

载流条形薄板的非线性应力与变形分析^{*}

NONLINEAR STRESS AND DEFORMATION ANALYSIS OF THIN CURRENT-CARRYING STRIP PLATES

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摘要 在所建立的载流条形薄板的非线性磁弹性基本方程——运动方程、几何方程、物理方程和电动力学方程的基础上,通过变量代换,整理成含有10个基本未知函数的标准柯西型方程。采用差分及准线性化方法,将含有10个基本未知函数的偏微分方程组,变换能用离散正交法编程求解的准线性微分方程组。由此计算分析两边简支条形薄板在电磁场和机械载荷耦合作用下的应力与变形,研究侧向电流和外磁场强度对载流条形薄板的磁弹性效应。

关键词 载流条形薄板 磁弹性 非线性 应力 变形

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Abstract The nonlinear magneto-elastic basic equations of thin current-carrying strip-plate are built, based on the kinetic equations, the geometric equations, the physical equations and the electrodynamics equations of thin current-carrying plate, the normal Cauchy form nonlinear differential equations, which includes ten basic unknown functions in all, are obtained by means of variable replacement method. Using the difference method and quasi-linearization method, the nonlinear differential equations, which include ten basic unknown functions in all, are reduced to a sequence of quasi-linear differential equations, which can be solved by the method of discrete orthogonalization. The stresses and nonlinear deformations of thin strip-plate with two simply supported edges under the coupled action of the electromagnetic field and mechanical load are calculated, the magneto-elastic effects on thin current-carrying strip-plate by the side current and electromagnetic induction density are studied.

Key words Thin current-carrying strip plate; Magneto-elastic; Nonlinear; Stress; Deformation

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1 引言

20世纪末,由于理论和实际应用研究的需要,使耦合场理论的研究有了飞速发展,特别是电磁场和机械场相互作用的问题,在现代的力学发展中占有重要的位置。事实表明,耦合场理论的发展有着十分广泛的应用前途,它对于航空、航天、化工、供电器械、原子与核工业、地球物理等方面学科的强度研究,具有重要意义。

在研究一些处于强电磁场环境中工作的结构或部

件的应力与变形问题时,引入电磁场作用因素,更加合理地考虑影响结构或零部件强度、刚度和稳定性的各种外部载荷,对结构及零部件的设计和分析计算有着重要的理论价值和现实意义。20世纪70、80年代,前苏联学者^[1]、^[2]、^[3]、^[4]、^[5]、^[6]、^[7]、^[8]、^[9]、^[10]、^[11]、^[12]、^[13]、^[14]、^[15]、^[16]、^[17]、^[18]、^[19]、^[20]、^[21]、^[22]、^[23]、^[24]、^[25]、^[26]、^[27]、^[28]、^[29]、^[30]、^[31]较系统地从理论方面研究了板壳磁弹性问题,但很少给出具体算例。近些年来,许多学者在这方面进行了积极的探索。我国学者 Zhou Y H、Cao Y W、Zheng X J^[2,3]研究了铁磁材料的梁板在磁场中的屈曲及后屈曲问题。另外,Hasanyan D J^[4,5]给出了理想导

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** 边宇虹,女,1962年5月生,河北丰润人,汉族。教授,博士研究生,研究方向为板壳磁弹性理论及其应用。通信地址:秦皇岛市燕山大学建筑工程与力学学院。

体板在倾斜磁场中非线性振动的数学模型,并研究了有限导电性板条在磁场中的非线性振动问题。Hany H Sherie、Kamal A Hekmy^[6]借助于拉普拉斯及指数富利叶变换方法,解决了二维热磁弹性问题,给出半空间导体中的温度、应力以及感应磁场的分布。Magdy A Ezzat、Ahmed S El-Karamany^[7]借助于状态空间和拉普拉斯变换解决导电介质在两次热松弛情况下的二维热磁弹性问题。Liang Y、Shen Y P、Zhao M^[8]运用能量密度断裂准则研究了软铁磁材料中的共线裂纹问题。Chun-Bo Lin 等^[9~11]研究了磁场中的二维裂纹问题,给出封闭形式的磁弹性应力,并得出裂纹尖端附近奇异性的应力强度。Gaganidze E^[12]研究了铁磁材料在磁场中的动态响应。杨丽敏等^[13]将有限元法和解析法相结合,研究含孔压电板的力学集中问题,给出含孔压电板方程的通解。这些成果为磁弹性理论及其应用的研究打下了良好的基础。但是,目前非线性理论研究还不够完备,除梁、板、壳在电磁场中的振动、稳定性及屈曲问题外,其应力应变状态的分析非常少见,而且通常还仅仅停留在解一维^[14~17](或轴对称)问题的数值解上。在一维或轴对称的磁弹性研究的问题中,一般基本未知函数是 8 个,对于二维或准二维磁弹性问题的应力与变形的研究,至今尚未见到有关报道。

