

## The Evaluation and Numerical Simulation of Uniformity of Surface Roughness

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**Abstract**—To study the uniformity of surface roughness, the height amplitudes  $SRa$  based on area-direction character is defined in the paper. And on the foundation of this parameter, assessed uniformity indexes of surface roughness are established. The result of the numerical simulation of three special curved surfaces indicates that  $SRa$  can describe preferably the quantitative relationship among the height amplitude, assessed area and assessed direction. The assessed area-uniformity indexes, assessed direction-uniformity indexes and assessed total-uniformity indexes can be used to assess the area-uniformity, direction-uniformity and total-uniformity of height amplitude.

**Keywords**-Surface Roughness; Area-direction Character; Evaluation; Numerical Simulation; Uniformity

## I. INTRODUCTION

$Ra$  is the 2D assessing parameter used frequently. Actual engineering surface profile has a close relationship with assessed area and assessed direction. The experimental result show that the  $Ra$  measured from different area, or from different direction in same area is often different. It has great significance to study these differences.

Presently, there are mainly 5 types of assessing method of the surface roughness. They are 2D parameter assessing method, 3D parameter assessing method, fractal assessing method, graphics assessing method and image assessing method<sup>[1]</sup>. In image assessing method, digital image such as color image, gray image or black-and-white image(two value image) are used to describe 3D curved surface<sup>[2-4]</sup>. The color value of a pixel presents the average height value of a small area. The position of the pixel presents 2D coordinate of the small area. The image provides quantitative 3D value of the surface. All kinds of parameters can be calculated in image processing method.

In essence, the normal engineering has a 3D tiny structure. So 2D profile cannot comprehensively and accurately describe the surface of the parts.

$Sa$  is the 3D assessing parameter used frequently. The different direction and different area are not considered when determining datum plane, so  $Sa$  cannot describe the relation-ship among the surface roughness, assessed area and assessed direction.

The height amplitude  $SRa$  based on area-direction character is defined in the paper. And on the foundation of this parameter, assessed uniformity indexes is established. The result of the numerical simulation of three special curved surfaces indicates that the assessed uniformity

indexes can be used to assess the uniformity of surface roughness.

## II. AREA-DIRECTIONAL SURFACE ARITHMETIC MEAN DEVIATION AND ASSESSED UNIFORMITY INDEXES

### A. Area-directional Surface Arithmetic Mean Deviation

$Sra$  is area-directional surface arithmetic mean deviation. General 3D assessed parameter  $Sa$  cannot reflect and assess directional character, because there is only one benchmark, that is , datum plane. Statistical method used in this paper is to average directional profile family parameter twice. The procedure of the method is as following.

First, calculate each datum line of n profiles in same direction in assessed area, and calculate  $Ra$ .

Second, calculate directional assessed parameter  $SRa$  in this assessed area by averaging  $Ra$ .

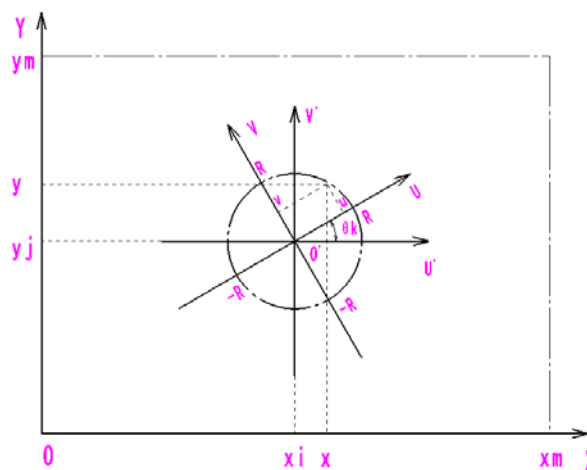


Fig. 1 Assessed coordinate system

Figure 1 is assessed coordinate system. OXYZ is base coordinate system(Z axis is vertical paper outward). Rectangle  $x_m \times y_m$  is assessed surface. Height amplitude coordinate  $z(x, y)$  on the surface is known.  $(x_i, y_j)$  is assessed area center coordinate.  $\theta_k$  is assessed angle.  $O'U'V'W'$  is assessed position coordinates( $W'$  axis is vertical paper outward).  $O'UVW$  is assessed direction coordinates( $W$  axis is vertical paper outward) on which  $w(u, v)$  is height amplitude coordinate. The assessed area is a circle which center is  $O'$ , and radius is  $R$ .  $(u, v)$  and  $(x, y)$  is the same point. There is a relationship as following.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \cos \theta_k & -\sin \theta_k \\ \sin \theta_k & \cos \theta_k \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} + \begin{bmatrix} x_i \\ y_j \end{bmatrix}$$

$$w(u, v) = z(x, y)$$

Suppose

$$T(u, v) = w(u, v) - \frac{1}{f2(v) - f1(v)} \int_{f1(v)}^{f2(v)} w(u, v) du$$

The definition of area-directional surface arithmetic mean deviation:

$$\begin{aligned} SRA(l) &= SRA(x_i, y_j, \theta_k) \\ &= \frac{1}{2R} \int_{-R}^R \left[ \frac{1}{f2(v) - f1(v)} \int_{f1(v)}^{f2(v)} |T(u, v)| du \right] dv \end{aligned}$$

In these formula,  $i=1, 2, \dots, r$  is the number of horizontal areas.

