Chapter 17

A CRITERION FOR THERMO-PLASTIC SHEAR INSTABILITY

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A criterion for thermo-plastic shear instability which includes heat transfer is derived from a system of equations describing plastic deformation, the first law of thermodynamics and Fourier's heat transfer rule.

It is shown that the criterion is nearly the same as that for adiabatic case formally and thermal conductivity connects the characteristic time and length of the instability phenomenon. Both strain-rate hardening and current strain rate control the time during which the instability develops fully.

The criterion is applied to three metals: titanium, mild steel and an aluminium alloy. It is shown that titanium becomes unstable at small strains and instability develops fully at high rates but for mild steel this is reversed.

I. INTRODUCTION

It is accepted that the flow stress of a metal is dependent not only on the strain but the strain rate and temperature. Thus in pure shear the flow stress is written as

$$\tau = f(\gamma, \dot{\gamma}, \theta) \qquad [1.1]$$

where τ is stress, γ is strain, $\dot{\gamma}$ is strain-rate and θ is temperature. Increasing strain and strain-rate generally increases the

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flow stress whereas increasing the temperature decreases the flow stress. This temperature effect is complicated by the heat generated during plastic deformation. At low strain rates, part of this heat is dissipated due to conduction. However at high strain rates such as occur in projectile impact, punching and machining, heat conduction is minimized. Under these circumstances catastropic failure has been observed. Many authors have described adiabatic shear instability and some have proposed criteria to determine the onset of adiabatic instability. For instance, Culver (1) proposed the condition

$$d\tau = 0 \qquad [1.2]$$

to deduce this criterion. Culver further assumed that the constitutive relation of the material had the form

$$\tau = \beta\gamma^n \qquad [1.3]$$

and obtained a simple critical strain criterion

$$\gamma_c = \frac{n\rho c_v}{K\left|\frac{\partial\tau}{\partial\theta}\right|} \qquad [1.4]$$

where ρ is the density, c_v is the specific heat, $\left|\frac{\partial\tau}{\partial\theta}\right| = \left|\left(\frac{\partial\tau}{\partial\theta}\right)_{\dot{\gamma},\gamma}\right|$ is the thermal softening and K is a constant equalling the quotient of the heat generated by plastic work and the total plastic work.

Here we start with a system of differential equations describing mechanical deformation, the first law of thermodynamics and Fourier's heat transfer rule to derive the criterion of thermoplastic shear instability. It is found that generally the criterion is approximately and formally the same as the above one for adiabatic conditions. But heat transfer controls the relationship between the characteristic time and length of the instability phenomenon explicitly, whereas strain-rate hardening and current strain-rate control the time during which the instability develops fully.

II. CRITERION

We confine ourselves to pure shear deformation and make three initial assumptions: (1) the material is incompressible; (2) it is possible to neglect elastic energy, since it is small in comparison with the energy of plastic deformation; (3) the relationship between the plastic work W_p and the heat q generated by it, is given

$$q = KW_p \tag{2.1}$$

where $K \simeq 0.9$. In this case, the dynamic equation is

$$\rho \frac{\partial \ddot{u}}{\partial y} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\tau + \frac{\partial^2}{\partial x \partial y}(\sigma_x + \sigma_y) \tag{2.2}$$

where u is the displacement in the x direction, σ_x and σ_y are the Cauchy normal stresses in the x and y directions respectively and $\tau = \tau_{xy}$ is the shear stress.

Bearing in mind that there is a thermal effect involved in plastic deformation, the first law of thermodynamics and Fourier's heat transfer rule are introduced. The latter is

$$h_i = -\lambda\theta,_i \tag{2.3}$$

where h_i is the heat flux and λ is the thermal conductivity. Substituting eq.[2.1] and [2.3] into the first law of thermodynamics the energy equation, including heat transfer, is obtained

$$K\left(\frac{\partial W_p}{\partial t} + \dot{u}\frac{\partial W_p}{\partial x}\right) = \rho c_v\left(\frac{\partial \theta}{\partial t} + \dot{u}\frac{\partial \theta}{\partial x}\right) - \lambda\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\theta \tag{2.4}$$

where c_v is the specific heat.