本文针对非定常电磁场和机械场耦合作用下载流条形薄板的非线性变形问题进行分析研究。在导出载流薄板的磁弹性非线性运动方程、几何方程、物理方程和电动力学方程的基础上,通过变量代换,整理成含有 10 个基本未知函数的标准柯西型方程。通过采用差分及准线性化方法,将含有 10 个基本未知函数的偏微分方程组,变换成为能够用离散正交法编程求解的准线性微分方程组,并以两边简支条形薄板为例,计算该板在非定常电磁场和机械载荷耦合作用下的应力及变形,讨论其应力及变形与外加电磁参量之间的关系。为改变在电磁场环境下的工程结构中条形薄板的工作状态及强度研究提供理论分析和数值计算方法。

2 基本方程

如图 1 所示,在时变磁场中,载流柔性体满足直法线假设及纵向纤维无挤压的吉尔霍夫—乐甫假设。在满足磁弹性假说^[18]的基础上,根据弹性力学理论、电磁场理论中的欧姆定律和 Maxwell 方程,给出电磁场本构关系、电流密度本构关系、电磁场的边界条件和洛仑兹力的表达式,导出载流条形薄板的二维电动力学方程、磁弹性运动方程、几何方程和物理方程,并建立可解的偏微分方程组。

2.1 电磁场的本构关系

对于电磁材料有

$$\begin{aligned} D &= \epsilon_0(1 + \epsilon_r) E = E \\ B &= \mu_0(1 + \mu_m) H = \mu H \end{aligned} \quad (1)$$

式中 D 为电感应强度或电位移矢量, ϵ_0 为真空中的介电常数, ϵ_r 为电极化率, E 为电场强度, μ 为电介质材料的绝对介电常数, B 为磁感应强度, μ_0 为真空中的磁导率, μ_m 为磁化系数, μ 为电介质材料的绝对磁导率, H 为磁场强度。

2.2 电流密度的本构关系

介质中电流密度的本构关系为

$$J = (\mathbf{E} + \mathbf{v} \times \mathbf{B}) + q\mathbf{v} \quad (2)$$

式中 J 为电流密度, σ 为电导率, q 为电荷密度, v 为电荷运动速度。

2.3 电磁场的边界条件

电磁场边界条件为^[20]

$$\mathbf{n} \times (\mathbf{B}_2 - \mathbf{B}_1) = 0 \quad \mathbf{n} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0 \quad (3)$$

式中 \mathbf{B}_2 和 \mathbf{B}_1 分别是介质 2 和介质 1 的磁感应强度矢量, \mathbf{E}_2 和 \mathbf{E}_1 分别是介质 2 和介质 1 的电场强度矢量, \mathbf{n} 为边界处的法向矢量。

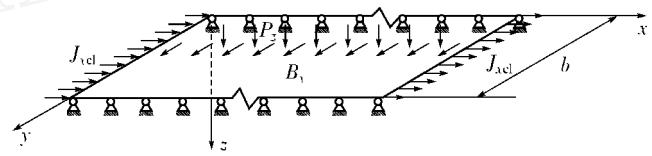


图 1 载流条形薄板

Fig. 1 Thin current-carrying strip-plate

2.4 载流条形薄板的二维电动力学方程

$$\begin{aligned} -\frac{\partial \mathbf{B}_z}{\partial t} &= \frac{\partial \mathbf{E}_y}{\partial x} - \frac{\partial \mathbf{E}_x}{\partial y} \\ \left[E_x + \frac{\partial v}{\partial t} B_z - \frac{1}{2} \frac{\partial w}{\partial t} (B_y^+ + B_y^-) \right] &= \frac{\partial H_z}{\partial y} - \frac{H_y^+ - H_y^-}{h} \\ \left[E_y - \frac{\partial u}{\partial t} B_z + \frac{1}{2} \frac{\partial w}{\partial t} (B_x^+ + B_x^-) \right] &= -\frac{\partial H_z}{\partial x} + \frac{H_x^+ - H_x^-}{h} \end{aligned} \quad (4)$$

