# Modeling of Acoustic Propagation across a Warm-Core Eddy in South China Sea

*Lili Ji; Mian Lin* Laboratory of Environment Mechanics, Institute of Mechanics, Chinese Academy of Sciences Beijing, China

# ABSTRACT

A three dimensional parabolic equation, including the current effect in both x and y axis, has been derived to investigate the effect of mesoscale eddies on acoustic propagation. The sound structure of a warm-core ring in the southwest of South China Sea is presented. Then propagation through the warm-core eddy is discussed. Further, the transmission loss for different cross angles has been examined. It is found that the influence of eddy on acoustic propagation decreases greatly with cross angle. In addition, the position of the eddy relative to the source causes changes in transmission loss of as much as 18dB.

KEY WORDS: Parabolic approximation; warm-core ring; sound propagation; three dimensions.

# INTRODUCTION

In recent years there has been considerable interest in ocean eddies, both from oceanographic and acoustic viewpoints. The primary effects of eddies are in general observed in the upper region of the ocean with diminishing effect in lower regions (Saunders, 1971; Fuglister, 1972). A striking environmental characteristic of eddies is a large distortion of the normally horizontal isotherms (Andrews, 1976). Eddies have been shown to be responsible for isothermal uplifts of 500 m or more, resulting in elevations of SOFAR axes of a few hundred meters (Parker, 1971). Thus, it is important to study the effect of eddies on acoustic propagation.

Several studies of acoustic transmission through ocean eddies have been performed. Numerical acoustical results have been obtained for particular eddy sizes, using both ray theory and the parabolic equation method (Vastano and Owens, 1973; Gemmill and Khedouri, 1974; Nysen and Power, 1978; Baer *et al.*, 1980; Henrick *et al.*, 1980; Itzikowitz *et al.*, 1982). Especially, Robertson et al. have derived a family of parabolic equations to study both current and sound-speed effects of cyclonic eddies of arbitrary size and strength on acoustic propagation (Robertson *et al.*, 1985). However, these parabolic equations are restricted to two dimensions. Eddies, as well as many other oceanic features, are three-dimensional phenomena. The azimuthal variations may significantly influence the propagation, especially when propagation long ranges (Baer *et al.*, 1981; Mellberg *et al.*, 1991). Thus, to predict the effects correctly, one must derive parabolic equations in three dimensions.

An anticyclonic ring detached from Kuroshio in the South China Sea is reported (Li *et al.*, 1998; Wu *et al.*, 2001). Several studies of acoustic transmission through the cross section of the warm-core ring center have been performed. However, previous studies of propagation through the warm-core eddy in the South China Sea have been restricted to two dimensions (Jian *et al.*, 2009). In this paper, the parabolic equations derived by Robertson et al. are generalized to three dimension including the current speed in both x and y axis. Then the three-dimensional effects of the warm-core eddy in both vertical and horizontal planes are examined. Further, the variations of transmission loss as the eddy progressing through the region between an acoustic source and receiver have been examined.

# THE 3D PARABOLIC APPROXIMATION FOR INHOMOGENEOUS MOVING WATER

The general governing equations for the motion of an adiabatic and nondissipative fluid are:

$$\frac{\partial \hat{\rho}_{w}}{\partial \hat{t}} + \hat{\nabla} \cdot (\hat{\rho}_{w} \hat{\vec{v}}) = 0$$
(1a)

$$\hat{\rho}_{w} \frac{D\vec{v}}{D\hat{t}} = -\hat{\nabla}\hat{P}_{w}$$
(1b)

$$\hat{c}_{w}^{2} \frac{D\hat{\rho}_{w}}{D\hat{t}} = \frac{D\hat{P}_{w}}{D\hat{t}}$$
(1c)

where  $\hat{\nabla}$  is the gradient operator,  $D / D\hat{t} = (\partial / \partial \hat{t} + \hat{\vec{v}} \cdot \hat{\nabla})$ ,  $\hat{\rho}_w$  is fluid density,  $\hat{\vec{v}}$  is the fluid velocity vector,  $\hat{P}_w$  is pressure in water,  $\hat{c}_w$  is sound speed in water.

