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Universal Biot number determining stress duration and susceptibility of ceramic cylinders to quenching

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Universal Biot number determining stress duration and susceptibility of ceramic cylinders to quenching

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A universal Biot number, which not only describes the susceptibility of ceramic cylinders to quenching but also determines the duration that ceramic cylinders are subjected to thermal stress during thermal shock, is theoretically obtained. The analysis proves that thermal shock failure of ceramic cylinders with a Biot number greater than the critical value is a rapid process, which only occurs in the initial heat conduction regime. The results provide a guide to the selection of ceramic materials for thermostructural engineering, with particular reference to thermal shock.

Keywords: thermal properties; ceramics; thermal shock; Biot number

1. Introduction

Thermal shock, which, in ceramics, causes a catastrophic decrease in strength and instantaneous failure at a very low critical temperature compared with their melting points, has been a major challenge in their thermostructural application [1,2]. The primary failure mechanism is traditionally considered as crack initiation in the ceramics during thermal shock, when the stresses imposed by a thermal gradient exceed the strength of the materials [3–8]. Previous studies have shown that the thermal shock fracture resistance of ceramic materials is dependent, besides their physical properties and the environmental medium, not only on their geometrical characteristics, e.g. plate, cylinder and sphere, but also on the stress duration during thermal shock [9,10], which effectively impacts the thermal stress level and distribution inside the ceramics. Because stress duration during thermal shock is closely associated with variable rapid heat conduction, which is a problem for heat transfer, the characteristics of stress duration are not well understood [9–11].

In existing studies on thermal shock in ceramics, the Biot number, which is the ratio of inter-conduction and surface-convection resistances to heat transfer, plays a key role in determining the transient temperature and thermal stress fields of the ceramics [7–13]. However, the relationships between the Biot number and stress duration or the geometrical characteristic during thermal shock still remain unclear.

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In the present study, the equation for heat conduction is used initially to obtain the temperature-wave penetration time in a ceramic cylinder. Secondly, stress duration, in which surface tensile stress under cold shock and the central axial stress under hot shock separately reach their maximum values, is obtained using thermoelasticity. Thirdly, by comparing the temperature-wave penetration time with stress duration, a universal Biot number is obtained that determines both the susceptibility and failure process of the ceramic cylinder to thermal shock. Finally, the relationship between the Biot number and the properties of the ceramic cylinder in thermal shock is discussed.

2. Results and discussions

Consider an infinitely long cylinder of radius R , which has a uniform initial temperature T_0 . At initial time $\tau=0$, the surface of the cylinder is suddenly exposed to a convective-medium temperature T_∞ . Obviously, the central axis of the cylinder is symmetrical for heat propagating from the surface to the interior, and the cylindrical coordinates are embedded in the central axis, as shown in Figure 1.

The transient temperature field in the interior of the cylinder $T(r, \tau)$ satisfies the heat transfer equation:

$$\frac{\partial T}{\partial \tau} = a \frac{1}{r} \left(r \frac{\partial T}{\partial r} \right), \quad (1)$$

where $a = k/\rho c_p$ is the thermal diffusivity; k , ρ and c_p are the thermal conductivity, density and specific heat at constant pressure of the cylinder material, respectively. The initial and boundary conditions of Equation (1) are

$$T(r, 0) = T_0, \quad (2)$$

$$\left. \frac{\partial T}{\partial r} \right|_{r=0} = 0, \quad (3)$$

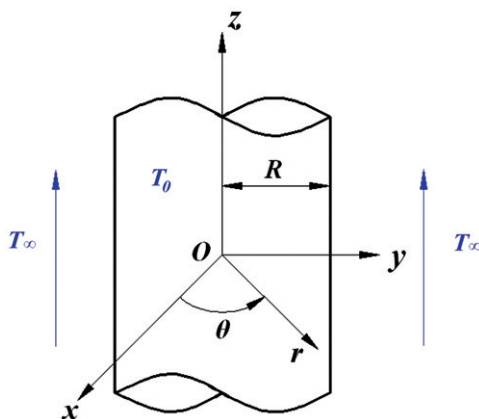


Figure 1. Cylinder of infinite length and radius R , at an initial temperature T_0 , suddenly exposed to a convective-medium temperature T_∞ .

