theta)$. For example, the dipole field is

$$
\begin{equation*}
\mathbf{B}_{0}=\frac{\mu}{r^{3}}(2 \cos \theta, \sin \theta, 0) \tag{2.8}
\end{equation*}
$$

which may satisfy Equation (2.1) with the condition

$$
\begin{equation*}
\nabla p_{0}=-\rho_{0} \mathbf{g} \tag{2.9}
\end{equation*}
$$

In this case, Equation (2.4) is satisfied. However, formally, if we use (2.8), Equation (2.6) or (2.7) requires

$$
\begin{equation*}
a(r, \theta)=a_{0}(r)-\frac{3 \mu_{0}^{2}}{2 r^{6}} \sin ^{2} \theta \tag{2.10}
\end{equation*}
$$

where

$$
\begin{equation*}
a_{0}(r)=p_{0}(r)+\frac{4 \mu^{2}}{r^{6}} \tag{2.11}
\end{equation*}
$$

This simple example implies that the function $a(r \cdot \theta) / 4 \pi$ is associated with the thermodynamic and magnetic pressure, and should be given reasonably.

Equation (2.7) may be considered as a first-order differential equation for function $(1 / \sin \theta)(\partial \psi / \partial r)$, and its characteristic equation is

$$
\begin{equation*}
\frac{\mathrm{d} r}{\partial \psi / \partial \theta}=\frac{\mathrm{d} \theta}{-\partial \psi / \partial r}=\frac{\mathrm{d}[(1 / \sin \theta) \partial \psi / \partial r]}{-r^{2} \sin \theta \partial a(r, \theta) / \partial \theta} \tag{2.12}
\end{equation*}
$$

The former two relations give

$$
\begin{equation*}
\psi(r, \theta, \phi)=c_{1} \tag{2.13}
\end{equation*}
$$

where $c_{1}$ is an integration constant. The latter two relations give

$$
\begin{equation*}
\frac{1}{2} \mathrm{~d}\left(\frac{1}{\sin \theta} \frac{\partial \psi}{\partial r}\right)^{2}=r^{2} \frac{\partial a(r, \theta)}{\partial \theta} \mathrm{d} \theta \tag{2.14}
\end{equation*}
$$

We put the function $a(r, \theta)$ as

$$
\begin{equation*}
a(r, \theta)=a_{0}(r)+\frac{1}{2 r^{2}} \frac{\mathrm{~d} a_{1}(\theta)}{\mathrm{d} \theta} \tag{2.15}
\end{equation*}
$$

In this case, Equation (2.14) reduces to

$$
\begin{equation*}
\left(\frac{1}{\sin \theta} \frac{\partial \psi}{\partial r}\right)^{2}=a_{1}(\theta)+c_{2} \tag{2.16}
\end{equation*}
$$

According to both integral relationships (2.13) and (2.16), the solution of Equation (2.7) will satisfy the condition

$$
\begin{equation*}
\left(\frac{\partial \psi}{\partial r}\right)^{2}=\sin ^{2} \theta a_{1}(\theta)+\sin ^{2} \theta F_{1}(\psi, \phi) \tag{2.17}
\end{equation*}
$$

where $F_{1}$ is an arbitrary function of $\psi$ and $\phi$. For instance, if the function $a_{1}(\theta)$ is adopted as

$$
\begin{equation*}
a_{1}(\theta)=0 \tag{2.18}
\end{equation*}
$$

Equation (2.19) gives the general solution

$$
\begin{equation*}
\psi(r, \theta, \phi)=F[r \sin \theta+f(\theta, \phi), \phi] \tag{2.19}
\end{equation*}
$$

where $F$ and $f$ are arbitrary functions. Relationship (2.19) is the general solution of Equation (2.7) under conditions (2.16) and (2.18). Obviously, these conditions restrict the magnetic field in some special configurations. For example, the dipole magnetic field (2.8) is excluded in this special solution (2.19). However, some basic features of this special solution may apply to the large-scale solar magnetic field, and to the configuration of the stellar magnetic field.