Supposing that the variation of all the physical variables in the y direction is much greater than that in the x direction, we obtain the following system of equations

$$\begin{cases} \rho \dfrac{\partial^2 \gamma}{\partial t^2} = \dfrac{\partial^2 \tau}{\partial y^2} & [2.5] \\[2ex] K\tau \dfrac{\partial \gamma}{\partial t} = \rho c_v \dfrac{\partial \theta}{\partial t} - \lambda \dfrac{\partial^2 \theta}{\partial y^2} & [2.6] \end{cases}$$

We suppose that the material exhibit no strain-rate history effects, then differentiating [1.1] we obtain

$$d\tau = Qd\gamma + Rd\dot{\gamma} - Pd\theta \tag{2.7}$$

where $Q = \left(\frac{\partial \tau}{\partial \gamma}\right)_{\dot{\gamma},\theta}$ is work-hardening, $R = \left(\frac{\partial \theta}{\partial \dot{\gamma}}\right)_{\theta,\gamma}$ is strain-rate hardening and $P = -\left(\frac{\partial \tau}{\partial \theta}\right)_{\gamma,\dot{\gamma}}$ is thermal softening.

The perturbation method is used to solve the equations [2.5] and [2.6]. Supposing

$$\gamma = \gamma_0 + \gamma_* e^{\alpha t + iky} \qquad [2.8]$$

$$\theta = \theta_0 + \theta_* e^{\alpha t + iky} \qquad [2.9]$$

$$\tau = \tau_0 + (Q_0\gamma_* + R_0\alpha\gamma_* - P_0\theta_*)e^{\alpha t + iky} \qquad [2.10]$$

where γ_0, $\dot{\theta}_0$ and τ_0 are solutions of the equations [2.5] and [2.6], γ_* and θ_* are the perturbations and are much smaller than γ_0 and θ_0 respectively, k is the wave number and α is the reciprocal of characteristic time. Substituting [2.8], [2.9] and [2.10] into equations [2.5] and [2.6], and considering the necessary condition for a non-trivial solution of an homogeneous system of equations is that the determinant of the coefficients should be zero, we can obtain the so-called "spectral equation" connecting k and α:

$$\rho^2 c_v \alpha^3 + \rho[KP_0\dot{\gamma}_0 + (\lambda + c_v R_0)k^2]\alpha^2$$

$$+ (\lambda R_0 k^2 + \rho c_v Q_0 - K\tau_0 P_0)k^2\alpha + \lambda Q_0 k^4 = 0 \qquad [2.11]$$

The condition $\lambda \to 0$ is equivalent to the adiabatic case. In this situation it is simple to verify that if

$$B = \frac{K\tau_0 P_0}{\rho c_v Q_0} > 1 \qquad [2.12]$$

α must have a positive root and therefore shear instability will occur.

Physically, the inequality [2.12] implies that when the thermal softening caused by plastic work becomes more dominant than work-hardening, thermo-plastic shear instability must occur.

As far as non-adiabatic conditions are concerned, the situation is a bit different. In this case, there is a pair of values of α_m and k_m, at which α_m is the maximum. In addition to eq.[2.11], α_m and k_m must satisfy the following equation

$$\frac{d\alpha_m}{dk_m^2} = 0 \qquad [2.13]$$

By differentiation and rearrangement, the following criterion for positive real α_m can be found

$$B > 1 + \sqrt{4C} \qquad [2.14]$$

and α_m falls in the interval

$$0 < \tilde{\alpha}_m < \frac{B-1}{A+1} \qquad [2.15]$$

where

$$A = \frac{c_v R_o}{\lambda} \qquad B = \frac{K\tau_o P_o}{\rho c_v Q_o} \qquad C = \frac{K\lambda\dot{\gamma}_o P_o}{\rho c_v{}^2 Q_o}$$

$$\tilde{\alpha}_m = \frac{\lambda\alpha}{c_v Q_o} \qquad \tilde{k}_m{}^2 = \frac{\lambda^2 k^2}{\rho c_v{}^2 Q_o} \qquad [2.16]$$

Therefore, inequality [2.14] is the criterion of thermo-plastic shear instability under non-adiabatic conditions.

It is important that the thermal conductivity λ and current strain rate only appear explicitly in dimensionless parameter C in the criterion [2.14]. If $C \ll 1$, both adiabatic and non-adiabatic deformations would have the same formal criterion $B > 1$.