The quantities  $\hat{\rho}_w$ ,  $\hat{P}_w$ ,  $\hat{\vec{v}}$  in Eq. 1 are regarded as composed of ambient components, describing the state of the medium in absence of an acoustic disturbance, plus acoustic perturbation components caused by eddies. The former are indicated by zero subscript and the latter by a unit subscript, so that

$$\hat{p}_{w} = \hat{p}_{w0} + \hat{p}_{w1}, \hat{\rho}_{w} = \hat{\rho}_{w0} + \hat{\rho}_{w1}, \hat{v}_{w} = \hat{v}_{w0} + \hat{v}_{w1}$$
(2)

We assumed that the fluid density and the velocities  $\hat{c}_w$  and  $\overleftarrow{v}_w$  of sound and fluid, respectively, are time-independent but depend arbitrary on the vertical coordinate  $\hat{z}$  and weakly on the horizontal coordinates  $\hat{x}$ .

To nondimensionalize Eqs.1~2, we let (Robertson et al., 1985):

$$\hat{x} = xL, \hat{z} = zL, \hat{t} = tT$$

$$\hat{y} = (\hat{z}) - \hat{U}\hat{y}(z) - \hat{U}u(z)\hat{i}$$
(3)
(3)

$$\hat{\vec{v}}_{w0}(\hat{z}) = \hat{U}_0 \hat{\vec{v}}(z) = \hat{U}_0 u_0(z) \hat{\vec{i}},$$
(4a)
$$\hat{\vec{v}}_w(\hat{z}, \hat{z}, \hat{t}) = \delta \hat{z} \hat{\vec{v}}(z, z, t)$$
(4b)

$$\hat{\vec{v}}_{w1}(\hat{x},\hat{z},\hat{t}) = \delta \hat{c}_0 \vec{v}_1(x,z,t),$$
(4b)
$$\hat{\rho}_{w0}(\hat{x},\hat{z}) = \hat{\rho}_{00} \rho_0(x,z),$$
(4c)

$$\hat{\rho}_{w1}(\hat{x},\hat{z},\hat{t}) = \delta \hat{\rho}_{00} \rho_1(x,z,t), \tag{4d}$$

$$\hat{p}_{0}(\hat{x},\hat{z}) = \hat{\rho}_{00}\hat{U}_{0}^{2}p_{0}(x,z)$$
(4e)

$$\hat{p}_1(\hat{x},\hat{z},\hat{t}) = \delta \hat{\rho}_{00} \hat{c}_0 \hat{L} \hat{T}^{-1} p_1(x,z,t)$$
(4f)

where  $\hat{L}$  and  $\hat{T}$  are characteristic length and time scales to be specified later,  $\hat{\rho}_{00}$ ,  $\hat{U}_0$  and  $\hat{c}_0$  are a reference density, current speed and sound speed, respectively.  $u_0, \vec{v}_1, p_0, p_1, \rho_0$  and  $\rho_1$  are dimensionless quantities of order of magnitude unity,  $\vec{i}$  is the unit vector along the *x* axis, and  $\delta$  which represents the acoustic perturbation is a small dimensionless number.

We substitute Eqs. 2~4 into Eq. 1a to obtain  

$$\delta \hat{L} \hat{c}_0^{-1} T^{-1} \frac{\partial \rho_1}{\partial t} + \qquad (5)$$

$$\nabla \cdot \left( M \rho_0 \vec{v}_0 + \delta \rho_0 \vec{v}_1 + M \delta \rho_1 \vec{v}_0 + \delta^2 \rho_1 \vec{v}_1 \right) = 0$$

where *M* denotes the Mach number  $U_0 / \hat{c}_0$ .