式中 u, v, w 分别为 x, y, z 方向的位移, E_x, E_y, E_z 分别为 x, y, z 方向的电场强度, H_x, H_y, H_z 分别为 x, y, z 方向的磁场强度, B_x, B_y, B_z 分别为 x, y, z 方向的磁感应强度, h 为板的厚度, t 为时间变量, $B_i^\pm, H_i^\pm (i = x, y)$ 分别为相应量在板上、下表面的量值。

2.5 载流条形薄板的磁弹性运动方程

$$\begin{aligned} \frac{\partial N_x}{\partial x} + \frac{\partial S}{\partial y} + P_x + n_x + f_x &= h \frac{\partial^2 u}{\partial t^2} \\ \frac{\partial N_y}{\partial y} + \frac{\partial S}{\partial x} + P_y + n_y + f_y &= h \frac{\partial^2 v}{\partial t^2} \\ \frac{\partial (Q_x - N_{x,x} - S_{y,y})}{\partial x} + \frac{\partial (Q_y - N_{y,y} - S_{x,x})}{\partial y} + \\ P_z + n_z + f_z &= h \frac{\partial^2 w}{\partial t^2} \end{aligned}$$

$$\begin{aligned}\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x &= \frac{h^3}{12} \frac{\partial^2 w}{\partial t^2} \\ \frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y &= \frac{h^3}{12} \frac{\partial^2 w}{\partial t^2}\end{aligned}\quad (5)$$

式中

$$\begin{aligned}f_x &= J_{y \text{ cl}} h B_z + h E_y B_z - h \frac{\partial u}{\partial t} B_z^2 + \frac{h}{2} \frac{\partial w}{\partial t} (B_x^+ + B_x^-) B_z \\ f_y &= - J_{x \text{ cl}} h B_z - h E_x B_z - h \frac{\partial v}{\partial t} B_z^2 + \frac{h}{2} \frac{\partial w}{\partial t} (B_y^+ + B_y^-) B_z \\ f_z &= \frac{h}{2} [J_{x \text{ cl}} (B_y^+ + B_y^-) - J_{y \text{ cl}} (B_x^+ + B_x^-)] + \\ &\quad \frac{h}{2} E_x (B_y^+ + B_y^-) - \frac{h}{2} E_y (B_x^+ + B_x^-) + \\ &\quad h \left[\frac{1}{2} \frac{\partial v}{\partial t} (B_y^+ + B_y^-) + \frac{h}{12} \frac{\partial w}{\partial t} (B_y^+ - B_y^-) \right] \times \\ &\quad B_z - h \frac{\partial w}{\partial t} \left[\frac{1}{4} (B_y^+ + B_y^-)^2 + \frac{1}{12} (B_y^+ - B_y^-)^2 + \right. \\ &\quad \left. \frac{1}{4} (B_x^+ + B_x^-)^2 + \frac{1}{12} (B_x^+ - B_x^-)^2 \right] + \frac{h}{2} \frac{\partial u}{\partial t} B_z (B_x^+ + B_x^-) + \\ &\quad B_z + \frac{h^2}{12} \frac{\partial}{\partial t} B_z (B_x^+ - B_x^-)\end{aligned}\quad (6)$$

式中 N_x 、 N_y 、 Q_x 、 Q_y 、 S 、 M_x 、 M_y 和 M_{xy} 分别为板内相应方向上的内力和力矩， P_x 、 P_y 、 P_z 分别为机械载荷， n_x 、 n_y 、 n_z 分别为体积力， f_x 、 f_y 、 f_z 分别为相应方向上的洛伦兹力， α_x 、 α_y 分别为相应方向的转角， $J_{x \text{ cl}}$ 、 $J_{y \text{ cl}}$ 分别为相应方向的侧向电流密度， ρ 为板的质量密度。

2.6 载流条形薄板的几何方程和物理方程

$$\begin{aligned}x &= \frac{\partial u}{\partial x} + \frac{1}{2} \frac{\partial^2 w}{\partial x^2}, \quad y = \frac{\partial v}{\partial y} + \frac{1}{2} \frac{\partial^2 w}{\partial y^2}, \quad z = \frac{\partial w}{\partial z} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \\ x &= \frac{\partial}{\partial x}, \quad y = \frac{\partial}{\partial y}, \quad x y = -2 \frac{\partial^2 w}{\partial x \partial y} \\ x &= -\frac{\partial w}{\partial x}, \quad y = -\frac{\partial w}{\partial y} \\ N_x &= D_N (\alpha_x + \alpha_y), \quad N_y = D_N (\alpha_y + \alpha_x) \\ S &= D_N \frac{1 - \nu^2}{2}, \quad M_{xy} = M_{yx} = \frac{1}{2} D_M (1 - \nu^2) \alpha_{xy} \\ M_x &= D_M (\alpha_x + \alpha_y), \quad M_y = D_M (\alpha_y + \alpha_x)\end{aligned}\quad (7)$$