$$-k \frac{\partial T}{\partial r} \bigg|_{r=R} = h(T - T_\infty), \quad (4)$$

where h is the surface heat transfer coefficient. Based on a standard separation-of-variables technique, the solution to Equation (1) under conditions (2), (3) and (4) is written as [12]

$$\frac{T(r, \tau) - T_0}{T_\infty - T_0} = 1 - 2 \sum_{n=1}^{\infty} \frac{J_1(\beta_n)}{\beta_n [J_0^2(\beta_n) + J_1^2(\beta_n)]} \exp(-\beta_n^2 \cdot f) J_0(\beta_n \cdot r^*). \quad (5)$$

In Equation (5), J_0 and J_1 are the zero- and first-order Bessel function of the first kind, respectively; $r^* = r/R$ stands for the dimensionless radius of the cylinder; $f = \alpha\tau/R^2$ is the Fourier number that expresses the dimensionless characteristic time during temperature propagation; and β_n are the roots of the equation:

$$\beta_n \frac{J_1(\beta_n)}{J_0(\beta_n)} = \beta, \quad (6)$$

where $\beta = hR/k$ is the Biot number of the cylinder, which is a dimensionless characteristic number that includes the physical and geometric properties of the cylinder together with the property of the convective medium.

Equation (5) is used to study the temperature-wave penetration time in the cylinder, f_p . The penetration time is defined as the duration during which the temperature change propagating from the surface of the cylinder just reaches its central axis [14]. For the sake of convenience, here we take penetration time as the duration when the temperature change in the central axis occurs and reaches 0.1% of the initial temperature of the cylinder. In terms of different Biot moduli, the penetration time in the cylinder was obtained under cold shock conditions, i.e. $T_0 > T_\infty$, as shown in Figure 2. It indicates that the temperature-wave penetration

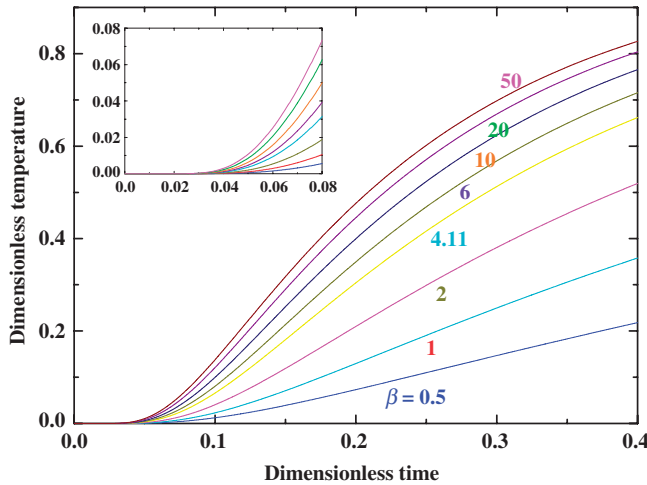


Figure 2. (Colour online). Temperature profiles of the central axis of the cylinder under cold shock conditions. Inset shows the temperature-wave penetration evolution time.

time is closely associated with the Biot number of the cylinder – the greater the Biot number, the shorter the penetration time.

We also investigated the thermal stress field in the interior of the cylinder. The dimensionless thermal stress of the cylinder during thermal shock is defined by [9,15]

$$\sigma^* = \frac{\sigma(r, \tau)(1 - \nu)}{\alpha E(T_\infty - T_0)}, \quad (7)$$

where E , ν and α are Young's modulus, Poisson's ratio and the coefficient of thermal expansion of the cylinder material, respectively; $\sigma(r, \tau)$ is the actual thermal stress field in the cylinder, and is expressed as [16]

$$\sigma_r(r, \tau) = \frac{\alpha E}{1 - \nu} \left[\frac{1}{R^2} \int_0^R (T - T_0) r \, dr - \frac{1}{r^2} \int_0^r (T - T_0) r \, dr \right], \quad (8)$$