We note that in the absence of any acoustic perturbation,  $\delta = 0$  and Eq.5 is identically zero since  $\hat{\vec{v}}_0$  depends only on z and has no vertical component. Keeping terms of  $O(\delta)$ , we obtain

$$\mu \frac{\partial \rho_1}{\partial t} + \nabla \cdot \left( \rho_0 \vec{v}_1 + M \rho_1 \vec{v}_0 \right) = 0$$
(6)
where
$$\mu = \hat{L} \hat{c}_0^{-1} \hat{T}^{-1}$$
(7)

Since we anticipate that the parameter  $\mu$  is order unity for acoustic waves, Eq.6 represents a scaled conservation of mass equation.

Substituting Eqs.2~4 into Eq.1b and algebraically simplifying, we obtain

$$(\rho_0 + \delta \rho_1) [\mu \delta \frac{\partial \dot{v}_1}{\partial t} + M^2 (\vec{v}_0 \cdot \nabla) \vec{v}_0 + M \delta (\vec{v}_0 \cdot \nabla) \vec{v}_1 + M \delta (\vec{v}_1 \cdot \nabla) \vec{v}_0 + \delta^2 (\vec{v}_1 \cdot \nabla) \vec{v}_1] = -\nabla (M^2 p_0 + \mu \delta p_1)$$

$$(8)$$

Further, in the absence of any acoustic perturbation  $\delta = 0$ , and the left side of Eq. 8 is identically zero since  $\vec{v}_0$  depends only on *z* and has no vertical component. Eliminating ambient terms from the linearized version of Eq. 8, we find

$$\mu \frac{\partial \vec{v}_1}{\partial t} + M \left( \vec{v}_0 \cdot \nabla \right) \vec{v}_1 + M \left( \vec{v}_1 \cdot \nabla \right) \vec{v}_0 = -\frac{\mu}{\rho_0} \nabla p_1 \tag{9}$$

Finally, we scale Eq. 1c in the same manner, using  $n = \hat{c}_0 / \hat{c}$  as the index of refraction. With the scaled variables already defined, Eq. 1c becomes

$$n^{-2} \left( \mu \delta \frac{\partial \rho_1}{\partial t} + \delta \vec{v}_1 \cdot \nabla \rho_0 + \delta M \vec{v}_0 \cdot \nabla \rho_1 + \delta^2 \vec{v}_1 \cdot \nabla \rho_1 \right)$$
  
=  $M^3 \vec{v}_0 \cdot \nabla p_0 + \mu M \delta \vec{v}_0 \cdot \nabla p_1 + M^2 \delta \vec{v}_1 \cdot \nabla p_0$  (10)  
 $+ \mu \delta^2 \vec{v}_1 \cdot \nabla p_1 + \mu^2 \delta \frac{\partial p_1}{\partial t}$ 

where  $\delta = 0$ , the reduced equation is the balance condition to be satisfied identically by the ambient terms. Under the condition  $\delta = O(M^2)$ , where, as usual,  $M \ll 1$ , the appropriate simplification of Eq. 1c is

$$n^{-2} \left( \mu \frac{\partial \rho_1}{\partial t} + \vec{v}_1 \cdot \nabla \rho_0 + M \vec{v}_0 \cdot \nabla \rho_1 \right) =$$

$$\mu M \vec{v}_0 \cdot \nabla \rho_1 + \mu^2 \frac{\partial \rho_1}{\partial t}$$
(11)

That this condition on  $\delta$  is reasonable follows from Eq. 4b, which implies that

$$\delta = O\left(\left|\vec{v}_1\right| / \hat{c}_0\right) \tag{12}$$

Since *M* in the ocean is never bigger than  $10^{-3}$ , the condition  $\delta = O(M^2)$  and Eq. 12 imply that the magnitude of the velocity induced by the acoustic disturbance is no more than about  $10^{-3}ms^{-1}$ . This is a reasonable and conservation means the acoustically induced disturbance is smaller than the ambient flow by a scaling factor of order Mach number.

