Nanoadhesion of a Power-Law Graded Elastic Material *

CHEN Shao-Hua(陈少华)**, CHEN Pei-Jian(陈培见)
LNM, Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190

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The Dugdale–Barenblatt model is used to analyze the adhesion of graded elastic materials at the nanoscale with Young's modulus E varying with depth z according to a power law $E = E_0(z/c_0)^k$ $(0 < k < 1)$ while Poisson's ratio ν remains a constant, where E_0 is a referenced Young's modulus, k is the gradient exponent and c_0 is a characteristic length describing the variation rate of Young's modulus. We show that, when the size of a rigid punch becomes smaller than a critical length, the adhesive interface between the punch and the graded material detaches due to rupture with uniform stresses, rather than by crack propagation with stress concentration. The critical length can be reduced to the one for isotropic elastic materials only if the gradient exponent k vanishes.

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In recent years, the conception, flaw tolerance or flaw insensitivity, has been raised not only for biological materials,[1−7] but also for micro- or nano-scale electronic devices and mechanical devices.[8−10] This conception is now often used in interfacial adhesion problems. Generally speaking, the actual adhesion strength for an interface, which is defined as the force per unit contact area at pull-off, can be much lower than the theoretical adhesion strength due to the presence of crack-like flaws induced by surface roughness or contaminants. The breakage of adhesion is due to the crack propagation. Flaw tolerance in the adhesive interface will maximize the adhesion strength at the theoretical one via size reduction. The adhesive contact interface fracture is not due to the crack-like flaw propagation but to a uniform bonding rupture. For example, Persson[10] investigated the adhesive contact between a rigid disk and an elastic half space and showed the adhesion strength saturates for small contact objects. Gao *et al.*[3] studied the adhesion strength of a flat-ended cylindrical punch in contact with a rigid substrate and found that the adhesive strength will attain the theoretical one below a critical scale. Hui *et al.*[4] and Glassmaker *et al.*[5] studied a bio-mimetic fibrillar structure with slender elastic fibers and demonstrated that the adhesion strength can be enhanced in contrast to a non-fibrillar structure. Northen and Turner[11] reported significantly improved adhesion in hierarchical hairy adhesive materials.

In this Letter, we study the nanoadhesion between a small rigid punch and a graded elastic material with Young's modulus varying with depth according to a power law $E = E_0(z/c_0)^k$ $(0 < k < 1)$ while Poisson's ratio ν remains a constant, such as the adhesive pad in cicada. When $k = 0$, the gradient half-space degrades to an isotropic material. If $k = 1$, it reduces to be a Gibson material such as the soil.

The model is shown in Fig. 1. Perfect adhesion is assumed as that in Ref. [10] and the half-width of the punch is a. The adhesive interface is described by the Dugdale–Barenblatt model.

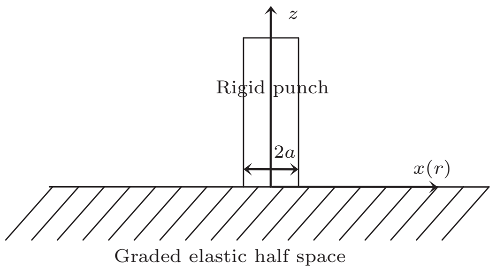

Fig. 1. Schematic diagram of a small-size rigid punch with half-width (plane strain) or radius (3D) a in adhesive contact with a power-law graded elastic half space.

From the solution of a plane strain graded elastic half space pulled by a homogeneous traction σ within the length region $2a$, the normal displacement $u_z(x)$ can be expressed as[12,13]

$$\frac{\beta c_0^k \sin(\pi\beta/2)\sigma}{2(1+k)I_k E^* k}\int_{-a}^{a}\frac{1}{|x-s|^k}ds = u_z(x), \quad (-a < x < a) \tag{1}$$

where

$$I_k = \frac{\pi\Gamma(3+k)}{2^{k+2}(2+k)\Gamma\left(\frac{3+k+\beta}{2}\right)\Gamma\left(\frac{3+k-\beta}{2}\right)},$$
$$\beta = \sqrt{(1+k)\left(1-\frac{k\nu}{1-\nu}\right)}, \quad E^* = \frac{E_0}{1-\nu^2}, \tag{2}$$

with Γ in the above formula being the Gamma function.

