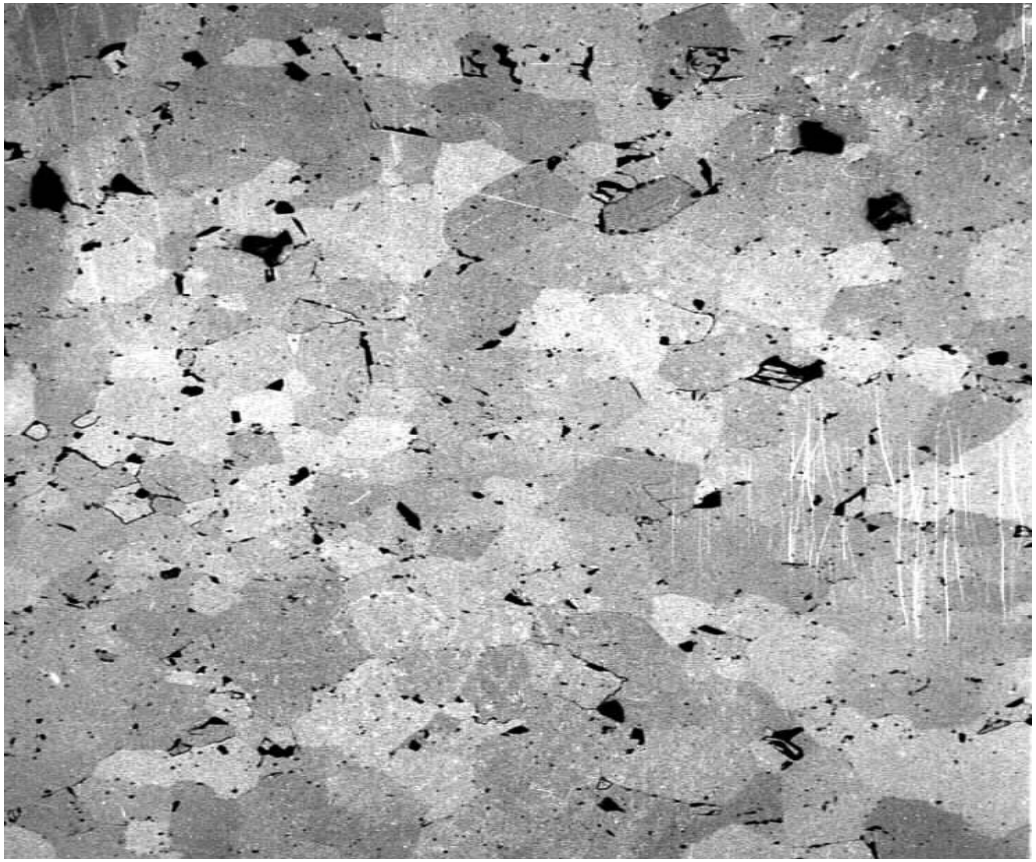
细观非均匀性的
跨尺度效应和新尺度涌现

- 非均匀性, 连续分叉, 渐进和灾变破坏

_白以龙，夏蒙棼，……………………

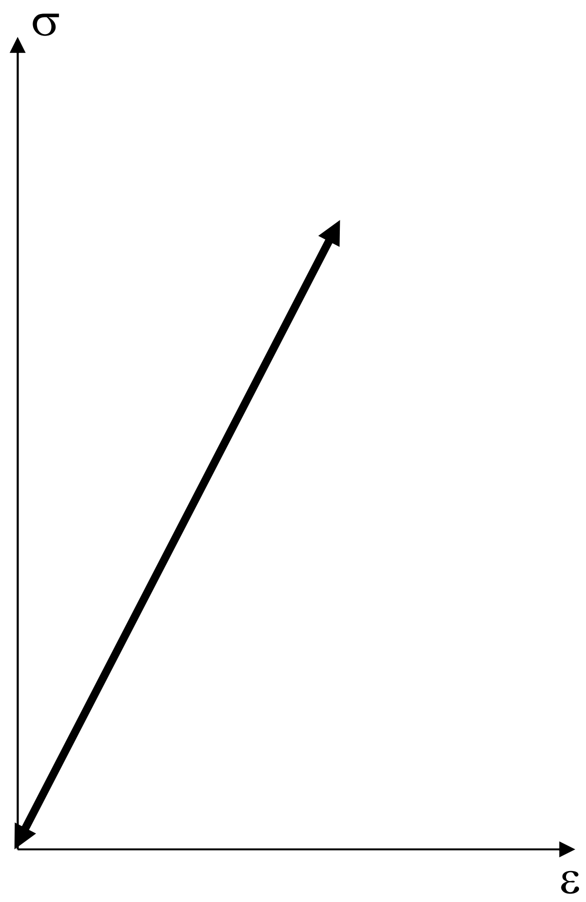
LNM

现状1:非均匀性在力学理论中几乎没有考虑

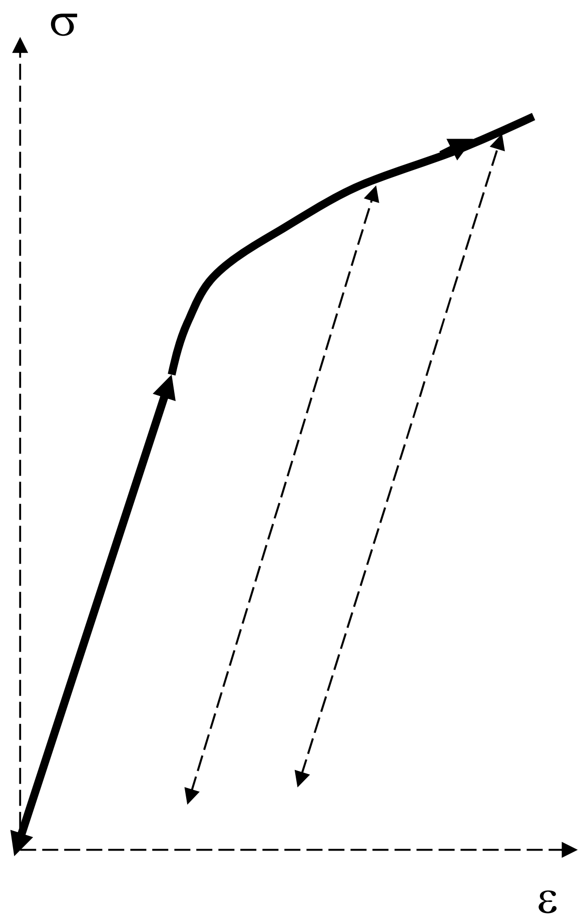


after Rong F

现状2: 固体力学理论中还缺什么



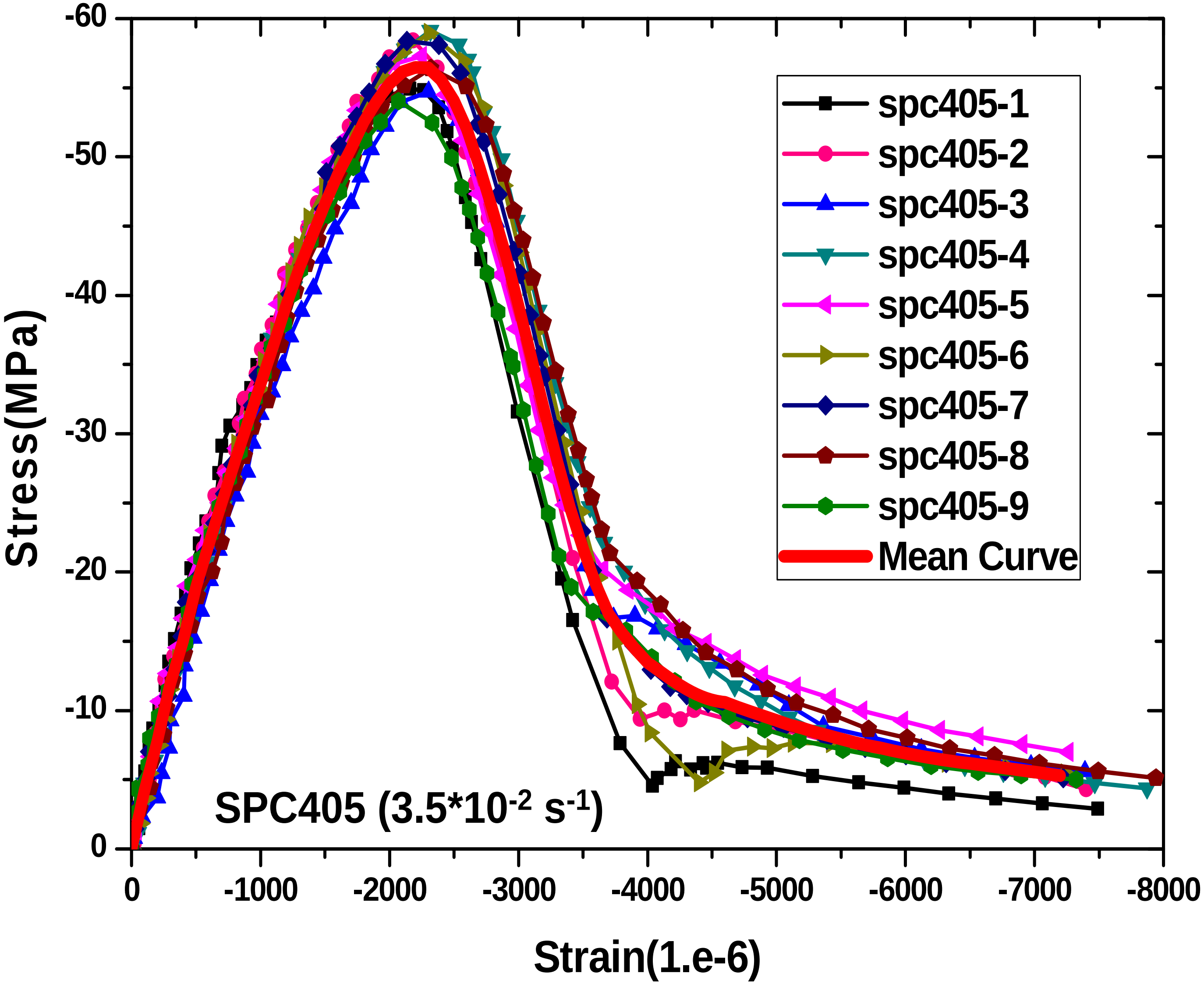
Elastic



Elastic-Plastic

?