$j = 1, 2, \dots, s$  is the number of vertical areas.

$k = 1, 2, \dots, t$  is the number of directions.

$l = i + r \times j + r \times s \times k$  is the number of areal directions.

$l = k + t \times i + t \times r \times j$  is the number of directional areas.

$$f1(v) = -\sqrt{R^2 - v^2}, f2(v) = \sqrt{R^2 - v^2}.$$

#### B. Assessed Area-uniformity Indexes.

Assessed area-uniformity indexes are used to describe the uniformity of  $SRA$  in each assessed area in special assessed direction. The definitions are:

$$SRA_{amean}(m) = \frac{1}{r \times s} \sum_{i=1}^r \sum_{j=1}^s SRA(x_i, y_j, \theta_k)$$

$$\begin{aligned} SRA_{astd}(m) &= \sqrt{\frac{1}{r \times s} \sum_{i=1}^r \sum_{j=1}^s SRA(x_i, y_j, \theta_k)^2 - SRA_{amean}(m)^2} \\ SRA_{acv}(m) &= \frac{SRA_{astd}(m)}{SRA_{amean}(m)} \end{aligned}$$

In these formula,  $i=1, 2, \dots, r$  is the number of horizontal areas.

$j = 1, 2, \dots, s$  is the number of vertical areas.

$k = 1, 2, \dots, t$  is the number of directions.

$SRA_{amean}(m)$  is the mean deviation of areal surface arithmetic mean deviation in assessed direction.

$SRA_{astd}(m)$  is the STD of areal surface arithmetic mean deviation in assessed direction.

$SRA_{acv}(m)$  is the CV of areal surface arithmetic mean deviation in assessed direction.

Coefficient of variation is a relative value. The smaller the value, the better the uniformity of  $SRA$  in each assessed area in assessed direction.

#### C. Assessed Direction-uniformity Indexes.

Assessed direction-uniformity indexes are used to describe the uniformity of  $SRA$  in each assessed direction in special assessed area. The definitions are:

$$SRA_{dmean}(n) = \frac{1}{t} \sum_{k=1}^t SRA(x_i, y_j, \theta_k)$$

$$SRA_{dstd}(n) = \sqrt{\frac{1}{t} \sum_{k=1}^t SRA(x_i, y_j, \theta_k)^2 - SRA_{dmean}(n)^2}$$

$$SRA_{dcv}(n) = \frac{SRA_{dstd}(n)}{SRA_{dmean}(n)}$$

In these formula,  $i=1, 2, \dots, r$  is the number of horizontal areas.

$j = 1, 2, \dots, s$  is the number of vertical areas.

$k = 1, 2, \dots, t$  is the number of directions.

$n = i + r \times j$  is the number of areas.

$SRA_{dmean}(n)$  is the mean deviation of directional surface arithmetic mean deviation in assessed area.

$SRA_{dstd}(n)$  is the STD of directional surface arithmetic mean deviation in assessed area.

$SRA_{dcv}(n)$  is the CV of directional surface arithmetic mean deviation in assessed area.

Coefficient of variation is a relative value. The smaller the value, the better the uniformity of  $SRA$  in each assessed area in assessed area.

#### D. Assessed Total-uniformity Indexes.

Assessed total-uniformity indexes are used to describe the whole uniformity of  $SRA$ . The definitions are:

$$SRA_{mean} = \frac{1}{r \times s \times t} \sum_{i=1}^r \sum_{j=1}^s \sum_{k=1}^t SRA(x_i, y_j, \theta_k)$$

$$\begin{aligned} SRA_{std} &= \sqrt{\frac{1}{r \times s \times t} \sum_{i=1}^r \sum_{j=1}^s \sum_{k=1}^t SRA(x_i, y_j, \theta_k)^2 - SRA_{mean}^2} \\ SRA_{cv} &= \frac{SRA_{std}}{SRA_{mean}} \end{aligned}$$

$$SRA_{acvm} = \frac{1}{t} \sum_{m=1}^t SRA_{acv}(m)$$

$$SRA_{dcvm} = \frac{1}{r \times s} \sum_{n=1}^{r \times s} SRA_{dcv}(n)$$

In these formula,  $i=1, 2, \dots, r$  is the number of horizontal areas.