Let  

$$p_1 = p(x, y, z)e^{-i\omega t}$$
(13)

where  $\omega = \hat{\omega}\hat{T}$  is scaled frequency. By keeping terms of O(M), we obtain:

$$i\omega p_{xx} + i\omega p_{yy} + i\omega \rho_0 \left(\frac{1}{\rho_0} p_z\right)_z + 2M u_{0x} p_{xx} + 2M v_{0y} p_{yy} + 2M u_{0z} p_{xz} + 2M v_{0z} p_{yz} - 2\omega^2 M n^2 \left(u_0 p_x + v_0 p_y\right) + i\omega^3 n^3 p = 0$$
(14)

Picking  $\hat{\omega}^{-1} = \hat{T}$  ensures that time derivatives are O(1), since the scaled frequency is  $\omega = 1$ . The choice  $\hat{L} = \hat{c}_0 / \hat{\omega}$  then gives us the inverse wavenumber  $\hat{k}_0$  as our length scale. Consequently, we obtain  $\mu = 1$ .

In order to generate a parabolic approximation to Eq. 14, we convert to cylindrical coordinates:

$$i\omega \left[ p_{rr} + \frac{1}{r} p_r + \rho \left( \frac{1}{\rho} p_z \right)_z + \frac{1}{r^2} p_{\theta\theta} + \omega^2 n^2 p \right] -2M \left( u_x \cos\theta + \omega^2 n^2 u \cos\theta + \omega^2 n^2 v \sin\theta \right) p_r +2M \left( u_z \cos\theta - v_z \sin\theta \right) p_{zr} - 4M u_x \sin 2\theta p_{r\theta}$$
(15)  
$$-2M u_x \frac{\cos 2\theta}{r^2} p_{\theta\theta} - 2\frac{M}{r} \left( u_z \sin\theta - v_z \cos\theta \right) p_{z\theta} +2M \omega^2 n^2 \frac{1}{r} \left( u \sin\theta - v \cos\theta \right) p_{\theta} + 4M u_x \sin 2\theta p_{\theta} = 0$$

Let (Robertson et al., 1985)

$$p = \psi(x, y, z)r^{\frac{1}{2}}e^{ir}$$

$$r = \delta^{-1}r'$$

$$z = \delta^{-\frac{1}{2}}z'$$

$$\theta = \delta^{\frac{1}{2}}\theta'$$
(16)

Then we have:

$$i\omega \left[ \delta^{2} \psi_{r'r'} + 2\delta i\psi_{r'} + \left( n^{2} - 1 + \frac{\delta^{2}}{4r'^{2}} \right) \psi + \delta \rho \left( \frac{1}{\rho} \psi_{z'} \right)_{z'} + \delta \frac{1}{r^{2}} \psi_{\theta'\theta'} \right] -2M \left( \delta u_{r} \cos \theta \cos 2\theta + \omega^{2} n^{2} u \cos \theta + \omega^{2} n^{2} v \sin \theta \right) \left[ \delta \psi_{r'} - \delta \frac{1}{2r'} \psi + i \psi \right] +2M \delta^{\frac{1}{2}} \left( u_{z} \cos \theta - v_{z'} \sin \theta \right) \left[ \delta^{\frac{2}{3}} \psi_{r'z'} - \delta^{\frac{2}{3}} \frac{1}{2r'} \psi_{r'} + \delta^{\frac{1}{2}} i \psi_{z'} \right] -4M \delta u_{r'} \cos \theta \sin 2\theta \left[ \delta^{\frac{1}{2}} \psi_{r'\theta'} - \delta^{\frac{1}{2}} \frac{1}{2r'} \psi_{\theta'} + \delta^{-\frac{1}{2}} i \psi_{\theta'} \right] -2\delta^{\frac{3}{2}} \frac{M}{r'} \left( u_{z'} \sin \theta - v_{z'} \cos \theta \right) \psi_{z'\theta'} + 2\delta^{\frac{1}{2}} M \omega^{2} n^{2} \frac{1}{r'} \left( u \sin \theta - v \cos \theta \right) \psi_{\theta'} -2\delta^{2} M u_{r'} \frac{\cos \theta \cos 2\theta}{\left( r' \right)^{2}} \psi_{\theta'\theta'} + 4\delta^{\frac{1}{2}} M u_{r'} \cos \theta \sin 2\theta \psi_{\theta'} + 2\delta M u_{r'} \cos \theta \cos 2\theta \left( \delta^{2} \psi_{r'r'} - \delta^{2} \frac{1}{r'} \psi_{r'} + 2i \delta \psi_{r'} + \delta^{2} \frac{3}{4(r')^{2}} \psi - \delta i \frac{1}{r'} \psi - \psi \right] = 0$$