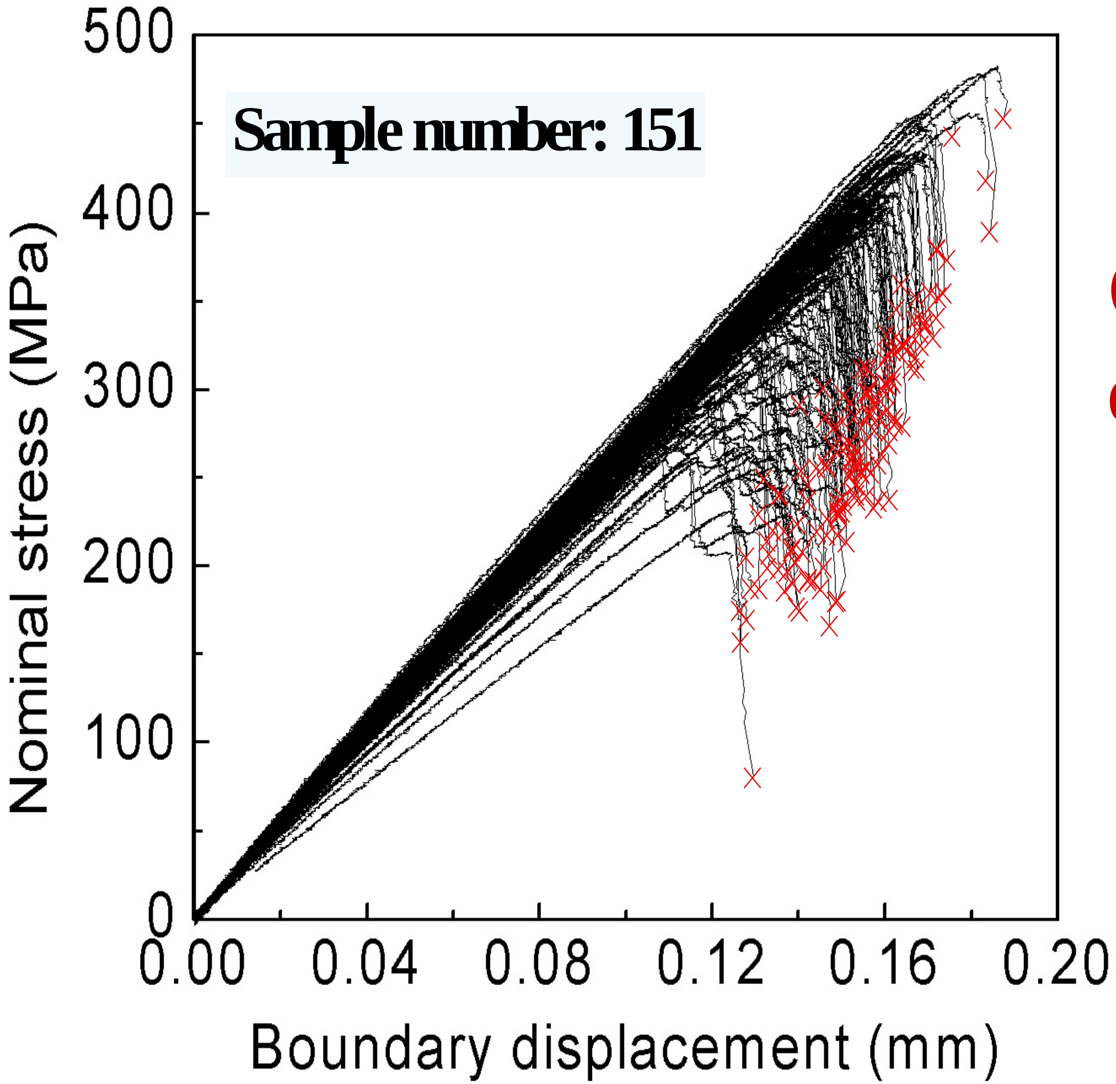
Case1



Gradual failure

Concrete
After Li Jie

Case 2



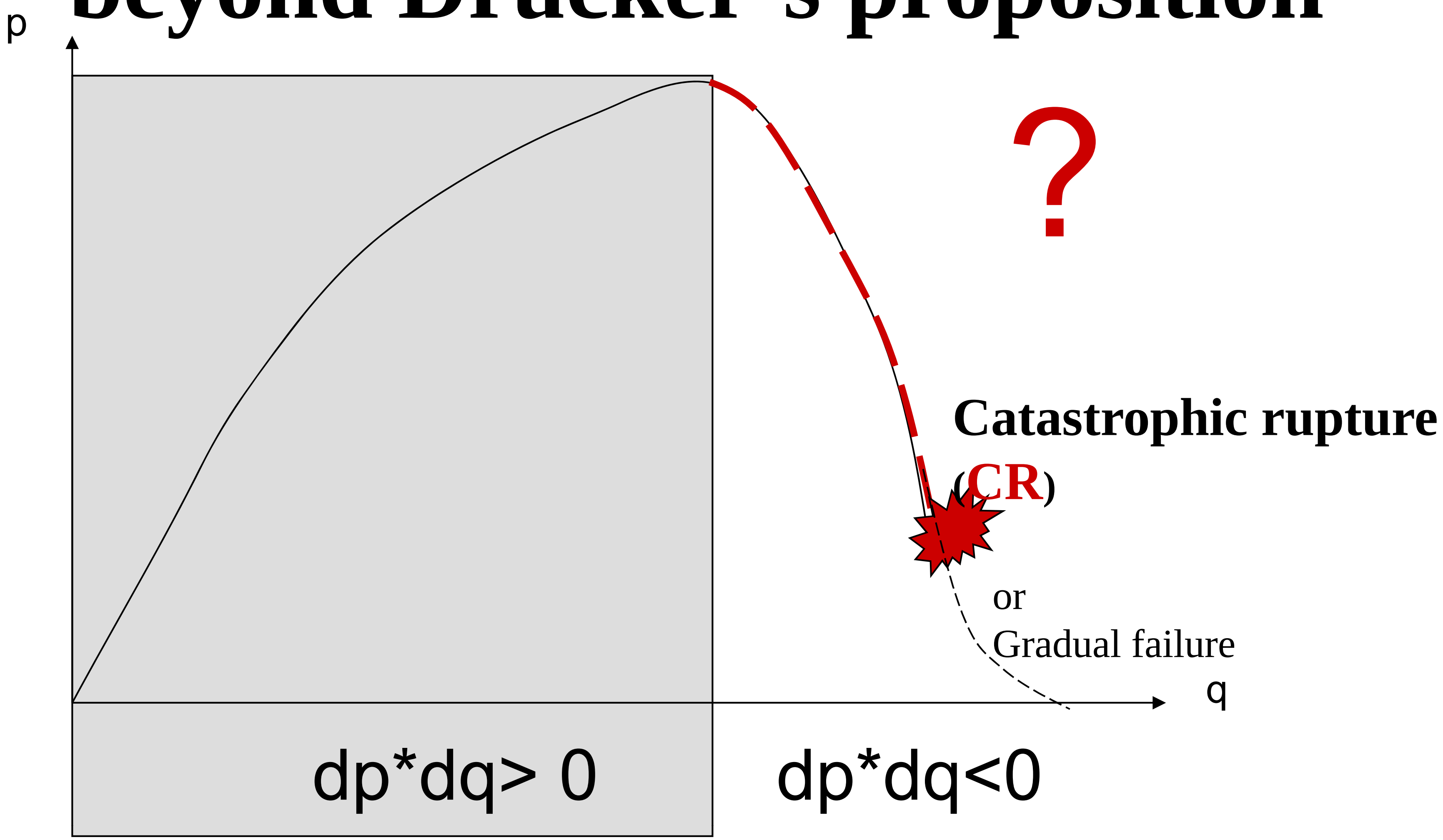
Catastrophic rupture occurs diversely!