式中 $D_N (= Eh/(1 - \nu^2))$ 和 $D_M (= Eh^3/[12(1 - \nu^2)])$ 分别为板的抗拉刚度和抗弯刚度， E 为弹性模量， ν 为泊松比。

2.7 方程组的建立

选择 u 、 v 、 w 、 α_x 、 α_y 、 Q_y 、 S 、 M_y 、 E_x 和 B_z 作为基本未知函数，综合方程式(4) ~ (7)，可得如下偏微分方

程组

$$\begin{aligned}\frac{\partial u}{\partial y} &= \frac{2S}{D_N(1 - \nu^2)} - \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial y} &= \frac{N_y}{D_N} - \frac{1}{2} \frac{\partial^2 w}{\partial y^2} - \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 - \frac{\partial u}{\partial x} \\ \frac{\partial w}{\partial y} &= - \frac{\partial^2 w}{\partial y^2} = \frac{1}{D_M} M_y + \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial N_y}{\partial y} &= -h \frac{\partial^2 v}{\partial t^2} - \frac{\partial S}{\partial x} - (P_y + n_y + f_y) \\ \frac{\partial Q_y}{\partial y} &= \frac{h^3}{12} \frac{\partial^2}{\partial t^2} \left(\frac{12}{h^2} w - \frac{\partial^2 w}{\partial x^2} \right) - (P_z + n_z + f_z) - \\ &\quad \frac{\partial N_y}{\partial x} \frac{\partial w}{\partial x} - N_y \frac{\partial^2 w}{\partial x^2} - \frac{\partial w}{\partial x} Eh \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial x^2} \right) - \\ &\quad \frac{\partial^2 w}{\partial x^2} Eh \left[\frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right] + S \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial x^2} \frac{\partial S}{\partial x} - \\ &\quad \frac{\partial^2 M_y}{\partial x^2} + \frac{Eh^3}{12} \frac{\partial^4 w}{\partial x^4} \\ \frac{\partial S}{\partial y} &= h \frac{\partial^2 u}{\partial t^2} - Eh \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial x^2} \right) - \frac{\partial N_y}{\partial x} - (P_x + n_x + f_x) \\ \frac{\partial M_y}{\partial y} &= Q_y + N_y - S_x + \frac{h^3}{12} \frac{\partial^2 w}{\partial t^2} - 2 \frac{\partial}{\partial x} \left[D_M (1 - \right. \\ &\quad \left. \nu^2) \frac{\partial^2 w}{\partial x^2} \right] \\ \frac{\partial E_x}{\partial y} &= -\frac{1}{\mu} \frac{\partial^2 B_z}{\partial x^2} + B_z \frac{\partial}{\partial t} \left(\frac{\partial u}{\partial x} \right) + \frac{\partial u}{\partial t} \frac{\partial B_z}{\partial x} - \\ &\quad \frac{(B_x^+ + B_x^-)}{2} \frac{\partial}{\partial t} \left(\frac{\partial w}{\partial x} \right) + \frac{\partial B_z}{\partial t} \\ \frac{\partial B_z}{\partial y} &= \mu \left[E_x + \frac{\partial v}{\partial t} B_z - \frac{\partial w}{\partial t} \frac{(B_y^+ + B_y^-)}{2} \right] + \frac{B_y^+ - B_y^-}{h}\end{aligned}\quad (8)$$

式中

$$Q_y = Q_y - N_y - S_x + \frac{\partial M_{xy}}{\partial x} \quad (9)$$

3 计算方法

为进行离散化处理，将式(8)及相应的边界条件写成

$$\frac{\partial N}{\partial y} = F(x, y, N) \quad D_1 N|_{y=d_1} = d_1 \quad D_2 N|_{y=d_2} = d_2 \quad (10)$$

