Correlation of crack growth rate and stress ratio for fatigue damage containing very high cycle fatigue regime

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Abstract A model is proposed to correlate the crack growth rate and stress ratio containing very high cycle fatigue regime. The model is verified by the experimental data in literature. Then a formula is derived for the effect of mean stress on fatigue strength, and it is used to estimate the fatigue strength of a bearing steel in very high cycle fatigue regime at different stress ratios. The estimated results are also compared with those by Goodman formula. [doi:10.1063/2.1203104]

Keywords very high cycle fatigue, crack growth rate, stress ratio, fatigue strength

The failure of very high cycle fatigue (VHCF) for high strength steels is usually caused by interior nonmetallic inclusions and a fish eye fracture mode often presents with a fine granular area (FGA)[1] observed around the inclusion at the fracture origin. Several models have been proposed for the formation of FGA, such as "hydrogen embrittlement" model,[2] "dispersive decohesion of spherical carbide" model,[3] and "polygonization and micro-debonding" model.[1] Zhao et al.[4] investigated the formation mechanism of FGA in high strength steels and presented a theoretical model to predict the threshold value of its formation based on the plastic zone at crack tip. In their results, it was indicated that the stress intensity factor range at the front of FGA kept constant that corresponding to the threshold value of the crack propagation ΔK_{th}. Duan et al.[5] proposed a model to predict the fish eye shape in VHCF regime using the energy approach. Although it is not very clear for the formation mechanism of FGA, it is widely accepted that the fatigue life in VHCF regime is primarily due to the crack initiation stage and most of fatigue life is spent on forming FGA. Hence, it is very important to investigate the evolvement process of crack initiation stage in order to predict the fatigue life of VHCF regime. Sun et al.[6] developed a model for estimating the fatigue life of high strength steels in high cycle and VHCF regimes with fish eye mode failure based on the cumulative fatigue damage, and estimated the crack growth rate of FGA which was of the order of magnitude 10^{-14} to 10^{-11} m/cycle. The studies by Stanzl-Tschegg and Schönbauer[7] indicated that at very high number of cycles the fracture surfaces of fatigue crack growth specimens tested in vacuum showed similar features in the VHCF regime with specimens where fish eye fractures formed. This indicates that it may be an effective way to study the evolvement process of crack initiation stage by studying the law of the crack growth rate containing VHCF regime. In this paper, a crack growth rate model containing the VHCF regime is developed. The model takes into account the effect of stress ratio. Then a formula is derived for the effect of mean stress on the fatigue strength.

Prior to the model, it is assumed that the crack growth below the traditional threshold value of the crack propagation ΔK_{th} can also be described as a function of the stress intensify factor range ΔK. In the near threshold stage, the crack growth rate is usually approximated by the form introduced by Elber[8]

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C(\Delta K - \Delta K_{\text{th}})^n, \tag{1}$$

where C and n are constants.

For modeling the crack growth rate containing VHCF regime, it is assumed that it has the following form

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C(\Delta K - \Delta K'_{\text{th}})^n, \tag{2}$$

where $\Delta K'_{\text{th}}$ denotes the threshold value of the crack initiation.

For the validation of the supposition that Eq. (2) can describe the crack growth rate containing VHCF regime, Fig. 1 plots the experimental data for a chromium steel at stress ratio $R = 0.05$ in vacuum taken from Ref. 7 with the fitting result by Eq. (2). It is seen that the crack growth rate in the very high numbers of cycles is very well described by the relation of Eq. (2). This indicates that, for a certain given stress ratio, the stress intensify factor range can be used to describe the crack growth rate even when it is much lower than the traditional threshold value of the crack propagation ΔK_{th}. It is noted that, in the early stage of the VHCF process, the crack growth length per cycle is far below Burgers vector, so the crack growth should not be produced cycle by cycle in the early stage of the VHCF process. In other words, it is more reasonable to call it the "equivalent crack growth rate" and the corresponding "equivalent crack length" in the early stage of the VHCF process. Here, we also call it crack growth rate for convenience.