Granite

after Xu XH

LNM

What happens beyond Drucker's proposition



Content

A. Heterogeneity

Global mean field approximation – ESB model

Energy criterion

Size effect – statistical interpretation

B. 3 interrelated features

Continuous Bifurcation,

Damage Localization

Catastrophic Rupture (CR)

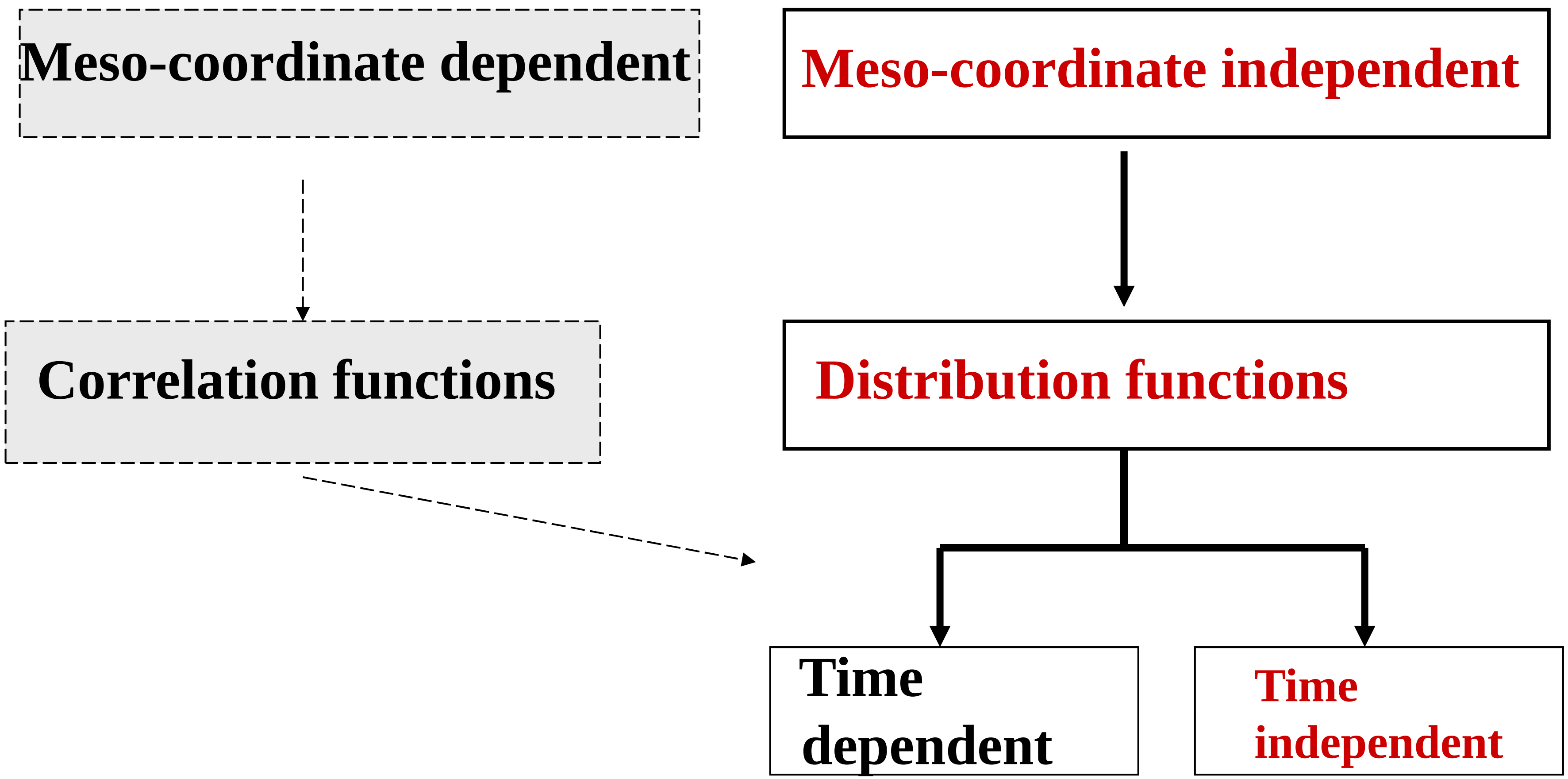
C. Preliminary application

M9 earthquake, 2011, Japan

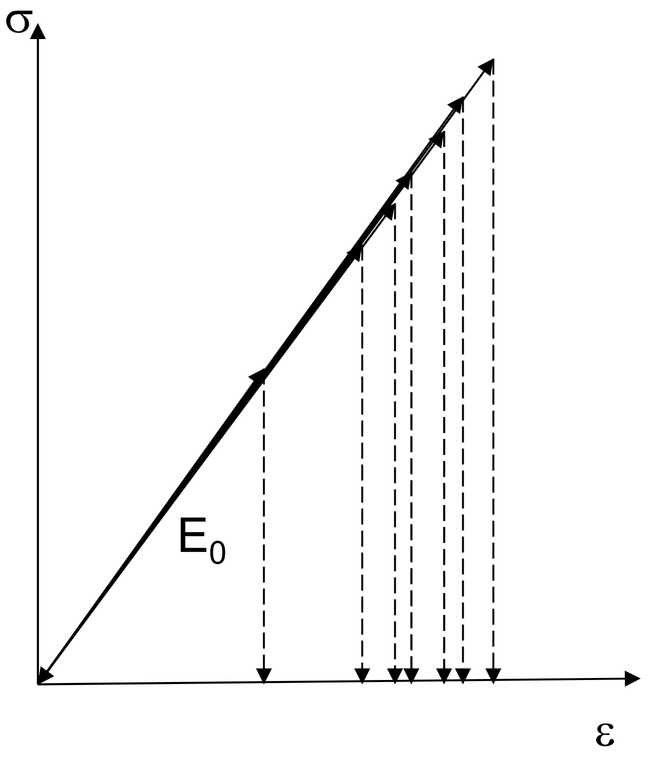
M8 earthquake, 2008, Wenchuan

LNM

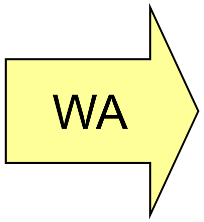
Statistical description of meso-scale heterogeneitis