## 3. Large-Scale Solar Magnetic Field

According to relationship (2.8), the potential function of the dipole field may be written as

$$
\begin{equation*}
\psi_{0}=\frac{\mu_{0} \sin ^{2} \theta}{r} \tag{3.1}
\end{equation*}
$$

We discuss the special solution of relation (2.19) as

$$
\begin{equation*}
\psi=\psi_{0} \frac{\sin ^{2} \theta}{r \sin ^{3} \theta+r_{0}+e(\theta, \phi)} \tag{3.2}
\end{equation*}
$$

where $\phi_{0}$ and $r_{0}$ are constants. Therefore, the function $f(\theta, \phi)$ in general solution (2.19) is adopted as

$$
\begin{equation*}
f(\theta, \phi)=\left[r_{0}+e(\theta, \phi)\right] / \sin ^{2} \theta, \tag{3.3}
\end{equation*}
$$

the regularity distribution requires the condition for the positive constant $r_{0}$

$$
r_{0}>|e(\theta, \phi)|
$$

The function $e(\theta, \phi)$ is a periodic function of $\phi$ with period $2 \pi$, and the condition of single value requires that

$$
\begin{equation*}
e(0, \phi)=0, \quad e(\pi, \phi)=0 . \tag{3.4}
\end{equation*}
$$

Then, the magnetic field could be determined if the function $e(\theta, \phi)$ is given.
Substituting (3.2) into (2.3), we obtain the distribution of the magnetic field as

$$
\begin{align*}
& B_{r}=\frac{\psi_{0}}{r^{2}} \frac{\left[2 r_{0}+2 e(\theta, \phi)-r \sin ^{3} \theta\right] \cos \theta-\sin \theta[\partial e(\theta, \phi) / \partial \theta]}{\left[r \sin ^{3} \theta+r_{0}+e(\theta, \phi)\right]^{2}},  \tag{3.5}\\
& B_{0}=\frac{\psi_{0}}{r} \frac{\sin ^{4} \theta}{\left[r \sin ^{3} \theta+r_{0}+e(\theta, \phi)\right]^{2}} . \tag{3.6}
\end{align*}
$$

The distribution of the magnetic field near the polar axis is

$$
\begin{equation*}
B_{r} \approx \pm \frac{2 \psi_{0}}{r^{2} r_{0}}, \quad B_{\theta}=0 \tag{3.7}
\end{equation*}
$$

and near the equator is

$$
\begin{align*}
B_{r} & =-\frac{\psi_{0} \frac{\partial e(\pi / 2, \phi)}{\partial \theta}}{r^{2}\left[r+r_{0}+e(\pi / 2, \phi)\right]^{2}} \\
B_{0} & =\frac{\psi_{0}}{r\left[r+r_{0}+e(\pi / 2, \phi)\right]^{2}} \tag{3.8}
\end{align*}
$$

Relationships (3.5), (3.6), and (3.7) show that the configuration of the magnetic field is similar to a dipole field near the polar region. Relationship (3.8) implies that the polarity of the field may reverse near the equator. This special solution combines both the properties of the dipole field near the polar region and the polarity reverses near the equator at the same time.

From results (3.5) and (3.6), the equation of the magnetic force line may be written as

$$
\begin{equation*}
\frac{\mathrm{d} r}{\left[2 r_{0}+2 e(\theta, \phi)-r \sin ^{3} \theta\right] \cos \theta-\frac{\partial e}{\partial \theta} \sin \theta}=\frac{r \mathrm{~d} \theta}{\sin ^{4} \theta} \tag{3.9}
\end{equation*}
$$

Equation (3.9) gives the solution

$$
\begin{equation*}
r=\frac{C_{0}}{\sin \theta}-\frac{r_{0}+e(\theta, \phi)}{\sin ^{3} \theta}, \tag{3.10}
\end{equation*}
$$

where $C_{0}$ is the integral constant.
Substituting solutions (3.5) and (3.6) into (2.5), and using assumptions (2.15) and (2.18), we derive the plasma pressure as

$$
\begin{equation*}
p(r, \theta, \phi)=\frac{a_{0}(r)}{4 \pi}-\frac{\psi_{0}^{2}}{8 \pi r^{2}} \frac{A(r, \theta)}{\left[r \sin ^{3} \theta+r_{0}+e(\theta, \phi)\right]^{4}}, \tag{3.11}
\end{equation*}
$$

where

$$
A(r, \theta)=\sin ^{8} \theta+\left[\sin ^{3} \theta \cos \theta+\frac{\sin \theta}{r} \frac{\partial e}{\partial \theta}-\frac{2 \cos \theta}{r}\left(e+2 r_{0}\right)\right]^{2}
$$

Both terms in the right-hand side of (3.11) are decreasing when $r$ is increasing. The pressure will be positive and a decreasing function of $r$, if $a_{0}(r) / 4 \pi$ is decreasing no faster than the magnetic pressure $B^{2} / 8 \pi$ and the initial value $a_{0}\left(r_{0}\right) / 4 \pi$ is larger than the initial magnetic pressure $B^{2}\left(r_{0}, \theta, \phi\right) / 8 \pi$.