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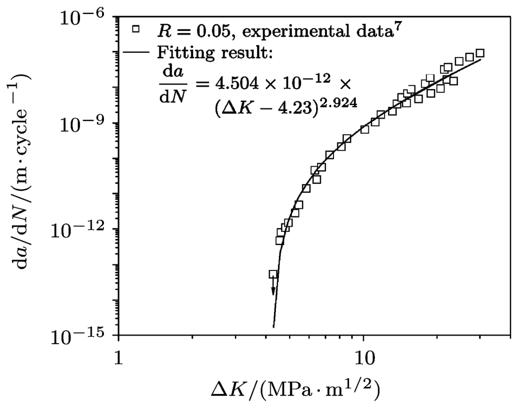

Fig. 1. Experimental data for a chromium steel at stress ratio $R = 0.05$ in vacuum taken from Ref. 7 with the fitting result by Eq. (2).

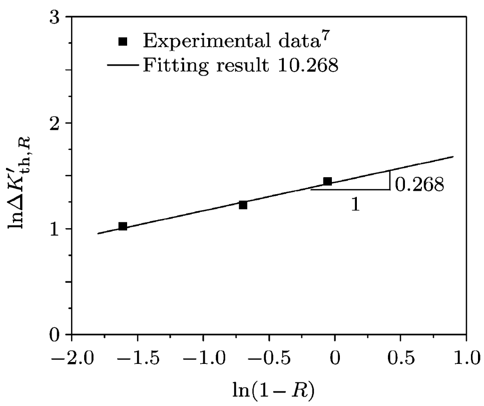

Fig. 2. Values of $\Delta K'_{\mathrm{th},R}$ for a chromium steel in vacuum tested in Ref. 7 versus $(1-R)$ in logarithm scale.

It is considered that the effect of the stress ratio R on the crack growth rate containing VHCF regime can be expressed as

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C[f(R)\Delta K_R - f(R)\Delta K'_{\mathrm{th},R}]^n, \tag{3}$$

where $f(R)$ is a function of the stress ratio R and the subscript R denotes the stress ratio.

Now, see the determination of function $f(R)$. Figure 2 shows the value of $\Delta K'_{\mathrm{th},R}$ for a chromium steel in vacuum tested in Ref. 7 versus $(1-R)$ in logarithm scale. It indicates that the expression of function $f(R)$ can be well approximated by

$$f(R) = (1-R)^\alpha = A/\Delta K'_{\mathrm{th},R},$$
$$\text{i.e.}\quad \Delta K'_{\mathrm{th},R} = (1-R)^{-\alpha}\Delta K'_{\mathrm{th},0}, \tag{4}$$

where A and α are constants.

Equation (4) has the same form as that for the influence of R on the traditional threshold value of the crack propagation ΔK_{th}.[9]

Thus, when considering the stress ratio, the crack growth rate containing the VHCF regime becomes

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C[(1-R)^\alpha \Delta K_R - (1-R)^\alpha \Delta K'_{\mathrm{th},R}]^n, \tag{5}$$

i.e.

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C(\Delta K_{\mathrm{eff}} - \Delta K'_{\mathrm{th},0})^n, \tag{6}$$

where $\Delta K_{\mathrm{eff}} = (1-R)^\alpha \Delta K_R$ denotes the effective stress intensify factor range.

It is seen that, if the effect of the stress ratio R on the crack growth rate containing VHCF regime can be described by Eq. (5), one will obtain the same crack growth rate $\mathrm{d}a/\mathrm{d}N$ for identical value of ΔK_{eff}. On the contrary, if the crack growth rate $\mathrm{d}a/\mathrm{d}N$ is the same for identical value of ΔK_{eff} at different R, Eq. (5) reflects the effect of the stress ratio R on the crack growth rate containing VHCF regime.