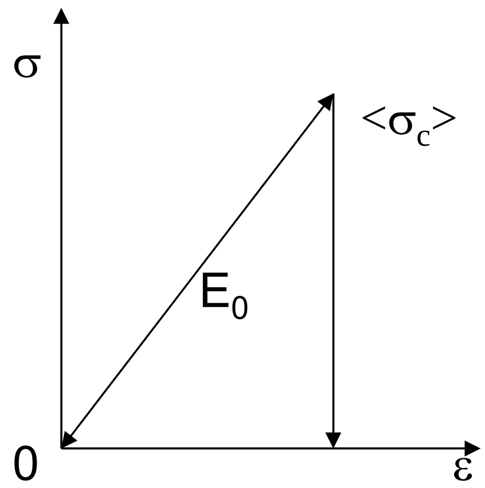


Heterogeneity vs Average

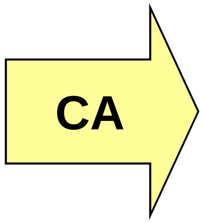


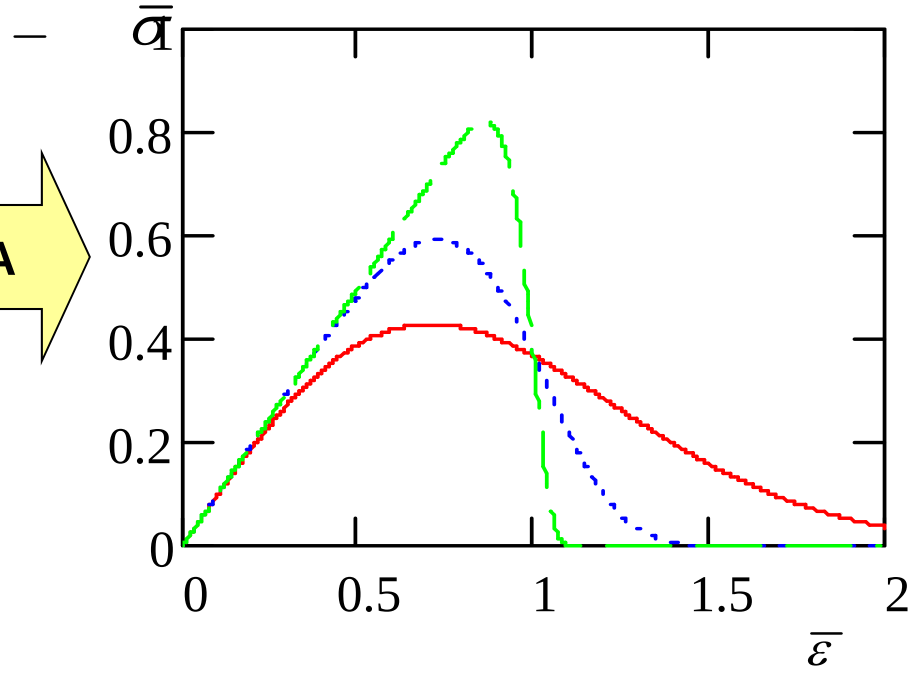
(a)





(b)

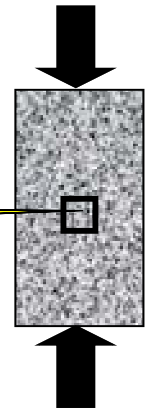


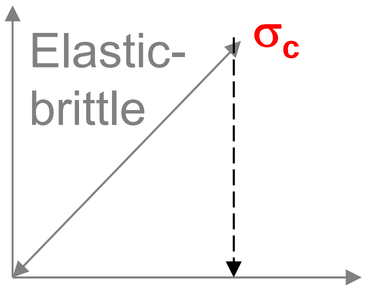


(c)

A-1 Elastic Statistically-Brittle (ESB) Model

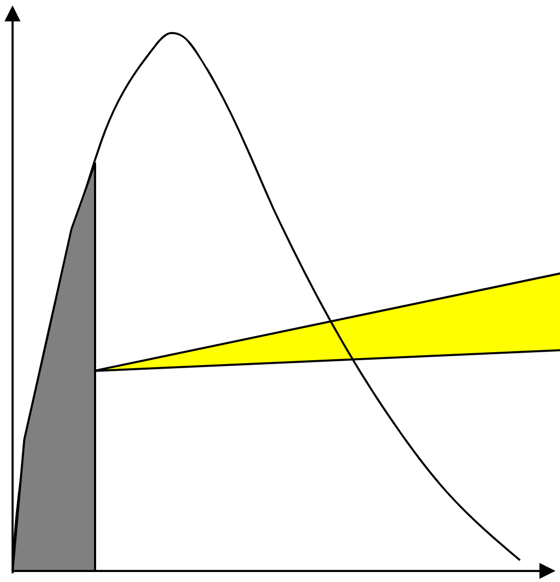
every meso element:





- **Same elastic constants**
- **Different strength σ_c with distribution $h(\sigma_c)$**

$h(\sigma_c)$



σ_S

Weibull distribution

$$D(\bar{\sigma}_s) = 1 - \exp[-\bar{\sigma}_s{}^m]$$

Coupling average (CA)

Meso-elastic $$\sigma_{meso} = E\varepsilon_{meso}$$

Average + Damage

$$\sigma =< \sigma_{meso} >=< \sigma_{meso}(1-D) + 0 \cdot D >=< \sigma_{meso,\neq 0} > (1-D)$$

Mean field approximation (MF) $$\sigma_{meso,\neq 0} = \sigma_t$$

Strain-equivalence

$$\sigma = \sigma_t(1-D)$$
$$= \sigma_{meso,\neq 0}(1-D) = E\varepsilon_{meso}(1-D)$$
$$= E\varepsilon(1-D)$$