$$\sigma_\theta(r, \tau) = \frac{\alpha E}{1 - \nu} \left[\frac{1}{R^2} \int_0^R (T - T_0) r \, dr + \frac{1}{r^2} \int_0^r (T - T_0) r \, dr - (T - T_0) \right], \quad (9)$$

$$\sigma_z(r, \tau) = \frac{\alpha E}{1 - \nu} \left[\frac{2}{R^2} \int_0^R (T - T_0) r \, dr - (T - T_0) \right], \quad (10)$$

where $\sigma_r(r, \tau)$, $\sigma_\theta(r, \tau)$ and $\sigma_z(r, \tau)$ stand for the radial, tangential and axial stress of $\sigma(r, \tau)$, respectively, and they satisfy:

$$\sigma_z = \mu(\sigma_r + \sigma_\theta) - \alpha E(T - T_0) = \sigma_r + \sigma_\theta, \quad (11)$$

$$\lim_{r \rightarrow 0} \frac{1}{r^2} \int_0^r (T - T_0) r \, dr = \frac{1}{2} [T(0, \tau) - T_0], \quad (12)$$

$$\lim_{r \rightarrow 0} \frac{1}{r} \int_0^r (T - T_0) r \, dr = 0. \quad (13)$$

According to Equations (5), (8), (9) and (10), the different stress fields of the cylinder are readily obtained, as shown in Figure 3. The results indicate that, under the cold shock conditions ($T_0 > T_\infty$), maximum tensile thermal stress occurs only on the surface of the cylinder, while under hot shock conditions ($T_\infty > T_0$), maximum tensile thermal stress occurs only on the central axis of the cylinder.

We now concentrate on the tensile stresses during both cooling and heating because ceramics are much weaker in tension than under compression; failure often occurs on the surface during cooling and on the central axis during heating [7,9,15].

In addition, according to Equations (8)–(13), the thermal stresses on the central axis of the cylinder satisfy