The $r$-component of Equation (2.1) is

$$
\begin{equation*}
\rho g=-\frac{\partial p}{\partial r}-\frac{B_{\theta}}{4 \pi r}\left(\frac{\partial r B_{\theta}}{\partial r}-\frac{\partial B_{r}}{\partial \theta}\right) . \tag{3.12}
\end{equation*}
$$

By using relationship (2.5), we reduce the above equation to

$$
\rho g=-\frac{1}{4 \pi} \frac{\partial a}{\partial r}+\frac{1}{4 \pi}\left(B_{r} \frac{\partial B_{r}}{\partial r}+\frac{B_{\theta}}{r} \frac{\partial B_{r}}{\partial \theta}-\frac{B_{\theta}^{2}}{r}\right)
$$

or

$$
\begin{equation*}
\rho g=-\frac{1}{4 \pi} \frac{\partial a}{\partial r}+\frac{1}{4 \pi r^{3} \sin ^{2} \theta} L_{1}(\psi) \tag{3.13}
\end{equation*}
$$

where the operator

$$
\begin{align*}
& L_{1}(\psi)= \frac{1}{r} \\
& \frac{\partial \psi}{\partial \theta} \frac{\partial^{2} \psi}{\partial r \partial \theta}-\frac{1}{r} \frac{\partial \psi}{\partial r} \frac{\partial^{2} \psi}{\partial \theta^{2}}-  \tag{3.14}\\
&-\frac{2}{r^{2}}\left(\frac{\partial \psi}{\partial \theta}\right)^{2}-\left(\frac{\partial \psi}{\partial r}\right)^{2}+\frac{\cot \theta}{r} \frac{\partial \psi}{\partial r} \frac{\partial \psi}{\partial \theta}
\end{align*}
$$

Equation (3.12) or (3.13) shows the equilibrium condition between the gravity, the pressure gradient, and the Lorentz force. As the first term in the right-hand side of (3.13) is positive, it is not difficult to get a reasonable density distribution by adopting the function $a(r \cdot \theta)$. By using conditions (2.15), (2.18) and solutions (3.5), (3.6), we obtain the density distribution

$$
\begin{align*}
\rho= & -\frac{r^{2}}{4 \pi G M} \frac{\mathrm{~d} a_{0}}{\mathrm{~d} r}-\frac{\psi_{0}^{2}}{4 \pi G M r\left(r \sin ^{3} \theta+r_{0}+e\right)^{4}}\left\{-2 \sin ^{6} \theta \cos ^{2} \theta+\right. \\
& +\frac{\sin ^{3} \theta}{r}\left[2\left(r_{0}+e\right)\left(1+4 \cos ^{2} \theta\right)-6 \frac{\partial e}{\partial \theta} \cos \theta \sin \theta+\frac{\partial^{2} e}{\partial \theta^{2}} \sin ^{2} \theta\right]+ \\
& \left.+\frac{2}{r^{2}}\left[\left(2 r_{0}+2 e-r \sin ^{3} \theta\right) \cos \theta-\frac{\partial e}{\partial \theta} \sin \theta\right]^{2}\right\}, \tag{3.15}
\end{align*}
$$

where the solar gravity is expressed as

$$
\begin{equation*}
g=\frac{M G}{r^{2}} \tag{3.16}
\end{equation*}
$$

$G$ is the gravitational constant, and $M$ is the mass of the Sun.

