RECENT PROGRESSES IN FLUID DYNAMICS RESEARCH
Proceedings of the Sixth International Conference on Fluid Mechanics
AIP Conf. Proc. Vol 1376, 382-385 (2011)
©2011 American Institute of Physics 978-0-7354-0936-1/$30.00

An Approach to Estimate the Flow Through an Irregular Fracture

Q. Q. Liu*, H. G. Fan

Laboratory of Environmental Mechanics, Institute of Mechanics, CAS, Beijing 100190, China

Email: qqliu@imech.ac.cn

Abstract A new model to estimate the flow in a fracture has been developed in this paper. This model used two sinusoidal-varying walls with different phases to replace the flat planes in the cubic law model. The steady laminar flow between non-symmetric sinusoidal surfaces was numerically solved. The relationships between the effective hydraulic apertures and the phase retardation for different amplitudes and wavelengths are investigated respectively. Finally, a formula of the effective hydraulic aperture of the fracture was carried out based on the numerical results.

Key words: fracture seepage, cubic law, effective hydraulic aperture, non-symmetric sinusoidal fracture

INTRODUCTION

Because of the complex surface of the natural fracture (Figure 1), it is not easy to obtain an evident law by direct analysis. Instead, the fracture is generalized to a geometrical model reflecting some main characteristics of the fractures. The parallel plate model is the most representative model, which ignores the roughness of the wall and the change of the fracture aperture and considers the fracture as a pair of smooth parallel plates. And the flow rate through a fracture varies as the cube of the joint aperture:$q = (h_{\mathrm{H}}^3/12\mu)(\Delta p/l)$, where, q is the flow rate, Δp is the pressure difference, l is the length of the fracture, h_{H} is the hydraulic aperture of the fracture, μ is the viscosity coefficient.

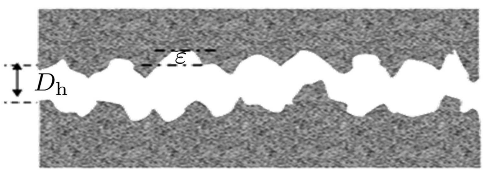

Figure 1: Schematic diagram of a nature fracture

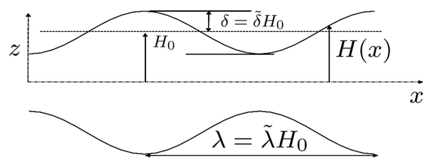

Figure 2: Symmetrical sinusoidal fracture model

However, experimental observations show that flow through a fracture decreases its speed more rapidly than that expected by the cube law of the mean aperture. So a symmetric sinusoidal fracture model is developed by Zimmermal, et al. [1]. The surface of fracture is simplified to sinusoidal form as shown in the Figure 2. Though this model is more realistic than the parallel plate model, it is limited in application because the real fracture is not always symmetrical. Therefore, by adding phase retardation to the symmetric sinusoidal fracture, a new model called non-symmetric sinusoidal fracture model is developed in this paper.

THEORETIC ANALYSIS

Figure 3 shows the geometrical pattern of fracture described by the unsymmetrical sinusoidal model. The expressions of top and bottom walls of fracture are given as follows, respectively

$$H(x) = H_0 \left[1 + \frac{\delta}{H_0} \sin\left(\frac{2\pi x}{\lambda}\right)\right] \tag{1}$$

$$H(x) = -H_0\left[1+\frac{\delta}{H_0}\sin\left(\frac{2\pi x}{\lambda}+\Delta\phi\right)\right] \tag{2}$$

Where, H_0 is the mean aperture of the fracture, δ, λ and $\Delta\varphi$ are the magnitude, wavelength and phase retardation of the sinusoidal wave, respectively. $\tilde{\delta}$ and $\tilde{\lambda}$ are normalized results. This model can reflect the characteristic of the real fracture much better than before.

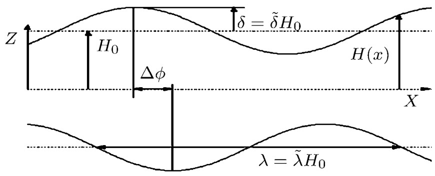

Figure 3: Unsymmetrical sinusoidal fracture model

The flow in the fracture is governed by the Navier-Stokes equations. However, for slow flow, the velocity gradient in flow direction is very small and can be ignored when the change of the fracture aperture is smooth. Therefore, the flow in a fracture can be described by following simplified forms

$$g\frac{\partial h}{\partial x} = v\frac{\partial^2 u}{\partial z^2} \tag{3}$$

$$g\frac{\partial h}{\partial z} = v\frac{\partial^2 w}{\partial z^2} \tag{4}$$

where, u and w are the velocity components in the x- and z-directions, respectively; v is the viscosity coefficient of water; h is the total hydraulic head. $h = p/\gamma + \varsigma$, p is the pressure and γ is the specific gravity of water, ς is the coordinate in g-direction.The boundary conditions can be described by the pressure condition as follows

$$x = 0, \quad h = h_\mathrm{l} \tag{5}$$

$$x = l, \quad h = h_\mathrm{r} \tag{6}$$

where, l is the fracture length, h_l and h_r are the height in the entrance and the exit, respectively.