$$\sigma_r = \sigma_\theta = \frac{\sigma_z}{2}, \quad (14)$$

and the thermal stresses on the surface of the cylinder satisfy

$$\sigma_r = 0; \quad \sigma_\theta = \sigma_z. \quad (15)$$

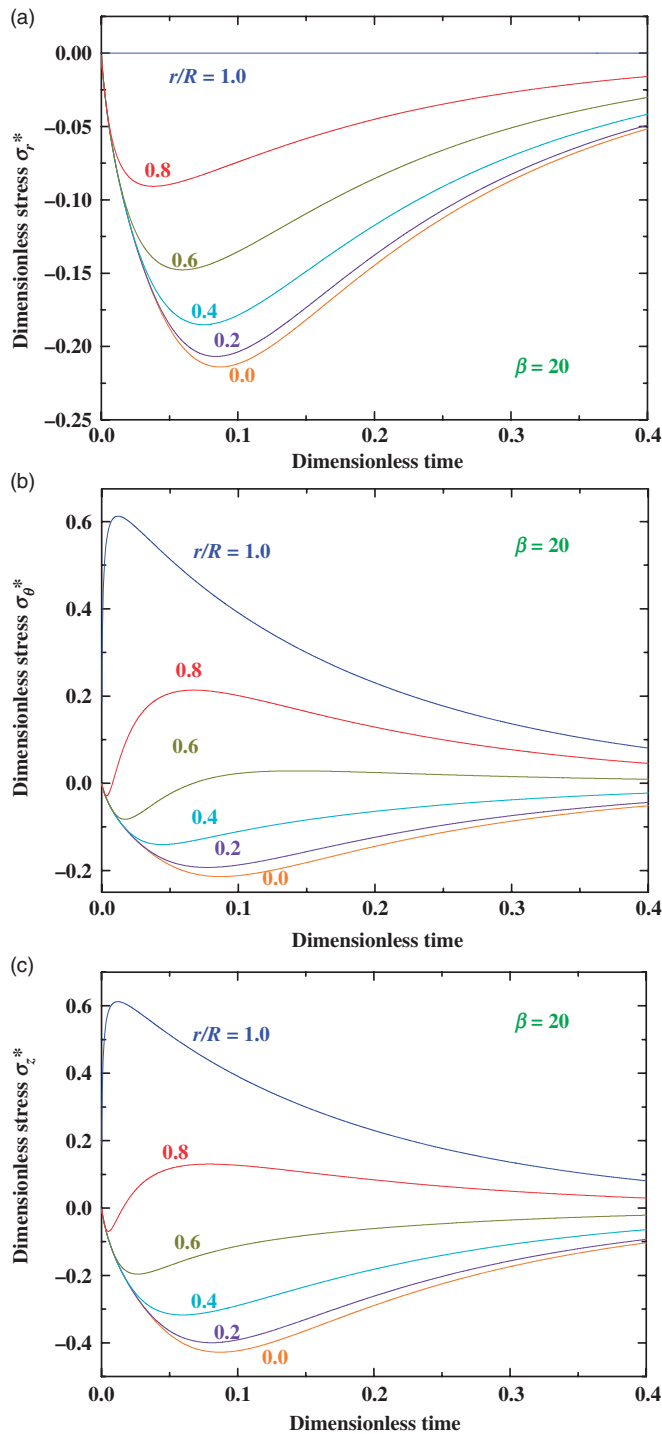


Figure 3. (Colour online). Evolution of stress fields in the cylinder ($\beta = 20$) under cold shock conditions: (a) radial stress, (b) tangential stress and (c) axial stress.

Therefore, axial stress σ_z will become the maximum tensile stress both on the surface of the cylinder during cold shock and on the central axis of the cylinder during hot shock. It is calculated in terms of the different Biot numbers of the cylinder according to Equations (5) and (10), as shown in Figure 4. This indicates that the maximum thermal stress in the cylinder, as for penetration time, is closely related to the Biot number and does not occur at the start of thermal shock, except when the Biot number is equal to infinity. In terms of each Biot number of the cylinder, there is an extremum in stress time f^* at which the thermal stress reaches its maximum value. The greater the Biot modulus, the shorter the stress time extremum, as shown in Figure 4. From the theory of thermal shock fracture [3,9,15], as the thermal stress

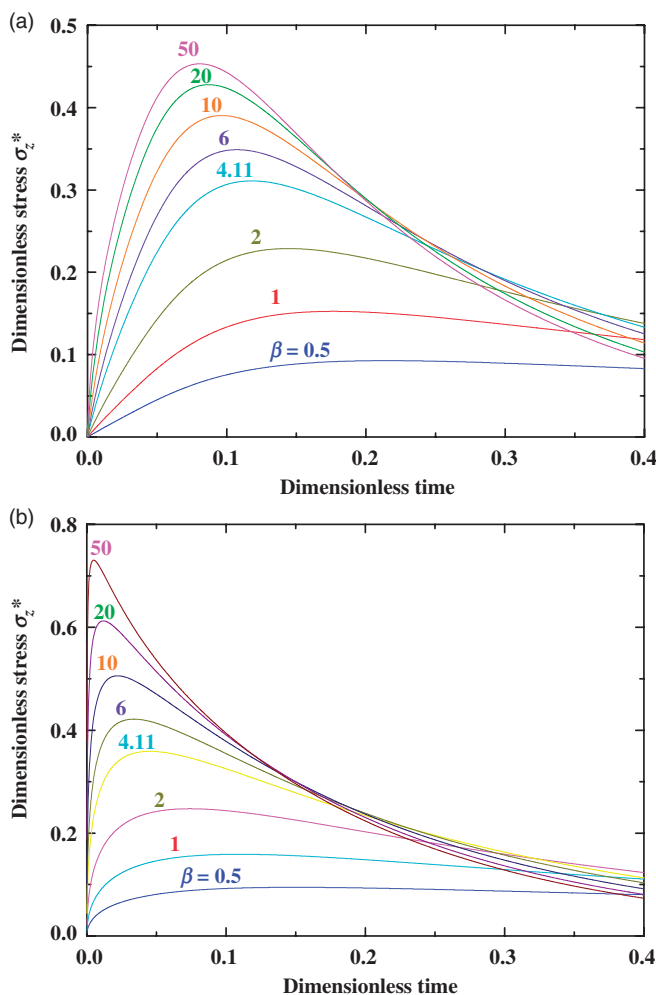


Figure 4. (Colour online). Changes in dimensionless stress fields with Biot numbers. (a) Axial tensile stresses on the central axis of the cylinder under hot shock conditions. (b) Axial tensile stresses on the surface of the cylinder under cold shock conditions.

in the cylinder during thermal shock is greater than the strength of the material, failure of the cylinder occurs. Therefore, failure of the ceramic cylinder occurs at or before the stress time extremum f^* . In addition, the tensile stress on the surface of the cylinder during cooling is much greater than on the central axis during heating under conditions of the same absolute temperature difference, $|T_0 - T_\infty|$. It shows that the failure in ceramic cylinder occurs more easily during cooling than heating.

