

# Numerical Study on the Distribution Characteristics of TP in Taihu Lake, China

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## Abstract

A depth-averaged two-dimensional hydrodynamics model and a two-dimensional water quality model, bases on the alternating direction implicit (ADI) method, is developed to study the distribution characteristics of total phosphorus (TP) in Taihu Lake. Wind stress, the pollution source from the inflow rivers, releasing rate of TP from the bottom sediment and water diversion from Yangtze River are the effecting factors of TP distribution. By using the model proposed in this paper, the concentration field of the total phosphorus was simulated, which leads to the conclusion that the flow field has a great influence on the spatial and temporal distribution of TP in Taihu Lake.

**Key words:** Taihu lake, eutrophication, Total Phosphorus (TP), water quality model, numerical simulation

serious. Transportation and diffusion of nutrients are one of the most important processes influencing the water quality and eutrophication in the large shallow lakes. The research on the characteristics of water quality has important directive significance to the management of lake pollution protection.

In this study, a depth-averaged 2-D water quality numerical model is developed. The model is coupled with a hydrodynamic model. The concentration field of TP induced by wind stress was simulated. The result is calibrated and verified by comparison of the available field data.

In the past few years, a lot of researches are carried out to study and predict the trends of eutrophication. When compared to other water quality models which have been developed previously, this model is superior in the following aspect. This water quality model is coupled with a hydrodynamic model, while other models are usually not coupled with hydrodynamic models.

## 1. Introduction

In recent decades, pollutions in water are becoming more and more serious. Eutrophication, which is the excessive growth of aquatic plants up to the level with interference to desirable water uses, has been increasingly acute all over the world. The problem is due to the excess discharge of nutrients, such as phosphorus, etc. It has been attended to seriously and many researches, including eutrophication control, modeling and so forth, have been conducted.

Taihu Lake is the third largest fresh water lake in China. It has been drastically polluted in the past decades due to many artificial factors. Drinking water crisis in Wuxi City in 2007, which was called a natural disaster, shows that eutrophication of Taihu Lake is becoming increasingly

## 2. Model framework

The developed model consists of a hydrodynamic model and a water quality model. Based on the hydrodynamic simulations, the spatial-temporal distributions of TP can be simulated by solving a series of advection-diffusion equations.

### 2.1 Hydrodynamics

The governing equations under orthogonal coordinates system are expressed as continuity equation

$$\frac{\partial \xi}{\partial t} + \frac{\partial}{\partial x}(Hu) + \frac{\partial}{\partial y}(Hv) = 0 \quad (0.1)$$

momentum equations

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - fv + g \frac{\partial \xi}{\partial x} - \varepsilon \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) - \frac{1}{\rho H} (\tau_x^s - \tau_x^b) = 0 \quad (0.2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + fu + g \frac{\partial \xi}{\partial y} - \varepsilon \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) - \frac{1}{\rho H} (\tau_y^s - \tau_y^b) = 0 \quad (0.3)$$

where  $u$  and  $v$  are the depth-averaged velocity components in the orthogonal coordinates;  $H (= h + \xi)$  is the water depth,  $h$  and  $\xi$  are the average water level and the water surface level;  $\rho$  is the density of water and  $g$  is the gravity acceleration;  $\varepsilon$  is the horizontal eddy viscosity coefficient;  $f$  is the Coriolis parameter;  $\tau_x^s$  and  $\tau_x^b$  are the  $x$ -components of shear stress on the water surface and on the bed.

$\tau^b$  is expressed empirically as:  $\tau^b = \rho g u \sqrt{u^2 + v^2} / C^2$  where  $C$  is the Chezy coefficient.

### 2.2 Transport equation of TP

A general transport equation for depth-averaged concentration of TP in the orthogonal coordinate can be written as

$$\frac{\partial(H\varphi)}{\partial t}+\frac{\partial(Hu\varphi)}{\partial x}+\frac{\partial(Hv\varphi)}{\partial y}=\frac{\partial}{\partial x}(HD_x\frac{\partial\varphi}{\partial x})+\frac{\partial}{\partial y}(HD_y\frac{\partial\varphi}{\partial y})+Q+F(\varphi)$$

(0.4)

where  $\varphi$  is the depth-averaged concentration of TP;  $D_x, D_y$  are the horizontal diffusive coefficients;  $Q$  represents external sources and sinks;  $F(\varphi)$  represents settling and sediment release.

Here, the term  $F(\varphi)$  is expressed as

$$F(\varphi_p)=d_p-v_p\cdot\varphi_p\cdot H$$

where  $d_p$  represents sediment release and  $v_p$  represents settling.

The parameters used in the model are showed below in Table 1.

Table 1  
Parameters used in the model

Parameter	Unit	value
$v_p$	$d^{-1}$	0.01
$d_p$	$g/(m^2\cdot d)$	0.0011
$\varepsilon$	$m^2/s$	5.0
$D_x、D_y$	$m^2/s$	5.0

3.Simulation of Taihu Lake

Flow field may be an important factor affecting TP distribution in Taihu Lake. The spatial distribution of TP

under different wind-induced current is simulated by using the water quality model developed.