Figure 3 plots the crack growth rate data for a chromium steel taken from Ref. 7 to show the effect of the stress ratio R. It is seen that the present model is in very good agreement with the experimental data. This indicates that Eq. (5) can correlate the crack growth rate and the stress ratio containing VHCF regime.

From Eq. (5), the fatigue life from the crack length a_1 to a_2 is obtained as follows

(a) For $n = 1$,

$$N = \frac{2}{C'(\Delta\sigma_{\mathrm{eff}})^n}\left[(\sqrt{a_2} - \sqrt{a_1}) + \sqrt{a'_{\mathrm{th},0}} \ln \frac{\sqrt{a_2} - \sqrt{a'_{\mathrm{th},0}}}{\sqrt{a_1} - \sqrt{a'_{\mathrm{th},0}}}\right], \tag{7}$$

(b) For $n = 2$,

$$N = \frac{2}{C'(\Delta\sigma_{\mathrm{eff}})^n}\left[\frac{\sqrt{a'_{\mathrm{th},0}}\,(\sqrt{a_2} - \sqrt{a_1})}{\left(\sqrt{a_2} - \sqrt{a'_{\mathrm{th},0}}\right)\left(\sqrt{a_1} - \sqrt{a'_{\mathrm{th},0}}\right)} + \ln \frac{\sqrt{a_2} - \sqrt{a'_{\mathrm{th},0}}}{\sqrt{a_1} - \sqrt{a'_{\mathrm{th},0}}}\right], \tag{8}$$

(c) For $n \neq 1$ and $n \neq 2$,

$$N = \frac{2}{C'(\Delta\sigma_{\text{eff}})^n} \frac{\left(\sqrt{a_2} - \sqrt{a'_{\text{th},0}}\right)^{1-n}\left[(1-n)\sqrt{a_2} + \sqrt{a'_{\text{th},0}}\right] - \left(\sqrt{a_1} - \sqrt{a'_{\text{th},0}}\right)^{1-n}\left[(1-n)\sqrt{a_1} + \sqrt{a'_{\text{th},0}}\right]}{(1-n)(2-n)}, \tag{9}$$

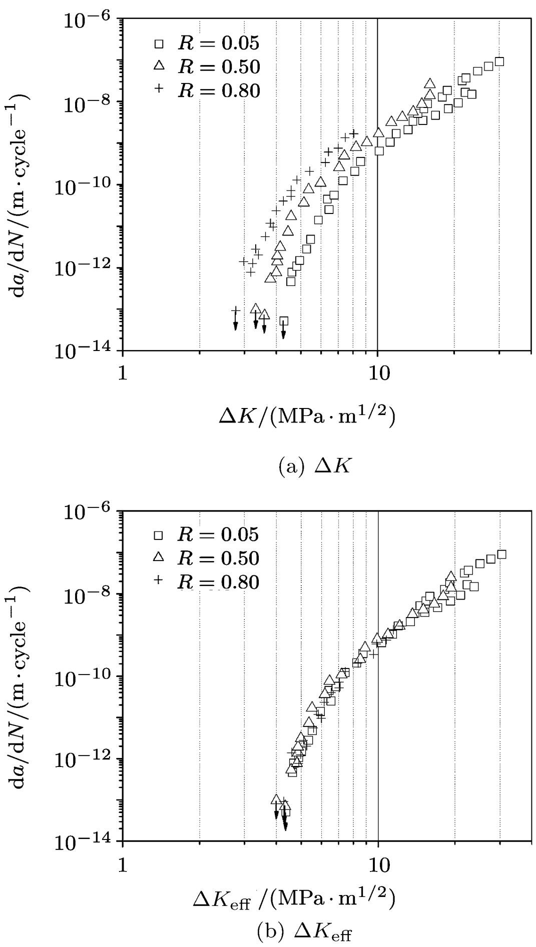

Fig. 3. Crack growth rate data for a chromium steel taken from Ref. 7.

where $\Delta\sigma_{\text{eff}} = \Delta\sigma_R(1-R)^\alpha$ denotes the effective stress range and $a'_{\text{th},0}$ denotes the crack length corresponding to the threshold value of the crack initiation (i.e. the crack length that the crack initiation occurs) at stress ratio $R = 0$.