## 4. An Example

Relationships (3.5) and (3.8) show that the polarity reverses near the equator require that the function $e(\theta, \phi)$ should be nonsymmetrical about the equator plane $\theta=\pi / 2$. Therefore, as an example, we choose an antisymmetric distribution, that is

$$
\begin{equation*}
e(\theta, \phi)=s(\phi) \sin (2 \theta) \tag{4.1}
\end{equation*}
$$

and the periodic function $s(\phi)$ may be written, for example, in the form

$$
\begin{equation*}
s(\phi)=s_{0} \sin (n \phi), \tag{4.2}
\end{equation*}
$$

where $s_{0}$ is a constant and $n$ is an integer. Expression (4.2) means that the polarity of the magnetic field will reverse $2 n$ times in the region near the equator. There are two sectors if $n=1$, and four sectors if $n=2$.


Fig. 1. The profiles of the magnetic force lines near the Sun or star in the plane $n \phi=3 \pi / 2$ and northern hemisphere $0 \leq \theta \leq \pi / 2$.

Using (4.1) and (4.2), we can rewrite Equation (3.10) of the magnetic force line as

$$
\begin{equation*}
\frac{r}{r_{*}}=\frac{\alpha \sin ^{2} \theta-s_{0}(b+\sin n \phi \sin 2 \theta)}{\sin ^{3} \theta} \tag{4.3}
\end{equation*}
$$

where $r_{*}$ is a typical length, and $\alpha$ and $b$ are nondimensional constants. Generally, Equation (4.3) describes a set of closed magnetic force lines because the term $-s_{0} b$ is more important than the term $\alpha \sin ^{2} \theta$ near the polar region. Figures 1 and 2 give the


Fig. 2. The profiles of the magnetic force lines for a greater distance from the Sun or star in the plane $n \phi=3 \pi / 2$ and northern hemisphere $0 \leq \theta \leq \pi / 2$.
magnetic force lines in the northern hemisphere $0 \leq \theta \leq \pi / 2$ near the solar surface and at a greater distance, respectively, for the case

$$
\begin{equation*}
\sin (n \phi)=-1 \quad \text { or } \quad n \phi=\frac{3 \pi}{2} \tag{4.4}
\end{equation*}
$$

Figure 3 gives both profiles satisfying condition (4.4) and the condition

$$
\begin{equation*}
\sin (n \phi)=1 \quad \text { or } \quad n \phi=\pi / 2 \tag{4.5}
\end{equation*}
$$

for $\alpha=3$. For comparison, the dipole field is also given in the same figure. Because the function $e(\theta, \phi)$ is not symmetric to the equator plane $\theta=\pi / 3$, neither is the magnetic


Fig. 3. The profiles of the magnetic force lines in the planes $n \Phi=\pi / 2$ and $3 \pi / 2$ for $\alpha=3$, and the lines of the dipole magnetic field.
force line. For example, the profile for case (4.4) in the southern-hemisphere $\pi / 2 \leq \theta \leq \pi$ is just the mirror reflection of the profile for case (4.5) in the northernhemisphere and vice versa. The profiles will be different for a different azimuthal angle $\phi$, and will construct a complicated configuration in three-dimensional space $(r, \theta, \Phi)$.


Fig. 4. The zero line of the line-of-sight component of the magnetic field $B_{i}$.

Substituting assumptions (4.1) and (4.2) into (3.5) and (3.6), we have

$$
\begin{align*}
& B_{r}=\frac{\psi_{0}}{r} \frac{2 r_{0} \cos \theta+2 s(\psi) \sin \theta-3 r \cos \theta \sin ^{2} \theta}{\left[r \sin ^{3} \theta+r_{0}+s(\phi) \sin 2 \theta\right]}  \tag{4.6}\\
& B_{0}=\frac{\psi_{0}}{r} \frac{\sin ^{4} \theta}{\left[r \sin ^{3} \theta+r_{0}+s(\phi) \sin 2 \theta\right]^{2}} \tag{4.7}
\end{align*}
$$

Near the equator, relationship (3.8) is

$$
\begin{gather*}
B_{r}=\frac{2 \psi_{0} s(\phi)}{r^{2}\left(r+r_{0}\right)^{2}} \\
B_{o}=\frac{\psi_{0}}{r\left(r+r_{0}\right)^{2}} \tag{4.8}
\end{gather*}
$$

These relationships show clearly that $B_{r}$ is positive near the north-polar $\theta \approx 0$, is negative near the south-polar $\theta \approx \pi$, and has polarity reverses near the equator. The line-of-sight component of the magnetic field is

$$
\begin{equation*}
B_{l}=B_{r} \sin \theta+B_{\theta} \cos \theta \tag{4.9}
\end{equation*}
$$