$j = 1, 2, \dots, s$  is the number of vertical areas.

$k = m = 1, 2, \dots, t$  is the number of directions.

$n = i + r \times j$  is the number of areas.

$SRA_{mean}$  is the mean deviation of total surface arithmetic mean deviation.

$SRA_{std}$  is the STD of total surface arithmetic mean deviation.

$SRa_{cv}$  is the CV of total surface arithmetic mean deviation.

$SRa_{acvm}$  is the mean CV of total surface arithmetic mean deviation in area.

$SRa_{dcvm}$  is the mean CV of total surface arithmetic mean deviation in direction.

Coefficient of variation is a relative value. The smaller the value, the better the uniformity of  $SRa$  in each assessed area in all assessed direction and all assessed area.

### III. NUMERICAL SIMULATION

Numerical simulation are on three types of curved surfaces including cylindrical surface, rotating surface and random surface. Raw data of the three surfaces are described by gray image which have  $1501^2$  pixels and 256 levels. Each pixel represents 100 square microns. The gray value of the pixel represents the mean height value of the area. Each gray level represents a height of 0.1 microns. There are a total of 25 assessed areas. These areas are placed in the middle of the surface, arranged from left to right, from bottom to top. The scope of assessed direction is from 0 to 180 degree, in which 0 degree is overlaid with 180 degree. The original degree is 0. The number of the assessed direction is 20, represent 9 to 180 degrees. The interval is 9 degree.

Because there are 25 assessed areas and 20 assessed direction,  $r = 5, s = 5, t = 20, 500 SRa$  can be calculated.

#### A. Cylindrical Surface

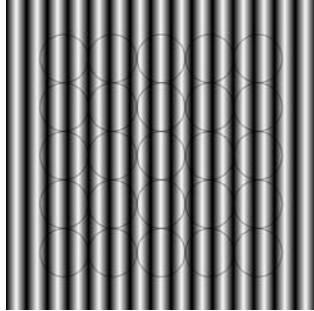


Fig. 2 Ideal cylindrical diagram

Raw data are shown in Figure 2. The gray image is made of groups of 100 pixels repeatedly. Each group of 100 pixels is arranged by gray value (The order is 0, 5, 10, ..., 245, 250, 245, ..., 10, 5 from left to right) of the pixels.

Figure 3 is the assessed graph of surface arithmetic mean deviation of the cylindrical surface. Upper left image and upper right image represent single area-directional height amplitude  $SRa$ . Middle left image and lower left image represent assessed area-uniformity index  $SRa_{amean}$  and  $SRa_{acv}$  in single direction. Middle right image and lower right represent assessed direction-uniformity index  $SRa_{dmean}$  and  $SRa_{dcv}$  in single area. Assessed total-uniformity indexes of the cylindrical surface are shown in Table 1.

The characters of  $SRa$  and relative assessed uniformity indexes of cylindrical surface are:

First,  $SRa$  which has the same angle to the direction of 90 degree in the same area is equal.  $SRa$  which has the same direction in every vertical area is equal.

Second,  $SRa_{acv}$  which has the same angle to the direction of 90 degree is equal.  $SRa_{dcv}$  in every vertical area is equal.

Third,  $SRa_{acvm}$  is far smaller than  $SRa_{dcvm}$ .

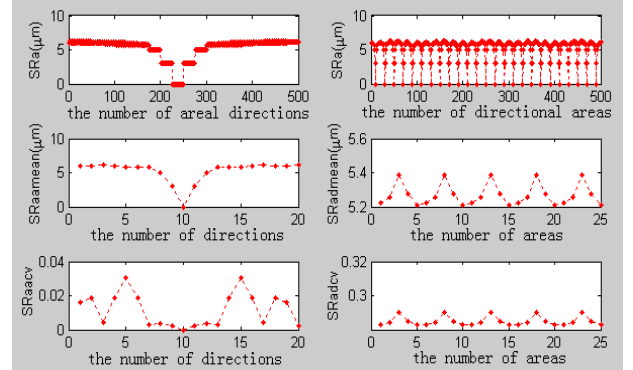


Fig. 3 Assessed graph of surface arithmetic mean deviation of the cylindrical surface

It is known from the characters above that the area-uniformity of the cylindrical surface is very good, while the direction-uniformity and total-uniformity are quite poor.

TABLE I. ASSESSED TOTAL-UNIFORMITY INDEXES

Curved Surfaces	$SRa_{acv}$	$SRa_{acvm}$	$SRa_{dcvm}$
Cylindrical Surface	0.285	0.012	0.285
Rotating Surface	0.233	0.228	0.225
Random Surface	0.004	0.004	0.002

#### B. Rotating Surface

Raw data are shown in Figure 4. To get the required gray image of  $1501^2$  pixels, steps are as follows:

First, draw concentric circles repeatedly. The number of the pixels of the most inside circle is 0. Each circle is a group of 100 concentric circles. The order of gray value of each group of concentric circles is 0, 5, 10, ..., 245, 250, 245, ..., 10, 5 from outside to inside.