Based on Reynolds equation, Zimmerman et al. [1] carried out an analytical solution of equivalent hydraulic aperture for symmetrical sinusoidal seepage [2]. Subsequently, by considering the effect of wavelength and amplitude of fracture walls, Sisavath et al. [3] obtained the second-order perturbation solution of the equivalent hydraulic aperture by using the Stokes equations

$$h_\mathrm{H}^3 = \frac{(2H_0)^3(1-\overline{\delta}^2)^{5/2}}{1+(\overline{\delta}^2/2)}\left[1+\frac{\overline{\delta}^2}{\overline{\lambda}^2}\left(\frac{36}{15}\right)\frac{\pi^2(1-\overline{\delta}^2)}{1+(\overline{\delta}^2/2)}\right]^{-1} \tag{7}$$

where, $\overline{\lambda}$ is the normalized wavelength, $\overline{\lambda} = \lambda/H_0$. H_0 is the half of the average distance between top and bottom boundaries; $\overline{\delta}$ is the normalized magnitude, $\overline{\delta} = \delta/H_0$, h_H is the equivalent hydraulic aperture.

Considering the effect of unsymmetrical sinusoidal fracture walls, we introduce a function of phase retardation to the formula (7). Consequently, when Reynolds number is low and the change of the fracture wall is smooth, the equivalent hydraulic aperture can be given in Eq.(8) for the unsymmetrical sinusoidal fracture model.

$$h_\mathrm{H} = f(\tilde{\lambda},\tilde{\delta})g(\Delta\varphi) \tag{8}$$

Where, $f(\tilde{\lambda},\tilde{\delta}) = \frac{(2H_0)^3(1-\overline{\delta}^2)^{5/2}}{1+(\overline{\delta}^2/2)}\left[1+\frac{\overline{\delta}^2}{\overline{\lambda}^2}\left(\frac{36}{15}\right)\frac{\pi^2(1-\overline{\delta}^2)}{1+(\overline{\delta}^2/2)}\right]^{-1}$, $g(\Delta\varphi)$ is the function of phase retardation $\Delta\varphi$.

NUMERICAL SIMULATION AND DISCUSSION

For ascertaining the forms of $g(\Delta\varphi)$, numerical method was employed here to fit the equivalent hydraulic aperture based on the formula (8). By means of the Fluent Software, the flows in fracture have been numerically simulated for different fractures.

The change law of $g(\Delta\varphi)$ is shown in Figure 4, in which $\Delta\varphi$ ranges from 0 to 4π and $\delta/H_0 = 0.2$, $\lambda/2H_0 = 0.2$. The geometrical pattern of fracture in Figure 3 shows when $\Delta\varphi = 0$, it degrades to the symmetrical sinusoidal fracture model, that means: $g(0) = 1$, $g(\Delta\varphi) = g(\Delta\varphi + 2\pi)$. Therefore, we can consider approximately the $g(\Delta\varphi)$ to be a sinusoidal function as following forms

$$g(\Delta\varphi) = 1 + a[\sin(\Delta\varphi + b) - \sin b] \tag{9}$$

In order to get the values of a and b in the Eq.(9), the fracture flows were simulated in the condition of low Reynolds number. In the numerical simulations, δ/H_0 ranges from 0.2 to 0.9 with interval 0.1, $\lambda/2H_0$ is from 2 to 8 with interval 1, and $\Delta\varphi$ is from 0 to 4π with interval 0.25π. Flow rates q in the fracture can be obtained for a given pressure difference Δp and different values of $\overline{\lambda}$, $\widetilde{\delta}$ and $\nabla\varphi$. Therefore, h_{H}can be solved by the cubic law, and $g(\Delta\varphi)$ can be calculated from Eq.(8). Then the value of a and b in Eq.(9) can be fitted by combining Eq.(8). Figures 5 and 6 show the fitted results of b changing with wavelength and magnitude of sinusoidal function wall.