But the characteristic time is given by

$$t_m \sim \frac{1}{\alpha_m} \sim \frac{R_o{}^*}{\dot{\gamma}_o} \frac{\rho c_v}{K\tau_o P_o - \rho c_v Q_o} \qquad [2.17]$$

where $R_o{}^* = (\frac{\partial\tau}{\partial \ln\dot{\gamma}})_o$. t_m is in inverse proportion to the current strain-rate $\dot{\gamma}_o$. This might be one of the reasons for the occurrence of thermo-plastic shear instability at high strain rates. On the other hand, the thermal conductivity λ links the characteristic time t with the characteristic length ℓ as follows

$$\frac{\ell^2}{t} \sim \frac{\alpha_m}{k_m{}^2} \sim a(\frac{\tilde{\alpha}_m}{\tilde{k}_m{}^2}) \qquad [2.18]$$

where the thermal diffusivity $a = \frac{\lambda}{\rho c_v}$.

III. APPLICATION OF CRITERION

The instability criterion

$$B = \frac{K\tau_o P_o}{\rho c_v Q_o} > 1 \qquad [3.1]$$

connects both state and material parameters. It is always desirable in practice to rearrange a criterion in a way such that on one side there are only state parameters and on the other there are only material parameters which form a new applicable material constant. In the case considered, $K/\rho c_v$ is obviously a material parameter, but τ_o, P_o and Q_o are state parameters and connected with each other by the constitutive relation of the material concerned. For this reason, the form $B > 1$ is not convenient in practice.

Examining [3.1] carefully, we can find that dimensionless parameter B implicitly contains a strain, because of Q_o = stress/strain. So, if a definite constitutive relation is introduced into [3.1], the criterion must contain only one state parameter-strain. Inequality [3.1] means that a state parameter-strain should be greater than a critical strain.

We suppose no matter how complicated the constitutive relation is, for a definite material, a characteristic strain exists as a new material constant. We may suggest that this new material constant becomes a practical criterion for a type of ductile failure phenomenon.

If the isothermal and rate-constant stress and strain relation behaves as

$$\tau = \beta\gamma^n \qquad [3.2]$$

and $P_o = -(\frac{\partial\tau}{\partial\theta})_o = ct$, the prescribed practical criterion is

$$\gamma \geq \gamma_c = \frac{n\rho c_v}{KP_o} \qquad [3.3]$$

If the stress-strain relation is linear work-hardening, then

$$\gamma \geq \gamma_c = \frac{\rho c_v}{KP_o} - \frac{\tau_Y}{\beta} \qquad [3.4]$$

Although $\tau = \beta\gamma^n$ or $\tau = \tau_Y + \beta\gamma$ etc. are only approximate expressions and affected by strain rate and temperature, in addition, P_0 is by no means a material constant, we still can arrive at a rough estimate about dimensionless parameters, the critical strain and the characteristic time.

Taking mild steel, titanium and an aluminum alloy as examples and adapting the data from Culver (1) and others, we obtain the results listed in Table I.

It can be seen that dimensionless parameter C is very small and A very large. It implies that the heat transfer can affect the process before the occurrance of instability, but not appear in the instability criterion explicitly. The time t_m roughly characterizes the interval during which instability can fully develop. From the table above titanium will become unstable at smaller shear strains and instability can develop more quickly. On the other hand for mild steel instability can develop slowly and at high shear strains.

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V. REFERENCES

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TABLE I

	T_i		Mild Steel		Al alloy	
$\rho(g/cm^3)$	4.533		7.850		2.707	
$c_v(erg/g.^oc)10^7$	0.544		0.502		0.963	
$\lambda(1/cm.sec.^oc)10^7$	0.493		0.611		1.598	
n	0.17		0.28		0.075	
$\beta(dyn/cm^2)10^9$	8.888		5.168		4.134	
$P_o(dyn/cm^2.^oc)10^7$	1.426		0.633		0.496	
γ_c	0.326		1.94		0.438	
$\dot{\gamma}_o(1/sec)$	5×10^{-3}	8×10^2	5×10^{-3}	1×10^3	5×10^{-3}	8×10^2
$R^*_o(dyn/cm^2)10^7$	2.0	2.5	3.8	7.6	3.8	3.8
A	10^9	10^4	10^9	10^4	10^8	10^3
C	10^{-13}	10^{-7}	10^{-13}	10^{-7}	10^{-13}	10^{-7}
B	1.03					
$\tilde{\alpha}_m$	5.8×10^{-12}	7.9×10^{-7}	4.7×10^{-12}	5.7×10^{-7}	5.6×10^{-11}	9.5×10^{-6}
$t_m(sec)$	42	3.1×10^{-4}	300	2.4×10^{-3}	46	2.7×10^{-4}

1 The last three lines, B, $\tilde{\alpha}_m$ and t_m are calculated for $\gamma-\gamma_c/\gamma_c \simeq 3\%$.