ESB model

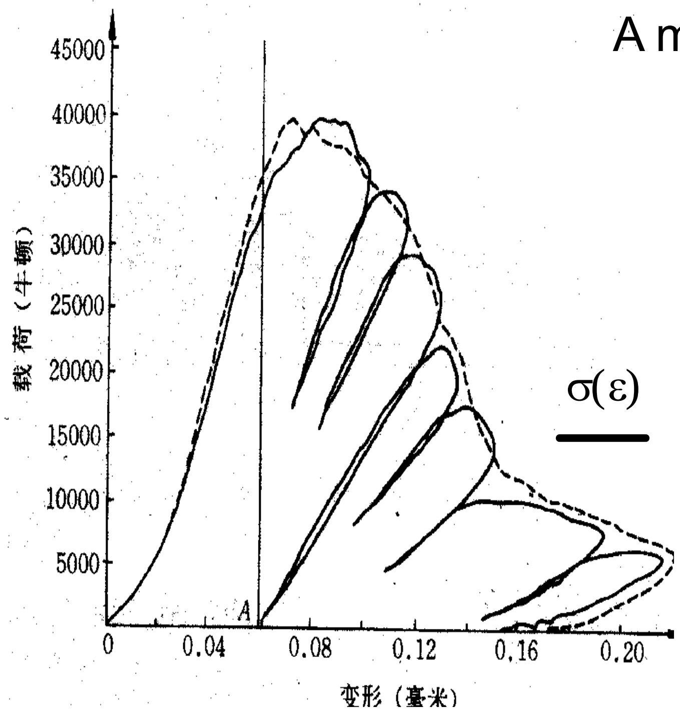
Uniaxial compression

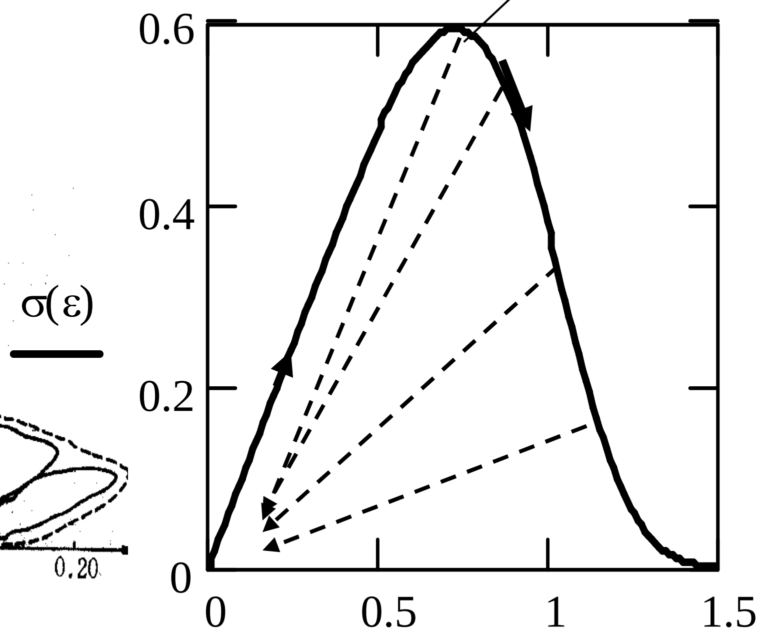
$$\bar{\sigma} = \bar{\varepsilon}(1-D) = \bar{\varepsilon} e^{-\bar{\varepsilon}^{m}}$$

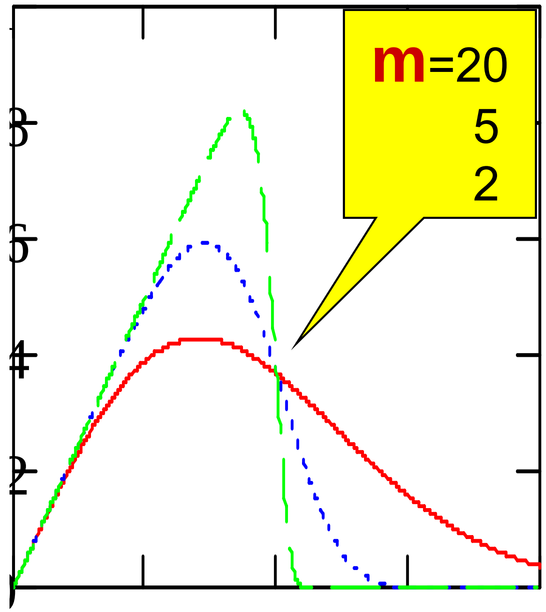
$$\bar{\sigma}_M = (me)^{-\frac{1}{m}}$$