As known, the crack growth rate increases with the increasing of the crack length. So for the fatigue life in the high cycle and VHCF regimes from an initiation crack length a_1 to failure, the fatigue life is mainly determined by the initiation crack length a_1. In other words, the effect of the variation of the crack length a_2 on the fatigue life can be negligible. Thus, for the same material with the same fatigue life and almost the same initiation crack length a_1, we have

$$\Delta\sigma_{R_2} = \Delta\sigma_{R_1}\left(1 - \frac{\sigma_{\text{avg},R_1}}{\sigma_{\max,R_1}}\right)^\alpha \cdot \left(1 - \frac{\sigma_{\text{avg},R_2}}{\sigma_{\max,R_2}}\right)^{-\alpha} = \Delta\sigma_{R_1}\left(1 - \frac{1+R_1}{2}\right)^\alpha \cdot \left(1 - \frac{1+R_2}{2}\right)^{-\alpha}. \tag{10}$$

Equation (10) indicates that the fatigue strength at stress ratio R_2 can be obtained through the fatigue strength at stress ratio R_1 for the same material with the same fatigue life. Especially, taking $R_1 = -1$, Eq. (10) changes to

$$\sigma_{a,R} = \sigma_{-1}\left(1 - \frac{\sigma_{\text{avg},R}}{\sigma_{\max,R}}\right)^{-\alpha} = \sigma_{-1}\left(\frac{1-R}{2}\right)^{-\alpha}. \tag{11}$$

In the following, Eq. (11) is attempted to study the effect of stress ratio on the fatigue strength in VHCF regime. It is known that the failure of VHCF is usually caused by interior non-metallic inclusions. For interior initiated failure mode, it is very hard to measure the evolvement process of the internal crack. Here, it is considered that the crack growth form is similar to that of the crack specimen[7] and the interior non-metallic inclusions are seen as equivalent crack.[2]

Figure 4 shows the estimated fatigue strengths by Eq. (11) with the experimental ones taken from Ref. 3, which is also compared with the estimated ones by Goodman formula. For the parameter α in Eq. (11), it is calculated by the fatigue strengths at fatigue life near 4×10^6 cycles for stress ratio $R = -1$ and $R = 0$, which has almost the same inclusion size at the fracture origin. It is seen that, for the VHCF regime, the trend of estimated fatigue strength by the present model with the fatigue life is in agreement with that of the experimental data. While the fatigue strength calculated from Goodman formula is a little higher than the experimental one in the trend for the VHCF regime.

In this paper, a model is proposed to study the effect of stress ratio on the crack growth rate containing VHCF regime by using the stress intensify factor range. Then, a formula is derived for estimating the effect of the mean stress on fatigue strength. The results are in agreement with the experimental data in literature.

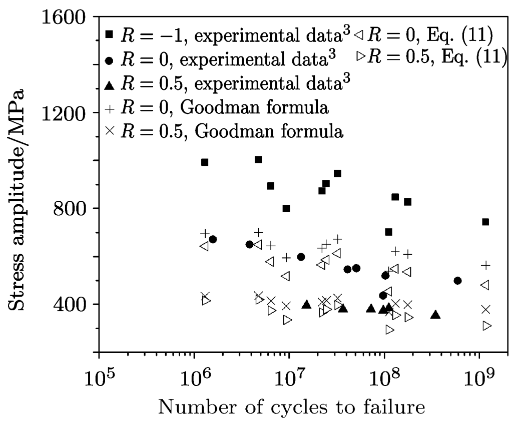

Fig. 4. Comparison of estimated values of fatigue strength by Eq. (11) with the experimental ones for a bearing steel under axial fatigue test taken from Ref. 3 and the estimated ones by Goodman formula.

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