From (4.6) and (4.7), the condition of zero $B_{I}$ is

$$
\begin{equation*}
2 r_{0} \cos \theta-r \sin ^{2} \theta \cos \theta(3-\sin \theta)+2 s(\phi) \sin \theta=0 \tag{4.10}
\end{equation*}
$$

Figure 4 gives the zero line of $B_{1}$ for the case that $s(\phi)$ is expressed as (4.2) and $s_{0}=r_{0}$ is much larger than $r$. The zero line may run roughly north-south in middle and low latitudes and east-west at high latitudes if function $s(\phi)$ is adopted as a step function. This large-scale feature agrees with observations (see, for example, Svalgaard and Wilcox, 1978).

Now, we discuss the thermodynamical parameters. The function $a_{0}(r)$ may be chosen as

$$
\begin{equation*}
a_{0}(r)=\frac{4 \pi p_{0}}{r^{m}} \tag{4.11}
\end{equation*}
$$

where $p_{0}$ is a positive constant and the index $m>1$. Then, relationship (3.11) gives the pressure distribution.

The constant $p_{0}$ satisfies the condition

$$
\begin{equation*}
p(r, \theta, \phi)=\frac{p_{0}}{r^{m}}-\frac{\psi_{0}^{2}}{8 \pi r^{2}} \frac{A(r, \theta)}{\left[r \sin ^{3} \theta+r_{0} e(\theta, \phi)\right]^{4}} \tag{4.12}
\end{equation*}
$$

The constant $p_{0}$ satisfies the condition

$$
\begin{equation*}
p_{0}>\frac{\psi_{0}^{2}}{8 \pi r_{*}^{2-m}} \frac{A_{2}}{\left(r_{0}-s_{0}\right)^{4}}, \tag{4.13}
\end{equation*}
$$

where

$$
A_{2}=1+\left(\frac{2 s_{0}}{r_{*}}+\frac{2 r_{0}}{r_{*}}+1\right)^{2}
$$

and the constant $m$ satisfies the inequality

$$
\begin{equation*}
1<m<2 . \tag{4.14}
\end{equation*}
$$

Similarly, the density distribution may be obtained from (3.15) as

$$
\begin{align*}
& \rho(r, \theta, \phi)=\frac{m}{G M} \frac{p_{0}}{r^{m-1}}-\frac{\psi_{0}^{2}}{4 \pi M G r\left[r \sin ^{3} \theta+r_{0}+s(\phi) \sin ^{2} \theta\right]^{4}} \times \\
& \quad \times\left\{-2 \sin ^{6} \theta \cos \theta+\frac{2 \sin ^{3} \theta}{r}\left[r_{0}\left(1+4 \cos ^{2} \theta\right)+2 s(\phi) \sin 2 \theta\right]+\right. \\
& \left.\quad+\frac{2}{r^{2}}\left[\left(2 r_{0}-r \sin ^{3} \theta\right) \cos \theta+2 s(\phi) \sin \theta\right]^{2}\right\} . \tag{4.15}
\end{align*}
$$

The Equation (2.3) of state gives the temperature distribution as

$$
\begin{equation*}
T(r, \theta, \phi)=\frac{p(r, \theta, \phi)}{\rho(r, \theta, \phi)} \tag{4.16}
\end{equation*}
$$

where $p(r, \theta, \phi)$ and $\rho(r, \theta, \phi)$ are given in (4.12) and (4.15), respectively. Obviously, the temperature is also decreasing when $r$ is increasing.

## 5. Discussion

The solutions of magnetostatic equations in three-dimensional space $(r, \theta, \phi)$ are given to describe the large-scale features of the solar or stellar magnetic fields. The coupling of the magnetic field and the plasma is considered by solving the nonlinear magnetostatic equations, and the nonlinear solutions are more complicated than the linear field or the simplified dipole field. The basic features of these nonlinear solutions agree qualitatively with observations.