m: Weibull modulus

A measure of meso-heterogeneity





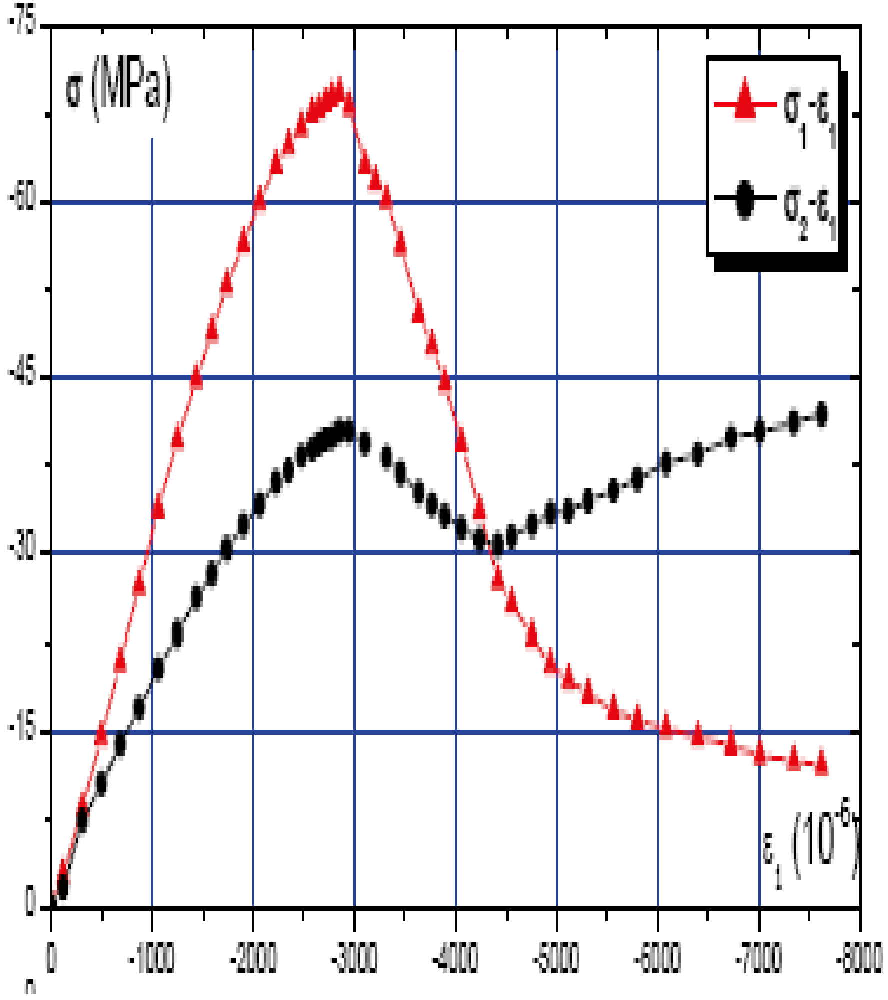


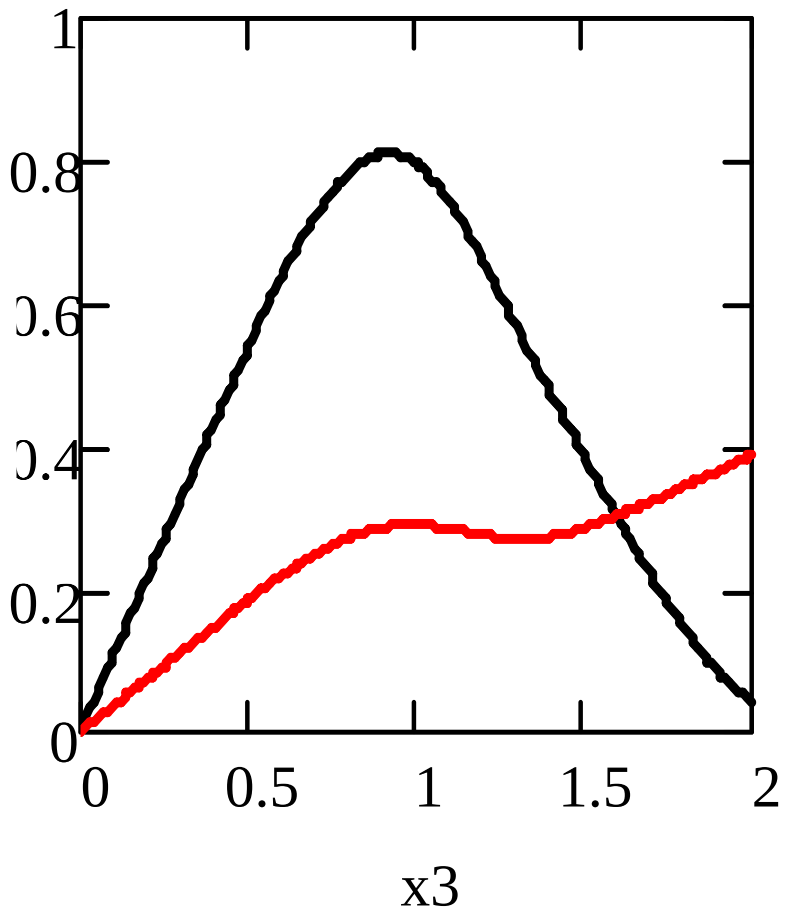
Exp observations

ESB model

ESB: Biaxial compression

: m=5, α=0.1, ν=0.25





Concrete after Li Jie

A-2. Catastrophic Rupture (CR) or Globally Stable deformation (GS)