$$(17)$$

Neglecting small terms, we obtain

$$2i\hat{k}_{0}\hat{\psi}_{\bar{r}} + \hat{\rho}\left(\frac{1}{\hat{\rho}}\hat{\psi}_{\bar{z}}\right)_{\bar{z}} + \hat{k}_{0}^{2}\left(n^{2} - 1\right)\hat{\psi} + \frac{1}{\hat{r}^{2}}\hat{\psi}_{\theta\theta} - 2\frac{\hat{k}_{0}^{2}}{\hat{c}_{0}}\left(\hat{u}\cos\theta + \hat{v}\sin\theta\right)\hat{\psi} = 0$$
(18)

By introducing an effective sound speed  $\tilde{c}$ , the above equation can be converted into a form more suitable for analysis. The result is:

$$2i\hat{k}_{0}\hat{\psi}_{\bar{r}} + \hat{\rho}\left(\frac{1}{\hat{\rho}}\hat{\psi}_{\bar{z}}\right)_{\bar{z}} + \hat{k}_{0}^{2}\left(\hat{n}^{2} - 1\right)\hat{\psi} + \frac{1}{\hat{r}^{2}}\hat{\psi}_{\theta\theta} = 0$$
(19)  
where  $\tilde{n} = \frac{\hat{c}_{0}}{\tilde{c}}, \tilde{c} \approx \hat{c} + \hat{u}\cos\theta + \hat{v}\sin\theta$ 

# PROPAGATION THROUGH THE 3-D WARM-CORE RING IN THE SOUTH CHINA SEA

An analytical model of the sound-speed structure of an ocean region containing an eddy has been derived by Henrick *et al.* (Watson *et al.*, 1976; Henrick *et al.*, 1977). Recently the analytical eddy model has been generalized to include the azimuth angle variation by Jian *et al.* In the following we will use the model to determine the sound speed in terms of a small set of parameter values of the warm-core eddy in the South China Sea.

An anticyclonic ring in the South China Sea has been reported by Li (Li *et al.*, 1998). Its position is located at the north latitude 21 degree, the depth of ocean is about 2200m, the effective depth of the eddy is about 1600m and the effective radius is about 150km in horizontal range. In addition, according to the situ observed data, maximum surface current speed  $U_0$ , surface sound speed  $c_0$  and density  $\rho_0$  are taken to be 1.0m/s,

1538m/s and 1021.5 kg /  $m^3$ , respectively. Using the above eddy model, we obtain the sound speed and current structure in the cross section through the center of the warm-core eddy (Fig. 1). The eddy has a

sound-speed minimum at its center and the minimum is 1483m/s. The SOFAR axis, or depth of minimum sound speed, increases in depth by approximately 200 m from the eddy's edge to its center.



Fig. 1 Sound speed (left) and current structure contours (right) through the center of the warm-core eddy

Let us consider a 25-Hz, cw source placed at the depth of 800m, 200km to the left of the eddy center. These parameters were picked to highlight the influence of the eddy. In Fig. 2, we schematically illustrate the positions of the source and the eddy in the horizontal plane, where  $\theta$  is the cross angle. In the following, we will investigate three-dimensional characteristics of acoustic propagation through the warm-core eddy.



Fig. 2 Eddy and source configuration in the horizontal plane

### Effects of the eddy on acoustic propagation at the zero cross angle

First, we consider the effect of the eddy on acoustic propagation at zero-degree cross angle over a range of 400km. Fig. 3 depicts contour map of transmission loss both in the region containing the warm-core eddy and in the same region where the perturbation of the eddy is absent. Although the two plots are superficially quite similar, there are major qualitative and quantitative differences. A striking feature is that the acoustic energy transform from the deeper channel induced by the warm-core eddy to the one in no-eddy case. This is because the sound channel axis bends downward from the sound speed structures in Fig. 1. Also, we can see that the transmission loss in the situation for having eddy is larger than that for no-eddy case.