The non-axisymmetric solutions can be applied to investigate what connection exists between the magnetic fields and the density structure of the corona. For example, the magnetic field near the equator is given as (4.8), where the boundaries of the sectors are

$$
\begin{equation*}
\sin (n \phi)=0 \quad \text { or } \quad n \phi=0, \pi \tag{5.1}
\end{equation*}
$$

Therefore, the tops of the closed magnetic fields (loops or arcades) are located at $n \psi=0$ and $\pi$ above the zero line of $B_{r}$. On the other hand, the density distribution near the equator may be given by (4.15) as

$$
\begin{equation*}
\rho\left(r, \frac{\pi}{2}, \phi\right)=\frac{m p_{0}}{G M r^{m-1}}-\frac{\psi_{0}^{2}}{4 \pi G M r\left(r+r_{0}\right)^{4}}\left[\frac{2 r_{0}}{r}+\frac{8 s_{0}^{2} \sin ^{2}(n \phi)}{r^{2}}\right] . \tag{5.2}
\end{equation*}
$$

Relationship (5.2) shows that conditions $n \psi=0$ and $\pi$ correspond to the density enhancement for the fixed $r$ near the equator. The plasma density is larger at the tops of the closed field than otherwise. This conclusion agrees with observations (see, for example, Newkirk and Altschuler, 1970). This conclusion can also be seen from relationship (2.5). As $a(r, \theta)$ is independent of azimuthal angle $\phi$, the larger the magnetic pressure, the smaller the plasma pressure. For a fixed $\theta$, for example $\theta=\pi / 2$, the plasma pressure at the tops of the closed field is high if the magnetic pressure is weak there.

Near the polar region, relationship (4.15) gives the density

$$
\begin{equation*}
\rho(r, 0, \phi)=\frac{m}{G M} \frac{p_{0}}{r^{m-1}}-\frac{2 \psi_{0}^{2}}{\pi M G r^{3} r_{0}^{2}} \tag{5.3}
\end{equation*}
$$

Comparing (4.2) and (4.3), the plasma density near the equator is usually larger than the density near the polar regions. Furthermore, from (4.12), the pressure distribution far from the Sun may be written approximately as

$$
\begin{equation*}
p(r, \theta, \phi)=\frac{p_{0}}{r^{m}}-\frac{\psi_{0}^{2}}{8 \pi r^{6}} \frac{1+\cot ^{2} \theta}{\sin ^{4} \theta} . \tag{5.4}
\end{equation*}
$$

The above expression is not accurate near the polar region $\theta=0$ and $\pi$. The function $\left(1+\cot ^{2} \theta\right) / \sin ^{4} \theta$ is minimal at the equator plane $\theta=\pi / 2$, and increases when $|\theta-\pi / 2|$ is increasing, that is, the pressure is highest at the equator. This feature is similar to the properties of the plasma sheet, and may describe the acceration disk near the equator.

In relation (4.1), the function $e(\theta, \phi)$ is antisymmetric about the equator. Therefore, the configurations of the fields still have some properties of symmetry. Of course, the function $e(\theta, \phi)$ may be without symmetry, and then the field is not symmetric. As an example, we can make

$$
\begin{equation*}
e(\theta, \phi)=s(\phi) \theta^{k}(\pi-\theta)^{\prime} \tag{5.5}
\end{equation*}
$$

where $k$ and $l$ are positive constants. Function (5.5) is symmetric only in the case $k=1$; otherwise, the distributions in the northern hemisphere are different from the ones in the southern hemisphere, and so is the magnetic flux. The solar magnetographic data show that a $7 \%$ difference exists in the total flux values between the two hemispheres (Howard, 1974).

These non-axisymmetric configurations may also apply to the stellar magnetic fields, although the major interest of the present paper is solar physics. Near the polar regions, the nonlinear stellar magnetic field may have a larger gradient than the dipole field, and the force lines will be more concentrated and extensive there. This means that there is a stronger radio source if high-energy electrons are ejected into the polar core.

It should be pointed out that a nonlinear solution is a special solution of the magnetostatic equations, and cannot easily satisfy a given boundary condition at the solar or stellar surface. Moreover, assumptions (2.15) and (2.18) limit the solutions to some special configuration. Dipole field (2.10) is excluded from the solution based on assumptions (2.15) and (2.18). Furthermore, the dynamical influences of solar wind and
stellar wind should be considered further. However, the configurations given by the present paper describe some basic features produced by the coupling between the plasma and the magnetic field, and can compare with observations of the large-scale solar magnetic field. The coupling relations are important for understanding the connections between the configuration of the magnetic field and the structure of plasma parameters.

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