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**To whom correspondence should be addressed. Email: chenshaohua72@hotmail.com

Solving Eq. (1) yields the normal displacement under tension of the homogeneous traction σ,

$$u_z(x) = \frac{\beta c_0^k \sin(\pi\beta/2)\sigma}{2(1-k^2)I_k E^* k}\left[(a+x)^{1-k} + (a-x)^{1-k}\right]. \tag{3}$$

The cohesive law in the Dugdale–Barenblatt model can be expressed as

$$\sigma(\delta) = \begin{cases} \sigma_{\rm th}, & \delta(x) \le \delta_0, \\ 0, & \delta(x) > \delta_0, \end{cases} \tag{4}$$

where $\sigma(\delta)$ is the normal traction on the adhesive contact interface, δ_0 is the maximum effective range of cohesive interaction, $\delta(x)$ is the separation between the contact surfaces. The interfacial energy is

$$\Delta\gamma = \sigma_{\rm th}\delta_0. \tag{5}$$

At the moment of pull-off, the maximum opening displacement at the contact edge should not be larger than the effective interaction range δ_0, i.e., $\delta(a) \le \delta_0$. Thus, the normal traction on the adhesion interface $\sigma(\delta)$ uniformly attains the interfacial theoretical strength $\sigma_{\rm th}$. The critical size $a_{\rm cr}$ can be obtained from

$$\delta(a_{\rm cr}) = \delta_0, \tag{6}$$

where

$$\delta(a) = u_z(0) - u_z(a). \tag{7}$$

Substituting Eq. (3) into Eq. (6) leads to the critical size

$$a_{\rm cr} = \left[\frac{2k(1-k^2)I_k \Delta\gamma E^*}{\sigma_{\rm th}^2 c_0^k \beta \sin(\pi\beta/2)(2-2^{1-k})}\right]^{\frac{1}{1-k}}. \tag{8}$$

When $a \le a_{\rm cr}$, the detachment of the adhesive interface will be due to the uniform rupture and the traction at the interface attains uniformly the theoretical strength $\sigma_{\rm th}$. For a special case, $k = 0$, the graded elastic half space reduces to be an isotropic elastic one. Then the critical size becomes

$$a_{\rm cr} = \frac{\pi E^* \delta_0}{2\sigma_{\rm th}\ln 4}, \tag{9}$$

which is consistent with the results of Chen and Soh.[14]

The non-dimensional relationship between $a_{\rm cr}$ and c_0 can be written as

$$\left(\frac{a_{\rm cr}}{\delta_0}\right)^{1-k} = \frac{2(1-k^2)kI_k}{\beta\sin(\pi\beta/2)(2-2^{1-k})}\left(\frac{E^*}{\sigma_{\rm th}}\right)\left(\frac{c_0}{\delta_0}\right)^{-k}. \tag{10}$$

Figure 2 displays the non-dimensional critical size $a_{\rm cr}$ as a function of the non-dimensional characteristic length c_0 for a determined parameter $E^*/\sigma_{\rm th}$ and different grading exponents k. One can see that for a given k, the critical size $a_{\rm cr}$ decreases with the increasing characteristic length c_0. When k tends to zero, $a_{\rm cr}$ tends to be a constant given in Eq. (9).

For the three-dimensional (3D) case, we can find the normal surface displacement solution[15] of a graded elastic half space loaded by a uniformly distributed load σ in a circle region with radius a,

$$u_z(r) = u_c F\left(\frac{k+1}{2}, \frac{k-1}{2}, 1, \frac{r^2}{a^2}\right), \tag{11}$$

where $F(a,b,c,d)$ denotes Gauss's hypergeometric function, and the center surface displacement at $r = 0$ is

$$u_c = \frac{2}{1-k}\frac{\pi\sigma B c_0^k}{E_0 a^{k-1}}, \tag{12}$$

where

$$B = \frac{(1-\nu^2)\beta\sin(\pi\beta/2)F_{\alpha\beta}}{k(1+k)\Gamma\left(\frac{1}{2}\right)\Gamma\left(\frac{k}{2}\right)},$$
$$\beta = \sqrt{[(1+k)(1-k\nu/(1-\nu))]}, \tag{13}$$

$$F_{\alpha\beta} = \frac{(2+k)\,2^{1+k}\Gamma\left(\frac{3+k+\beta}{2}\right)\Gamma\left(\frac{3+k-\beta}{2}\right)}{\pi\Gamma(3+k)}, \tag{14}$$

while the surface displacement at $r = a$ is given as

$$u_a = \frac{u_c\Gamma(1-k)}{\Gamma\left(\frac{1-k}{2}\right)\Gamma\left(\frac{3-k}{2}\right)}. \tag{15}$$

For a rigid punch of radius a in adhesive contact with a graded elastic half space, the maximum separation δ can be obtained to be

$$\delta(a) = u_z(0) - u_z(a) = u_c - u_a. \tag{16}$$

When the interfacial traction achieves the uniformly distributed interface theoretical strength $\sigma_{\rm th}$, we obtain

$$\delta(a) = \frac{2}{1-k}\frac{\pi\sigma_{\rm th}Bc_0^k a^{1-k}}{E_0}\left[1 - \frac{\Gamma(1-k)}{\Gamma\left(\frac{1-k}{2}\right)\Gamma\left(\frac{3-k}{2}\right)}\right]. \tag{17}$$

Flaw insensitivity condition requires $\delta(a) \le \delta_0$, then the critical contact radius $a_{\rm cr}$ can be derived from $\delta(a_{\rm cr}) = \delta_0$ as

$$a_{\rm cr} = \left[\frac{(1-k)E_0\delta_0}{2\pi\sigma_{\rm th}Bc_0^k}\right]^{\frac{1}{1-k}}\left[1 - \frac{\Gamma(1-k)}{\Gamma\left(\frac{1-k}{2}\right)\Gamma\left(\frac{3-k}{2}\right)}\right]^{\frac{-1}{1-k}}, \tag{18}$$

which means that if $a \le a_{\rm cr}$, the bond breaking may occur uniformly over that circle contact area and the traction at the interface achieves uniformly the theoretical strength of the interface.