Fig. 3 Contour map of transmission loss through the central section of the eddy (top) and transmission loss in the region containing no eddies (below)

#### Effects of the eddy on acoustic propagation for different $\theta$

In this section we will investigate the characteristic of acoustic propagation at different cross angle.

Fig.4 shows sound speed contours for different  $\theta$ . It can be found that the perturbation of sound speed caused by the eddy decreases with  $\theta$ . This phenomenon is obvious especially at the center of the eddy. For instance, the SOFAR axis rises by twenty meters from  $\theta = 2^{\circ}$  to  $\theta = 6^{\circ}$ .





Fig. 4 Sound speed contours at the cross angle of  $2^{\circ}$  (top),  $6^{\circ}$  (mid) and  $10^{\circ}$  (below)

Fig. 5 plots the contour maps of transmission loss with the warm-core eddy for different  $\theta$ . To observe the effect of the warm-core eddy more clearly, we extract the transmission loss versus range curves at the depth of 800m for both containing eddies and no-eddy cases (Fig. 6). The major difference noted in comparing the three pictures in Fig. 5 is that the transmission loss increases with  $\theta$ . Fig. 6 shows that the difference of transmission loss between with eddy and without eddy decreases with  $\theta$ . This result is obvious because the perturbation of sound speed caused by the eddy becomes smaller at larger cross angle as shown in Fig. 4.





Fig. 5 Contour maps of transmission loss with the warm-core eddy at the cross angle of  $2^{\circ}$  (top),  $6^{\circ}$  (mid) and  $10^{\circ}$  (below)



Fig. 6 the transmission loss versus range curves for both containing eddies and no-eddy cases at the cross angle of  $2^{\circ}$  (top),  $6^{\circ}$  (mid) and  $10^{\circ}$  (below)

In the transmission loss versus range curve the difference of transmission loss between with eddy and without eddy vary sharply with range. Thus it is difficult to measure the change of the effect of the eddy among different cross angle. Then we compute the variance of TL (transmission loss) difference for each picture of Fig. 6. Fig. 7 shows the variance of TL (transmission loss) difference versus cross angle curve. It reveals that the effect of eddy on acoustic propagation decreases faster at small cross angles than at large cross angles.



Fig. 7 The variance of TL (transmission loss) difference versus cross angle curve

#### Transmission through a moving eddy

In this section we will study the effect of the warm-core eddy on acoustic propagation as the eddy progressing through the region between an acoustic source and receiver. As show in Fig. 8, the origin of a coordinate system is fixed on the ocean surface, and a receiver is located at the range s to the right of the source. d/2=150km is the radius of the eddy. The initial effects of the eddy are felt when its center is at a range c, given by c=-d/2=-150km. At least part of the eddy is between source and receiver and thus influences the acoustic field, until c=s+d/2. Here the source-receiver separation is taken as 200km, thus c=s+d/2=350km.

Fig. 9 shows the variations in transmission loss as a function of eddy position for different depths of receiver. Particularly, when the receiver is at a depth of 800m which is the same depth as the source, the curve is approximately symmetric. This result is reasonable since the eddy is symmetric around its center. Also, we can obtain that maximum variations surpassing 15dB. It is necessary to point out that the warm-core eddy discussed in this paper is not very strong and for a stronger eddy the variations would be more rapid.



Fig. 8 Sketch of eddy traversing the source-receiver plane



Fig. 9 Transmission loss for an ocean region containing an eddy travelling between a sound source and a receiver for receiver depth of 50m (top), 800m (mid) and 2000m (below)

## DISCUSSION AND CONCLUSIONS

The purpose of this paper is to study the 3D effect of the eddy on acoustic propagation To highlights the inhomogeneous moving water effects, we derive an appropriate partial differential equation in three dimensions, including the current speed in both x and y axis. Based on numerical calculations, we finally obtain that:

1. Calculation of transmission Loss at zero-degree cross angle has

proven that there are more transmission loss than that of the no-eddy case. The acoustic energy leaks into the sound channel induced by the eddy.