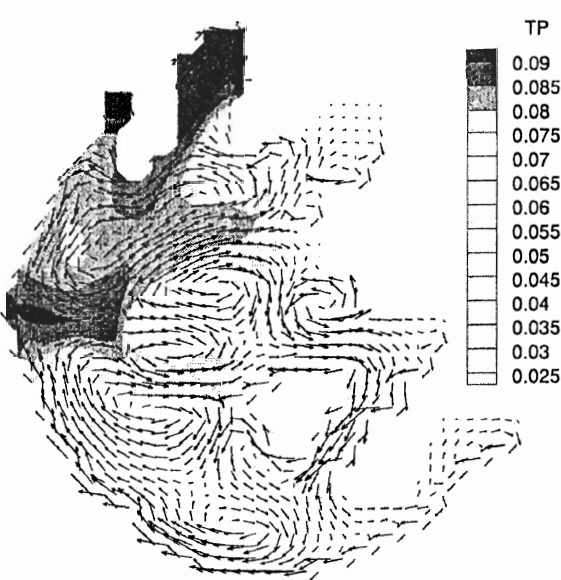


Figure 1. TP distribution under East wind(mg/L)

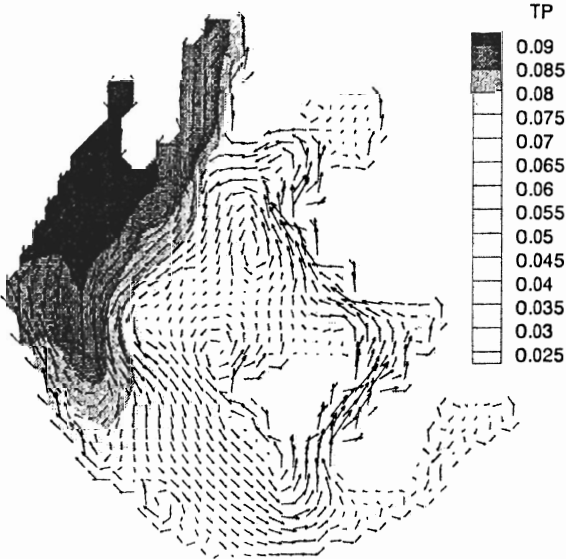


Figure 2. TP distribution under South Wind(mg/L)

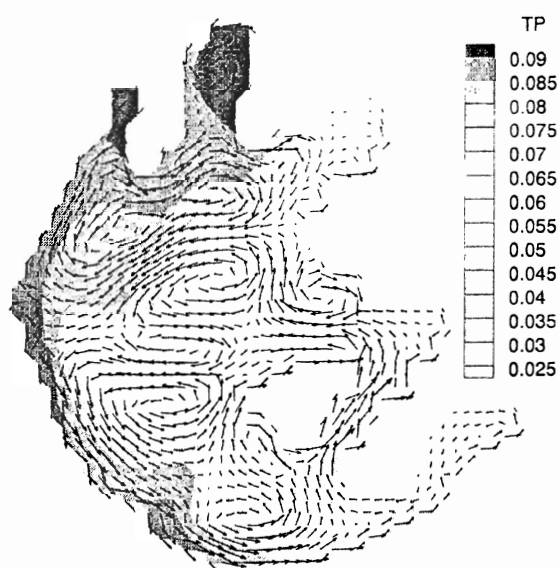


Figure 3. TP distribution under west wind(mg/L)

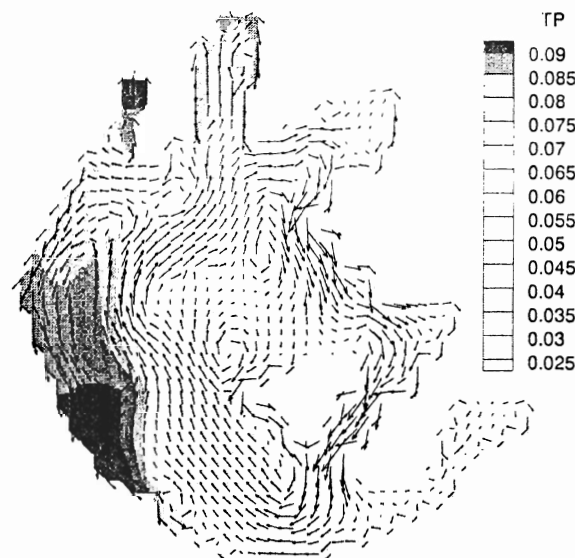


Figure 4. TP distribution under North Wind(mg/L)

The concentration field shows the distribution characteristics of TP in Taihu Lake. First, the concentration of TP in the west lake is higher than that in the east lake, and also the concentration of TP in the north lake is higher than that in the south lake, which is comparable with the observations; moreover, the flow field has a great influence on the spatial distribution of TP in Taihu Lake. Generally, anticlockwise current is more beneficial to the diffusion of TP than clockwise current.

#### Conclusions

Simulation of eutrophication phenomenon is important for the algae bloom disaster predication and waterbody management in Taihu Lake. Without a realistic simulation, measures to prevent the algae bloom disaster cannot be effected.

A depth-averaged 2D water quality model is developed. The model simulates the distribution characteristics of TP, which is associated with eutrophication in the waters. Of course, the model can be extended to other water quality constituents.

Distribution of TP in Taihu Lake is uneven. The concentration of TP in the west lake is much higher than that in the east lake. It is much easier to have a algae bloom disaster in the west Taihu lake. The lake current may be a beneficial factor for pollution diffusion.

More accurate simulations could be obtained by the

present method when more accurate observation data are available. It is meaningful that more water quality constituents are simulated in order to capture more details of eutrophication.

#### Acknowledgement

This research was supported by National Natural Science Funds of China for Distinguished Young Scholar (Grant No. 10825211).

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