式中 $N = \{u, v, w, \alpha_x, \alpha_y, Q_y, S, M_y, E_x, B_z\}^T$ ， D_1 、 D_2 为给定的直角矩阵， d_1 、 d_2 为给定的矢量。为求解式(8)的非线性问题，通常采用迭代法。在采用迭代法解方程组(8)之前，先应用差分通项^[21]对方程组(8)的

右侧部分进行差分,得到

$$\begin{aligned}
 \frac{\partial u^i}{\partial y} &= \frac{2S^i}{D_N(1-\beta)} - \frac{v^{i+1}-v^{i-1}}{2} + \frac{w^{i+1}-w^{i-1}}{2} \\
 \frac{\partial v^i}{\partial y} &= \frac{N_y^i}{D_N} - \frac{1}{2}(v_y^i)^2 - \frac{1}{2}\left(\frac{w^{i+1}-w^{i-1}}{2}\right)^2 - \\
 &\quad \left(\frac{u^{i+1}-u^{i-1}}{2}\right) \\
 \frac{\partial w^i}{\partial y} &= -\frac{v^i}{y} \quad \frac{\partial v^i}{\partial y} = \frac{1}{D_M} M_y^i + \frac{w^{i+1}-2w^i+w^{i-1}}{2} \\
 \frac{\partial N_y^i}{\partial y} &= h \frac{\partial^2 v^i}{\partial t^2} - \frac{S^{i+1}-S^{i-1}}{2} - (P_y^i + n_y^i + f_y^i) \\
 \frac{\partial Q_y^i}{\partial y} &= \frac{h^3}{12} \frac{\partial^2}{\partial t^2} \left(\frac{12}{h^2} w^i - \frac{w^{i+1}-2w^i+w^{i-1}}{2} \right) - \\
 &\quad \frac{N_y^{i+1}-N_y^{i-1}}{2} \frac{w^{i+1}-w^{i-1}}{2} - P_z^i - f_z^i - \\
 &\quad \frac{w^{i+1}-w^{i-1}}{2} Eh \left(\frac{u^{i+1}-2u^i+u^{i-1}}{2} + \frac{w^{i+1}-w^{i-1}}{2} \times \right. \\
 &\quad \left. \frac{w^{i+1}-2w^i+w^{i-1}}{2} \right) - N_y^i \frac{w^{i+1}-2w^i+w^{i-1}}{2} - \\
 &\quad \frac{w^{i+1}-2w^i+w^{i-1}}{2} Eh \left[\frac{u^{i+1}-u^{i-1}}{2} + \right. \\
 &\quad \left. \frac{1}{2} \left(\frac{w^{i+1}-w^{i-1}}{2} \right)^2 \right] + \\
 &\quad \frac{Eh^3}{12} \frac{w^{i+2}-4w^{i+1}+6w^i-4w^{i-1}+w^{i-2}}{4} - \\
 &\quad \frac{M_y^{i+1}-2M_y^i+M_y^{i-1}}{2} + S^i \frac{\frac{i+1}{y}-\frac{i-1}{y}}{2} + \\
 &\quad \frac{n_y^i S^{i+1}-S^{i-1}}{2} - n_z^i \\
 \frac{\partial S^i}{\partial y} &= h \frac{\partial^2 u^i}{\partial t^2} - (P_x^i + n_x^i + f_x^i) - Eh \frac{u^{i+1}-2u^i+u^{i-1}}{2} - \\
 &\quad \frac{N_y^{i+1}-N_y^{i-1}}{2} - Eh \frac{w^{i+1}-w^{i-1}}{2} \frac{w^{i+1}-2w^i+w^{i-1}}{2} \\
 \frac{\partial M_y^i}{\partial y} &= Q_y^i + N_y^i - S^i \frac{w^{i+1}-w^{i-1}}{2} - 2D_M(1-\beta) \\
 &\quad \left(\frac{i+1}{y}-\frac{2}{2} \frac{i}{y}+\frac{i-1}{y} \right) + \frac{h^3}{12} \frac{\partial^2}{\partial t^2} \frac{i}{y} \\
 \frac{\partial E_x^i}{\partial y} &= \frac{\partial B_z^i}{\partial t} - \frac{1}{\mu} \frac{B_z^{i+1}-2B_z^i+B_z^{i-1}}{2} - \\
 &\quad \frac{B_y^++B_x^-}{2} \frac{\partial}{\partial t} \left(\frac{w^{i+1}-w^{i-1}}{2} \right) + \\
 &\quad \frac{\partial}{\partial t} \left(\frac{u^{i+1}-u^{i-1}}{2} \right) B_z^i + \frac{\partial u^i}{\partial t} \frac{B_z^{i+1}-B_z^{i-1}}{2} \\
 \frac{\partial B_z^i}{\partial y} &= \mu \left[E_x^i + \frac{\partial v^i}{\partial t} B_z^i - \frac{1}{2} \frac{\partial w^i}{\partial t} (B_y^+ + B_y^-) \right] + \\
 &\quad \frac{B_y^+-B_y^-}{h} \tag{11}
 \end{aligned}$$