For a special case, $k = 0$, the critical size becomes

$$a_{\rm cr} = \frac{\pi E^*\Delta\gamma}{4(\pi/2-1)\sigma_{\rm th}^2}, \tag{19}$$

which is identical to the solution of Hui *et al.*[4] and Chen and Soh.[14] Comparing the critical radius in Eq. (19) with that obtained by Gao *et al.*,[3] one can find that the critical size predicted by the Dugdale–Barenblatt criterion is more conservative than that predicted by the Griffith criterion, which is similar to the flaw insensitivity conditions for a thin strip with central crack or double-edge cracks by Gao and Chen.[16]

The non-dimensional relationship between $a_{\rm cr}$ and c_0 can be written from Eq. (18) as

$$\left(\frac{a_{\rm cr}}{\delta_0}\right)^{1-k} = \frac{1-k}{2\pi B}\frac{E_0}{\sigma_{\rm th}}\left[1-\frac{\Gamma(1-k)}{\Gamma\left(\frac{1-k}{2}\right)\Gamma\left(\frac{3-k}{2}\right)}\right]^{-1} \times \left(\frac{c_0}{\delta_0}\right)^{-k}. \quad (20)$$

Comparing Eqs. (10) and (20), we can find an analogous tendency in the 3D case to the plane strain one. That is, for a determined grading exponent k, the critical size $a_{\rm cr}$ decreases also with an increasing c_0 in this 3D case. When the grading exponent k tends to zero, the critical size $a_{\rm cr}$ will be that in Eq. (19).

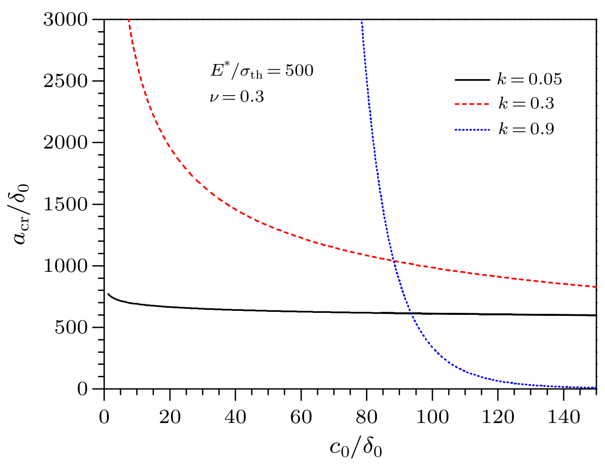

Fig. 2. The non-dimensional critical size $a_{\rm cr}/\delta_0$ as a function of the non-dimensional characteristic length c_0/δ_0 for determined parameters $E^*/\sigma_{\rm th}=500$, $\nu=0.3$ with different grading exponents k.

From Eqs. (8) and (18), one can see that the critical size of nano-adhesion depends on not only the feature of the solid surface, such as the surface energy $\Delta\gamma$ and the effective interaction stress $\sigma_{\rm th}$, but also the material constants, such as the referenced Young's modulus E_0, the gradient exponent k and the characteristically geometric length c_0, which describes the variation rate of Young's modulus near the surface. Nanoadhesion of flaw tolerance requires c_0 is comparable to the surface effective interaction distance δ_0 as shown in Fig. 2, which leads to the critical adhesion length $a_{\rm cr}$ comparable to δ_0. If c_0 is much larger than δ_0, the adhesion will be flaw-sensitive.

In conclusion, we have analyzed the size effects in both plane strain and three-dimensional nanoadhesion for small sized punches in adhesive contact with a graded elastic half space. We find a critical length scale, under which the bonding breaking occurs uniformly over the contact area, rather than by crack propagation as is almost the case for macroscopic bodies. The critical length depends on the grading exponent k, the ratio of $E_0/\sigma_{\rm th}$, as well as the characteristic length c_0 describing the gradient variation rate and the effective interaction distance δ_0. For this kind of gradient half space, surface nanoadhesion of flaw tolerance seems to depend largely on the material characteristics near the surface. When the graded elastic material reduces to an isotropic elastic one, the critical size of flaw insensitivity degrades consistently to the one found for an isotropic case.[4,14]

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