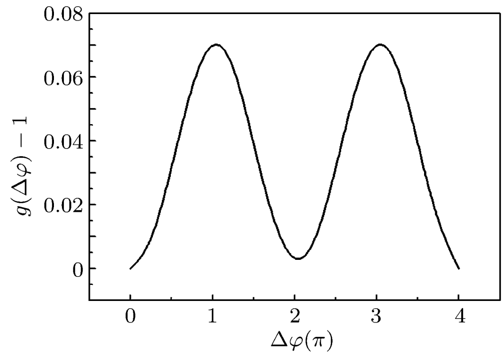

Figure 4: Function $g(\Delta\varphi)$

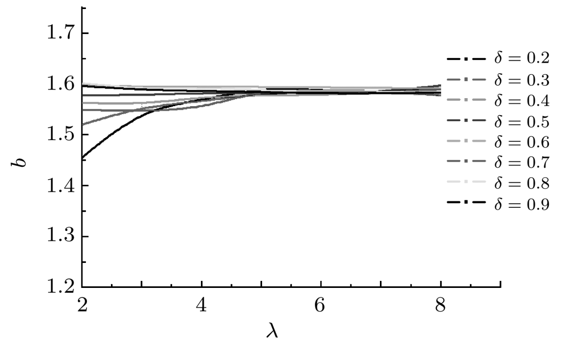

Figure 5: The relationship of parameter b and wavelength

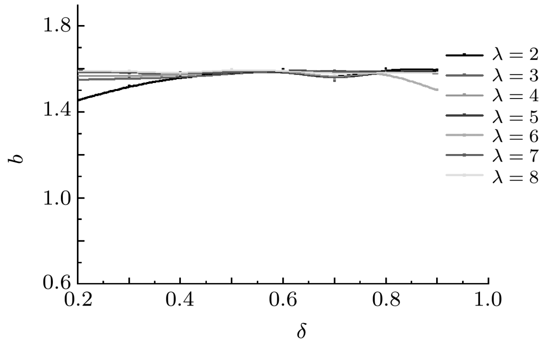

Figure 6: The relationship of parameter b and magnitude

Figures 5 and 6 show that b equals to $\pi/2$ approximately. Substituting $b = \pi/2$ into Eqs.(9) and (8), the equivalent hydraulic aperture for unsymmetrical sinusoidal fracture model is given as follows

$$h_{\mathrm{H}}^3 = \frac{(2H_0)^3(1-\overline{\delta}^2)^{5/2}}{1+(\overline{\delta}^2/2)}\left[1+\frac{\overline{\delta}^2}{\overline{\lambda}^2}\left(\frac{36}{15}\right)\frac{\pi^2(1-\overline{\delta}^2)}{1+(\overline{\delta}^2/2)}\right]^{-1}\cdot\{1+a[\cos(\Delta\phi)-1]\} \tag{10}$$

The numerical sumilated results shows that the parameter a is a complex function of $\overline{\lambda}$ and $\overline{\delta}$. Assuming the function of a with $\overline{\lambda}$ and $\overline{\delta}$ can be fitted to the following equation

$$a = k_1\overline{\delta}^{k2}\sqrt{1+\frac{k_3}{\overline{\lambda}+k_4}} \tag{11}$$

Then, we can obtained the fitting parameters $k_1 = -207.544\ 3$, $k_2 = 13.004\ 4$, $k_3 = -7.266\ 3$, $k_4 = 5.640\ 0$.

Consequently, the equivalent hydraulic aperture of unsymmetrical sinusoidal fracture seepage can be estimated by following formula

$$h_{\mathrm{H}}^{3}=\frac{(2H_0)^3(1-\overline{\delta}^2)^{5/2}}{1+(\overline{\delta}^2/2)}\left[1+\frac{\overline{\delta}^2}{\overline{\lambda}^2}\left(\frac{36}{15}\right)\frac{\pi^2(1-\overline{\delta}^2)}{1+(\overline{\delta}^2/2)}\right]^{-1}\left\{1+k_1\overline{\delta}^{k2}\sqrt{1+\frac{k_3}{\overline{\lambda}+k_4}}[\cos(\Delta\phi)-1]\right\} \tag{12}$$

When $\overline{\lambda}$ goes to infinite, formula (12) will degenerate to the analytical solution by Zimmerman et al. [1], and when $\overline{\delta}$ goes to zero, it can degenerate to the cubic law.

CONCLUSIONS

This paper developed a new model called non-symmetric sinusoidal fracture model to describe the flow in a fracture with complex geometry boundaries. This model is more close to the real nature fracture. The relationships between the effective hydraulic apertures and the phase retardation for different relative amplitudes and wavelengths are investigated respectively. In final, a formula for estimating the effective hydraulic aperture of the fracture was carried out. This result makes the cubic law can be applied to more general fracture with complex geometry boundaries.

Acknowledgements

The supports of the National Natural Science Funds of China for Distinguished Young Scholar (10825211) and the Key project of Natural Science Foundation of China (10932012) are gratefully acknowledged.

REFERENCES

1. Zimmermal RW, Kumar S, Bodvarsson GS. Lubrication theory analysis of rough-walled fractures. *Int. J. Rock. Mech.*, 1991; **28**: 325-331
2. Brown SR, Stockman HW, Reeves J. Applicability of the Reynolds equation for modeling fluid flow between rough surfaces. *Geophys. Res. Lett.*, 1995; **22**: 2537-2540
3. Sisavath S, Al-Yaarubi A, Pain CC, et al. A simple model for deviations from the cubic law for a fracture undergoing dilation or closure. *Pure appl. Geophys*., 2003; **160**: 1009-1020