2. The differences of transmission loss between having eddy and noeddy case decrease with the cross angle. Specially, at small cross angles the influence of eddy decreases more sharply.

3. As the position of the eddy varies relative to the source receiver configuration, the changes of transmission loss can reach 18dB.

In sum, the eddy plays an important role in acoustic propagation.

# ACKNOWLEDGEMENTS

The financial support from NSFC (Grant No. 41074097) and the "973" programs (No. 2009CB219405) are gratefully acknowledge.

# REFERENCES

- Andrews JC, Scully-Power PD (1976). "The structure of an East Australian currentanticyclonic eddy," J Phys Oceanogr, Vol 6, pp756–65.
- Baer, RN (1980). "Calculations of Sound-Propagation through an Eddy," *J Acoust Soc Am*, Vol 67, pp 1180~1185.
- Baer RN (1981). "Propagation through a three-dimensional eddy including effects on an array". J Acoust Soc Am, Vol 69, No 1, pp 70–75.
- Fuglister FC (1972). "Cyclonic rings formed by the Gulf Stream, 1965– 1966". In: Studies in physical oceanography, Vol 1, pp 137–68.
- Gemmill W, Khedouri E (1974). "A note on sound ray tracing through a Gulf Stream Eddy in the Sargosso Sea". US Naval Oceanogr Office, pp 21–74.
- Henrick, RF, Jacobson, MJ, Siegmann, WL (1980). "General Effects of Currents and Sound-Speed Variations on Short-Range Acoustic

Transmission in Cyclonic Eddies," *Journal of the Acoustical Society of America*, Vol 67, pp 121-134.

- Henrick, RF, Siegmann, WL, Jacobson, MJ (1977). "General Analysis of Ocean Eddy Effects for Sound-Transmission Applications," *Journal of the Acoustical Society of America*, Vol 62, pp 860-870.
- Itzikowitz S, Jacobson MJ, Siegmann WL (1982). "Short-range acoustic transmissions through cyclonic eddies between a submerged source and receiver". J Acoust Soc Am, Vol 71, pp1131–44.
- Jian YJ, Zhang J, Liu QS, Wang YF (2009). "Effect of mesoscale eddies on underwater sound propagation". *Applied Acoustics*, Vol70, pp 432-440.
- Li L, Nowlin JWD, Su JL (1998). "Anticyclonic rings from the Kuroshio in the South China Sea". Deep-Sea Res, Vol 45, pp1469– 1482.
- Mellberg LE, Robinson AR, Botseas G (1991). "Azimuthal variation of low-frequency acoustic propagation through Gulf Stream eddies". *Journal Acoust. Soc. Am.* Vol 89, No 5, pp2157-2167.
- Nysen PA, Power PS (1978). "Sound propagation through an East Australian Current eddy". *Journal Acoust. Soc. Am*, Vol 63, No 5, pp1381-1388.
- Parker CE (1971). "Gulf stream rings in the Sargasso Sea". *Deep-Sea Res*, Vol 18, pp981–93.
- Robertson JS, Siegmann WL, Jacobson MJ (1985). "Current and current shear effects in the parabolic approximation for underwater sound channels". *Journal Acoust. Soc. Am*, Vol 77, No 5, pp1768 1780.
- Saunders PM (1971). "Anticyclonic eddies formed from shoreward meanders of the Gulf Stream". *Deep-Sea Res*, Vol 18, pp1207–19.
- Vastano C, Owens GE (1973). "On the acoustic characteristics of Gulf Stream cyclonic rings". *J Phys Oceanogr*, Vol 3, pp470–478.
- Watson JG, Siegmann WL, Jacobson MJ (1976). "Deep-ocean dynamics for environmental acoustics models". J Acoust Soc Am, Vol 60, No 3, pp55–64.
- Wu PM, Guo XG, Wu RS (2001). "Analyses of sound velocity field in southwest sea area off Hengchun of Taiwan". J Oceanogr Taiwan Strait, Vol 20, No 3, pp279-286(in Chinese).