Two modes: GS and CR

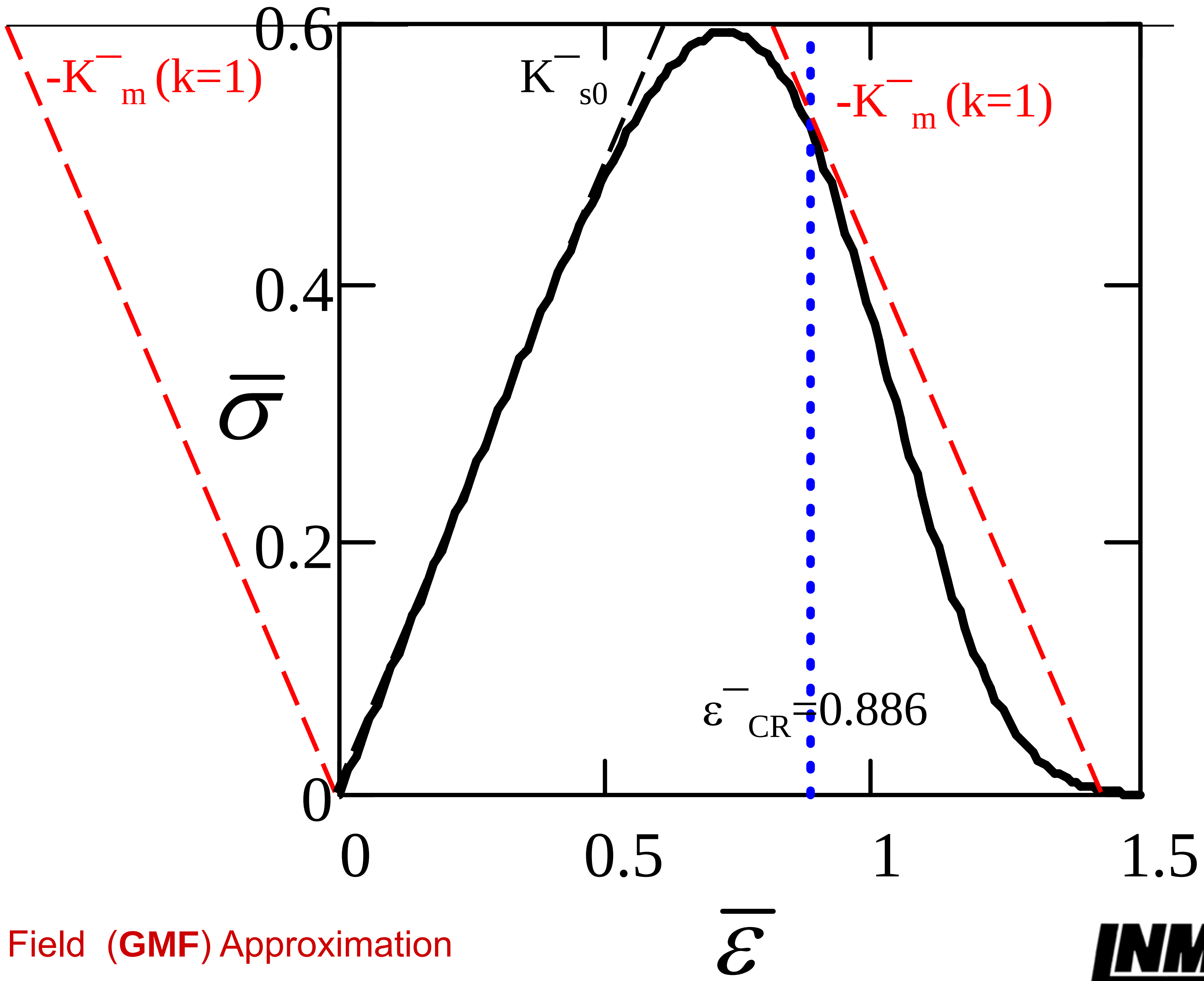
$$d\overline{W} > 0 \quad GS$$

$$d\overline{W} \approx 0 \quad CR$$

$$K_m \leq -K_s(u_s)$$