式中洛伦兹力分量的差分表达式为

$$\begin{aligned}
 f_x^i &= J_{yc1} h B_z^i - \frac{h}{\mu} B_z^i \left(\frac{B_z^{i+1}-B_z^{i-1}}{2} - \frac{B_x^+-B_x^-}{h} \right) \\
 f_y^i &= -J_{xc1} h B_z^i - h E_x^i B_z^i - h \frac{\partial v^i}{\partial t} (B_z^i)^2 + \\
 &\quad \frac{h}{2} \frac{\partial w^i}{\partial t} (B_y^+ + B_y^-) B_z^i \\
 f_z^i &= \frac{h}{2} [J_{yc1} (B_y^+ + B_y^-) - J_{xc1} (B_x^+ + B_x^-)] + \frac{h}{2} E_x^i (B_y^+ + \\
 &\quad B_y^-) + \frac{h}{2} (B_x^+ + B_x^-) \left[\frac{1}{\mu} \left(\frac{B_z^{i+1}-B_z^{i-1}}{2} - \frac{B_x^+-B_x^-}{h} \right) + \right. \\
 &\quad \left. \frac{1}{2} \frac{\partial w^i}{\partial t} (B_x^+ + B_x^-) \right] - h \frac{\partial w^i}{\partial t} \left[\frac{1}{4} (B_y^+ + B_y^-)^2 + \right. \\
 &\quad \left. \frac{1}{12} (B_y^+ - B_y^-)^2 + \frac{1}{4} (B_x^+ + B_x^-)^2 + \frac{1}{12} (B_x^+ - B_x^-)^2 \right] + \\
 &\quad h \left[\frac{1}{2} \frac{\partial v^i}{\partial t} (B_y^+ + B_y^-) + \frac{h}{12} \frac{\partial^2 v^i}{\partial t^2} (B_y^+ - B_y^-) \right] B_z^i - \\
 &\quad \frac{h^2}{12} \frac{\partial}{\partial t} \left(\frac{w^{i+1}-w^{i-1}}{2} \right) B_z^i (B_x^+ - B_x^-) \tag{12}
 \end{aligned}$$

式中 β 是沿 x 方向的差步长。对于时变电磁场和机械载荷,式(8)需按瞬态求解。采用纽马克稳定有限差式

$$\begin{aligned}
 \dot{u}^{t+1} &= \frac{u^{t+1}-u^t}{(\Delta t)^2} - \left[\frac{\dot{u}^t}{\Delta t} + \ddot{u}^t \left(\frac{1}{2} - \right) \right] \frac{1}{\Delta t} \\
 \dot{u}^{t+1} &= \dot{u}^t + \frac{\Delta t}{2} (\ddot{u}^t + \dot{u}^{t+1}) \tag{13}
 \end{aligned}$$

式中 Δt 为系统参数,一般取 $\Delta t = 0.25$; t 为时间增量。将式(12)代入式(11),在相应的初始条件及边界条件下,采用数值计算中的正交离散法即可求出基本未知函数的值。