Comparing the temperature-wave penetration time, f_p , with the stress time extremum during cooling, f^* , we find that there is a critical value for the Biot number of cylinder, $\beta_c = 4.11$, at which the two dimensionless times are the same, $f^* = f_p = 0.0452$, as shown in Figure 5. When the Biot number of a cylinder is greater than this critical value, $\beta > \beta_c$, the two types of dimensionless time satisfy $f^* < f_p$; when $\beta < \beta_c$, the dimensionless time $f^* > f_p$. This proves that if the Biot number of a ceramic cylinder is greater than this critical value, failure during cold shock occurs in the initial regime of the heat conduction process, in which the temperature-wave propagating from the surface does not arrive at the central axis of the cylinder [14,17]. This is clearly a very rapid material-failure process. It indicates that the critical Biot number determines the main duration characteristics for ceramic cylinders subjected to thermal shock and, therefore, can be applied to evaluate their susceptibility to thermal shock. For example, if the Biot number of a ceramic cylinder is greater than 4.11, the cylinder should be defined as sensitive to thermal shock. Whereas, if the Biot number of a ceramic cylinder is less than this critical value, its failure during cold shock can theoretically occur at any time before the stress time extremum f^* .

On the other hand, under hot shock conditions, there is no intersection point between the temperature-wave penetration time and stress time extremum, as shown

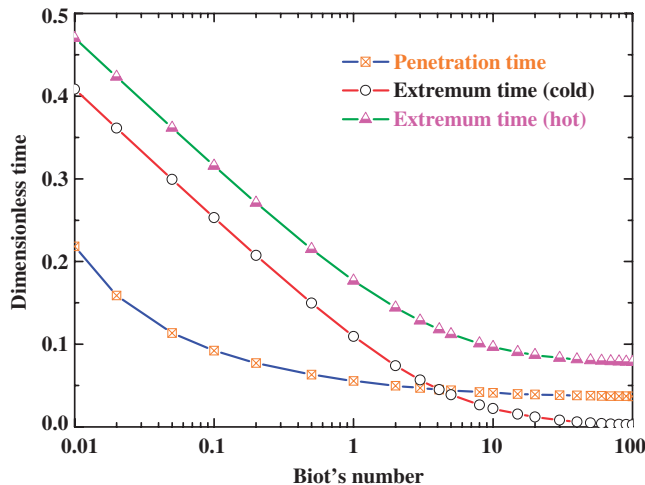


Figure 5. (Colour online). Variations in temperature-wave penetration time and stress time extrema under cold and hot shock with Biot number of the cylinder. Penetration and the cold extremum intersect where the Biot number is 4.11 and dimensionless time is 0.0452. However, penetration and the hot extremum do not intersect; the hot extremum time is always greater than penetration time.

in Figure 5. This is because the maximum thermal tensile stress appears only at the central axis of the cylinder under this condition [7,15], where penetration time is always less than the stress time extremum. Therefore, under hot shock conditions, the failure of a ceramic cylinder during thermal shock can theoretically occur at any time before the stress time extremum, which encompasses the initial or regular heat conduction regime.

3. Conclusions

A critical Biot number $\beta_c = 4.11$, which not only determines the susceptibility of ceramic cylinders to quenching but also indicates the duration when ceramic cylinders fail during thermal shock, is theoretically obtained under cold shock conditions. The results show that both stress duration and geometrical characteristic play significant roles in the failure of ceramic materials subjected to thermal shock. These results can provide a guide to the selection of the ceramics for thermostructural engineering, with particular reference to thermal shock.

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