EFFICIENT COEFFICIENT OF RESTITUTION AND LATTICE-BOLTZMANN SIMULATION OF GRAVITATIONAL SETTLING AND REBOUND IN A VISCOUS FLUID

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Summary Two fundamental problems in wall-bounded particle-laden flows are explored: how to correct the *unresolved* hydrodynamic force using the lubrication approximation when a finite grid size is used in the simulation and the particle is very close to the wall, and how to model the *effective* coefficient of restitution to account for the viscous dissipation of the fluid when a finite time step is used in the simulation. A new model for lubrication correction is proposed to account for the variation of the particle velocity during the impact. Based on the lubrication approximation, a semi-empirical model for the effective coefficient of restitution is proposed and integrated into the simulation. The numerical results obtained are in good agreement with the experimental results. Our method, therefore, provides a basis for investigating more complex wall-bounded particle-laden flows.

INTRODUCTION

Particle-laden flows are ubiquitous in nature and engineering applications [1]. A good understanding of the interaction between the particles and the walls is crucial for the performance of many engineering devices, such as the erosion of a pipe due to particle impact [2]. During the impact and rebound processes, there are rapid changes in both particle velocity and the fluid velocity surrounding the particle. The rapid change of fluid velocity leads to a large viscous dissipation which damps the particle velocity during a very short time duration immediately following the rebound [3]. The extent of the damping depends on the Stokes number at impact. Therefore, a model for the effective coefficient of restitution depending on the Stokes number at impact is needed to account for the viscous dissipation due to the fluid medium. We shall address two challenging issues: how to improve the lubrication approximation to account for the *unresolved* hydrodynamic force when a finite grid size is used and the particle is very close to the wall, and how to model the *effective* coefficient of restitution to account for the viscous dissipation in the particle-wall interaction using a finite time step? We will develop and validate our simulation method by comparing our numerical results with available experimental data.

THE COMPUTATIONAL APPROACH

Multiple-relaxation-time lattice-Boltzmann equation

The multiple-relaxation-time lattice-Boltzmann equation (MRT-LBE) [4] is used to simulate the flow,

$$\mathbf{f}\left(\mathbf{x}+\mathbf{e}_i\delta_t, t+\delta_t\right) = \mathbf{f}\left(\mathbf{x}, t\right) - \mathbf{M}^{-1}\cdot\mathbf{S}\cdot\left[\mathbf{m}-\mathbf{m}^{(eq)}\right], \tag{1}$$

where $\mathbf{f}$ is a vector indicating the distributions of 19 lattice particles in the D3Q19 model, $\mathbf{M}$ is a 19×19 orthogonal transformation matrix which converts the distribution function $\mathbf{f}$ from the discrete velocity space into the moment space $\mathbf{m}$, where the collision relaxation is performed. $\mathbf{m}^{(eq)}$ is the equilibrium value of the moment $\mathbf{m}$, δ_t is the time step. The transformations between the particle velocity space and the moment space are

$$\mathbf{m} = \mathbf{M}\cdot\mathbf{f}, \quad \mathbf{f} = \mathbf{M}^{-1}\cdot\mathbf{m}, \quad \mathbf{m}^{(eq)} = \mathbf{M}\cdot\mathbf{f}^{(eq)}. \tag{2}$$

Particle motion is governed by the gravity force and the hydrodynamic force of the fluid. An advantage of LBE is that the hydrodynamic force acting on the solid particle is directly calculated based on the impulses exerted on the lattice fluid particles and the Newton's third law. After obtaining the hydrodynamic force and torque, we update the transverse and rotational velocities and displacements of the particle using the Crank-Nicholson scheme.

Lubrication force correction for small gap distances

When the gap between the solid spherical particle and the wall is less than about 2 lattice units, the lubrication force correction is given as

$$\mathbf{F}^c_{lub} = -6\pi\mu r_p \mathbf{v}_p(h)\left[\frac{r_p}{h} - \frac{v_p(\Delta_0)}{v_p(h)}\frac{r_p}{\Delta_0}\right], \tag{3}$$

where r_p is the radius of the solid particle, $v_p(\Delta_0)$ and $v_p(h)$ are the components of the solid particle velocity perpendicular to the wall, Δ_0 is the critical distance to invoke the lubrication force correction and h is the distance between the sphere and the wall. The variation of the particle velocity during its approach towards the wall is considered in Eq. (3).