4 算例分析

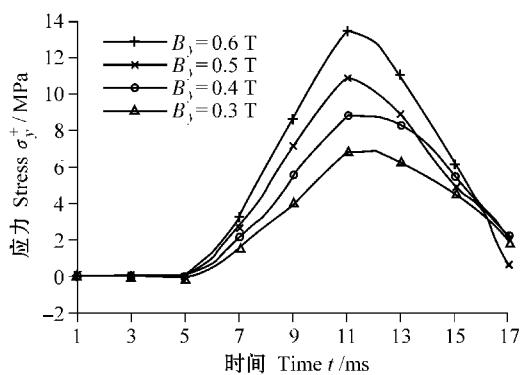
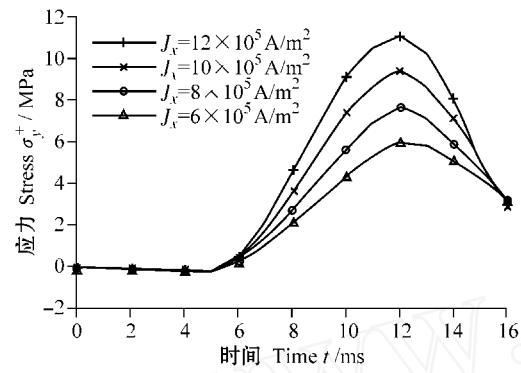
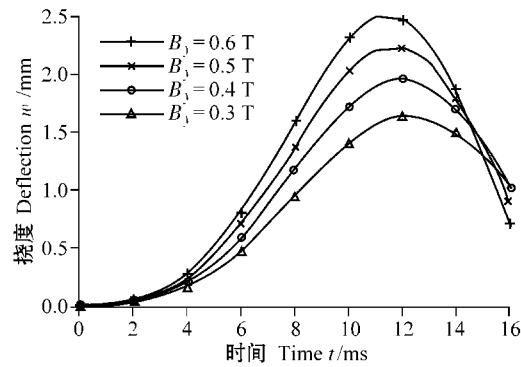
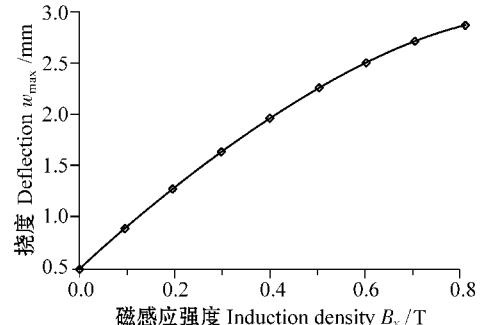
图 1 所示的铜制条形薄板处于磁场 $B = \{0, B_y, 0\}$ 中,通以密度为 $J_{yc1} = \{J_{xc1}, 0, 0\}$ 的侧向电流,机械载荷为 $P = \{0, 0, P_z\}$ 。已知 $E = 1.0 \times 10^{11} \text{ N/m}^2$, $\rho = 0.31$, $\nu = 0.960 \text{ kg/m}^3$, $\beta = 5.88 \times 10^{-7} (\text{A} \cdot \text{m})^{-1}$, $\mu = 1.25 \times 10^{-6} \text{ H/m}$, $J_{xc1} = J_x \sin(\omega t) \text{ A/m}^2$, $\omega = 10^2 \text{ s}^{-1}$, $P_z = 80 \text{ N/m}^2$, $h = 1 \times 10^{-3} \text{ m}$, 板宽 $b = 0.5 \text{ m}$ 。

边界条件 $y = 0$ 时, $B_z = 0.1 \sin(\omega t) \text{ T}$, $u = v = w = 0$, $M_y = 0$

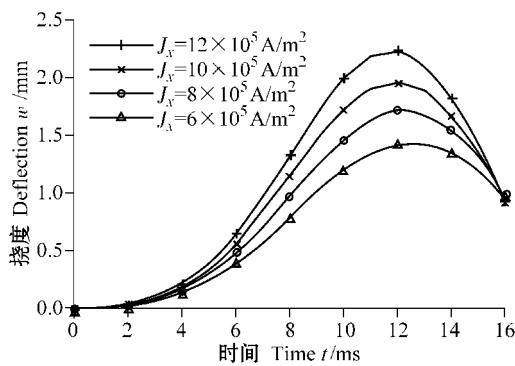
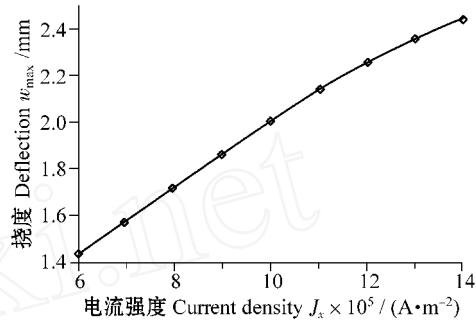
$y = 0.5 \text{ m}$ 时, $B_z = 0$, $u = v = w = 0$, $M_y = 0$

初始条件 $N(x, y, t)|_{t=0} = 0$, $\dot{u}(x, y, t)|_{t=0} = \dot{v}(x, y, t)|_{t=0} = \dot{w}(x, y, t)|_{t=0} = 0$

对式(11)编程并代入已知数据、初始条件和边界条件,在计算机上运算,解出 10 个基本未知函数 u 、 v 、 w 、 N_y 、 Q_y 、 S 、 M_y 、 E_x 和 B_z 。变换有关参数,可确定机械量与电磁量之间的关系及变化规律。由计算结果有