Second, cut out the concentric circles in the middle to a square of  $1501^2$  pixels.

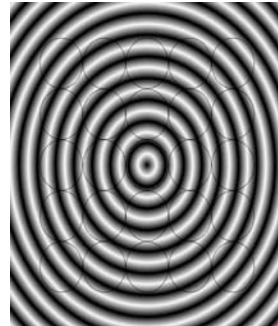


Fig. 4 Gray image of rotating surface

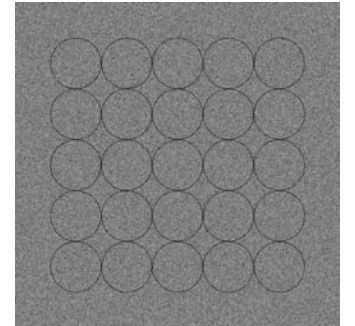


Fig. 5 Gray image of random surface

Figure 6 is the assessed graph of surface arithmetic mean deviation of the rotating surface. Assessed total-uniformity indexes of the rotating surface are shown in Table 1.

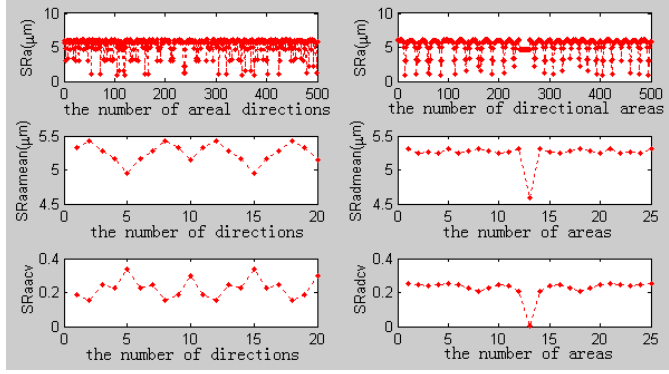


Fig. 6 Assessed graph of surface arithmetic mean deviation of the rotating surface

The characters of  $S Ra$  and relative assessed uniformity indexes of rotating surface are:

First,  $S Ra$  in center area is equal.

Second,  $S Ra_{acv}$  which has the same angle to the direction of 90 degree is equal.  $S Ra_{dcv}$  which area has the same distance to center is equal.

It is known from the characters above that the area-uniformity, direction-uniformity and total-uniformity of the rotating surface are quite poor.

### C. Random Surface

Raw data are shown in Figure 5. Each gray value of pixels is calculated by pseudo random function of uniform distribution. The scope of the gray value is from 0 to 255.

Figure 7 is the assessed graph of surface arithmetic mean deviation of the random surface. Assessed total-uniformity indexes of the random surface are shown in Table 1.

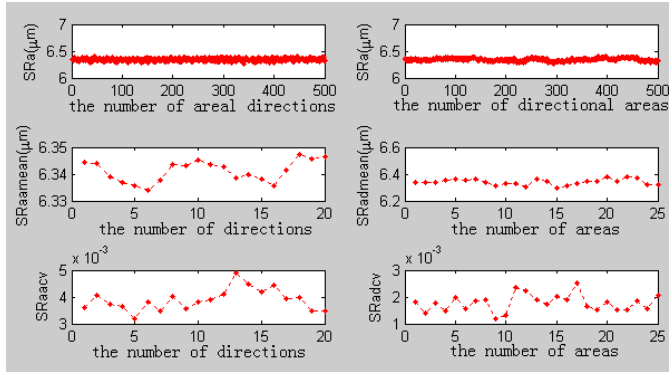


Fig. 7 Assessed graph of surface arithmetic meandeviation of the rotating surface

The characters of  $S Ra$  and relative assessed uniformity indexes of random surface are:

First,  $S Ra$  is approximately equal.

Second,  $S Ra_{acvm}$ ,  $S Ra_{dcvm}$  and  $S Ra_{cv}$  are close to 0.

It is known from the characters above that the area-uniformity, the direction-uniformity and total-uniformity of the random surface are very good.

## IV. CONCLUSIONS

$S Ra$  can describe preferably the quantitative relationship among the height amplitude, assessed area and assessed direction. The assessed area-uniformity indexes, assessed direction-uniformity indexes and assessed total-uniformity indexes can be used to assess the area-uniformity, direction-uniformity and total-uniformity of height amplitudes.

$S Ra$  has a close relationship with assessed area, assessed direction. The assessed uniformity indexes have close relationship with the number of assessed area, and the number of assessed direction. So the assessment should be used under the same condition.

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