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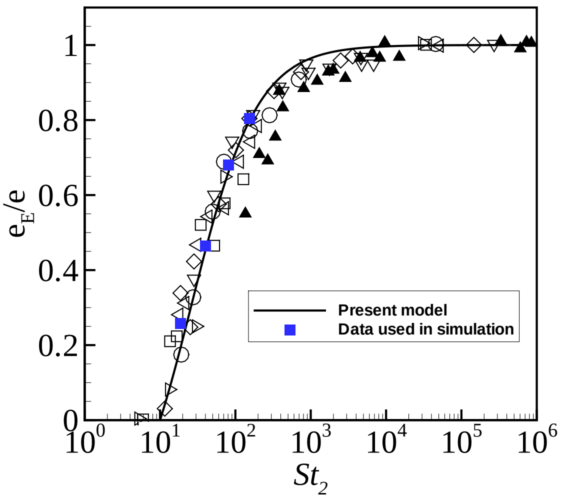
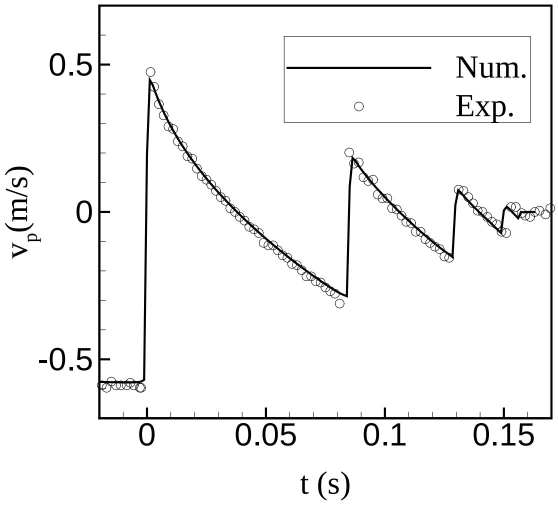


(a) Fig. 1 Variation of the effective coefficient of restitution with the particle Stokes number at impact. Symbols denote experimental data, line is for the model. Solid squares are the data used in current simulation.



(b) Fig. 2 Particle velocity as a function of time during repeated impact and rebound interactions in silicon oil. Lines are for numerical data and symbols for the experimental measurement by Gondret et al. [5].

The effective restitution coefficient for particle rebound

The effective restitution coefficient for particle rebound e_E is proposed as

$$\frac{e_E}{e} = 1 - \frac{18(1+e)}{18 + (8 + St)e}, \tag{4}$$

where e is the physical (dry) coefficient of restitution. The comparison between the model and the experimental data is shown in Fig. 1. We then use Eq. (4) in our simulation.

RESULTS

We shall numerically study the rebound process of a steel sphere in a tank experimentally performed by Gondret et al. [5]. In our simulations, 10 lattice grids are used to resolve the steel sphere of diameter 3 mm. The numerical results are compared with the experimental data in Fig. 2. An excellent agreement between them is demonstrated, showing that the present numerical method is able to capture the rebound process. Since the experimental data on the effective restitution coefficient and Eq. (4) prediction agree well, employing Eq. (4) directly into the simulation would give very similar results. Therefore, Eq. (4) could be used to treat particle-particle and particle-wall collision in a viscous fluid.

CONCLUSIONS

In this study, we have developed a particle-resolved simulation approach for a solid particle impacting on a solid wall in a viscous fluid. A revised formulation of the lubrication correction and a model for the effective coefficient of restitution were proposed. The proposed model provides a better physical interpretation of the rebound process and could be incorporated into both point-particle based approach and the particle-resolved simulation to model solid-particle impact on and rebound from a solid wall. The issues we have studied here represent the key building blocks towards accurate simulations of complex wall-bounded turbulent particle-laden flows. The advances made here can be used to extend the particle-resolved simulation approach to such complex flows, so that both the finite-size effects and the particle-wall interactions on particle dynamics and turbulence modulation can be studied.

Acknowledgments

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