图 2 应力 σ_y^+ 与 B_y 、 t 的关系Fig. 2 Relations between the stress σ_y^+ and B_y , t 图 3 应力 σ_y^+ 与 J_x 、 t 的关系Fig. 3 Relations between the stress σ_y^+ and J_x 图 4 挠度 w 与 B_y 、 t 的关系Fig. 4 Relations between the deflection w and B_y , t 图 5 挠度 w_{\max} 与 B_y 的关系Fig. 5 Relation between the deflection w_{\max} and B_y

1) 图 2 给出外加侧向电流幅值为 $J_x = 5 \times 10^5 \text{ A/m}^2$ 、其他参数同上、不同磁感应强度 B_y 作用下，

图 6 挠度 w 与 J_x 、 t 的关系Fig. 6 Relations between the deflection w and J_x , t 图 7 挠度 w_{\max} 与 J_x 的关系

板中 ($y = 0.25 \text{ m}$) 上表面处沿 y 轴方向的正应力 σ_y^+ 随时间的变化情况。在初始时刻, 随着 B_y 的增大应力变化很小。在 5 ms 后, 随着外加磁场 B_y 的增大, 应力峰值迅速增大, 且应力峰值出现的时刻有所提前, 说明改变外加磁感应强度 B_y 的大小, 可改变板的动特性。图 3 为板中 ($y = 0.25 \text{ m}$) 上表面处沿 y 轴方向的正应力 σ_y^+ 随时间的变化情况。在初始时刻, 应力随侧向电流变化很小。在 5 ms 后, 随着外加侧向电流的增大, 应力迅速增大, 应力峰值一般在 $t = 12 \text{ ms}$ 到达。

2) 图 4 为板中挠度随时间的变化情况。在初始时刻, 板中挠度随外加磁感应强度变化很小。随着时间的推移, 板中挠度随着 B_y 的增大而迅速增大, 且挠度峰值出现的时刻有所提前, 挠度峰值一般在 $t = 11 \text{ ms}$ 和 $t = 12 \text{ ms}$ 之间到达, 说明改变外加磁感应强度可改变板的动特性。图 5 为板中最大挠度与磁感应强度 B_y 的关系。板中最大挠度与 B_y 成非线性增长关系。图 6 给出在外加磁场强度 $B_y = 0.2 \text{ T}$ 、其他参数同上、不同侧向电流幅值 J_x 作用下, 板中 ($y = 0.25 \text{ m}$) 挠度随时间的变化情况。随着时间的推移, 板中挠度逐渐增大, 在 $t = 12 \text{ ms}$ 附近, 挠度达到峰值, 随后挠度开始减小。对比图 4 知, 改变外加侧向电流对板中挠度产生的影响与改变外加磁感应强度相似, 但改变外加侧向电流对板动特性的影响不如改变外加磁感应强度明显。图 7 为板中最大挠度与侧向电流的关系。在 $J_x = 9 \times 10^5 \text{ A/m}^2$ 作用下, 板中最大挠度与侧向电流成线性关系。

10^5 A/m^2 之前,板中最大挠度与外加侧向电流几乎成线性增长关系;在 $J_x = 9 \times 10^5 \text{ A/m}^2$ 后,最大挠度与外加侧向电流的关系由线性转变成非线性。

5 结论

采用差分及准线性化方法,将含有 10 个基本未知函数的偏微分方程组,变换成为能用离散正交法编程求解的准线性微分方程组,其解给出载流薄板在电磁场作用下的内力、应力及其位移计算数值。由此讨论该板在非定常电磁场和机械载荷耦合作用下的应力及变形与外加电磁参量之间的关系。为改变在电磁场环境下工程结构中条形薄板的工作状态及强度研究提供理论分析和数值计算方法。由算例得知:

- 1) 当环境中的电磁场强度较弱时,对结构元件的位移和应力影响不大,当环境中的电磁场相对较强时,对位移和应力的影响变得较为明显,说明对处于电磁场环境中的结构元件进行磁弹性分析的必要性和重要意义。
- 2) 改变外加磁感应强度和外加侧向电流都能改变板的动特性。
- 3) 载流条形薄板的应力及变形与外加磁感应强度呈现非线性增长趋势,而与外加侧向电流呈现的非线性趋势不如前者明显。
- 4) 通过电磁参量的变换能实现载流条形薄板的变形及其应力状态的控制。

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