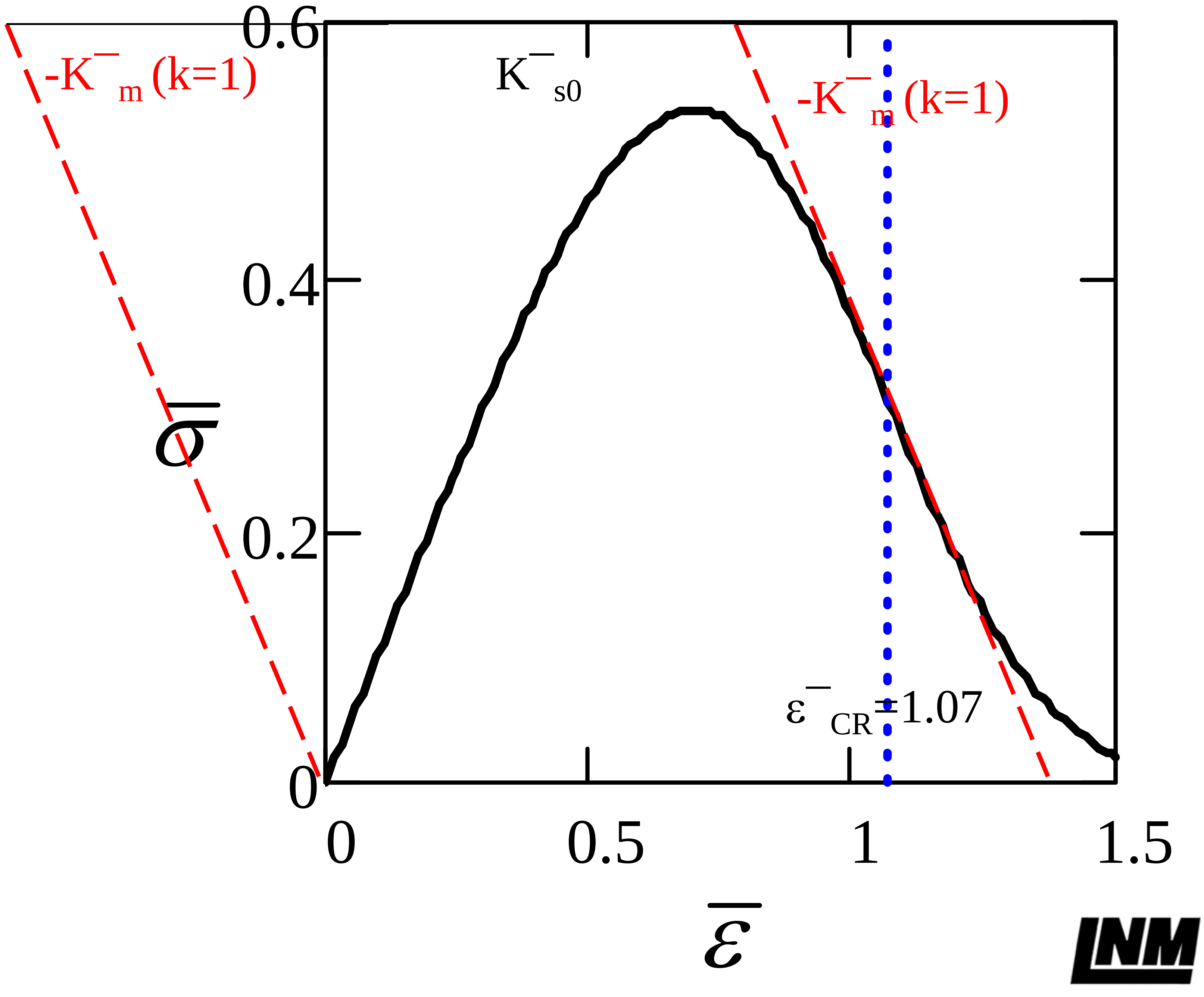
LNM

m=5



Global Mean Field (**GMF**) Approximation

m=3.591



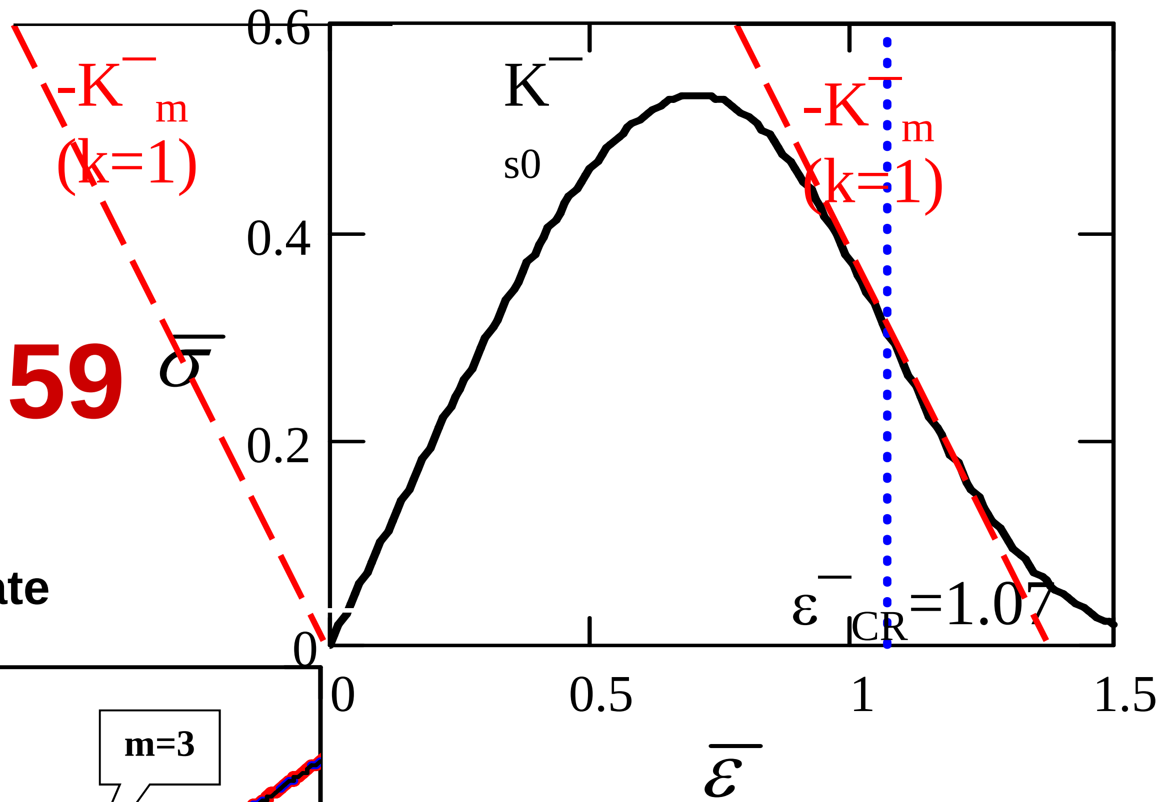
CR - Δw≤0 critical:

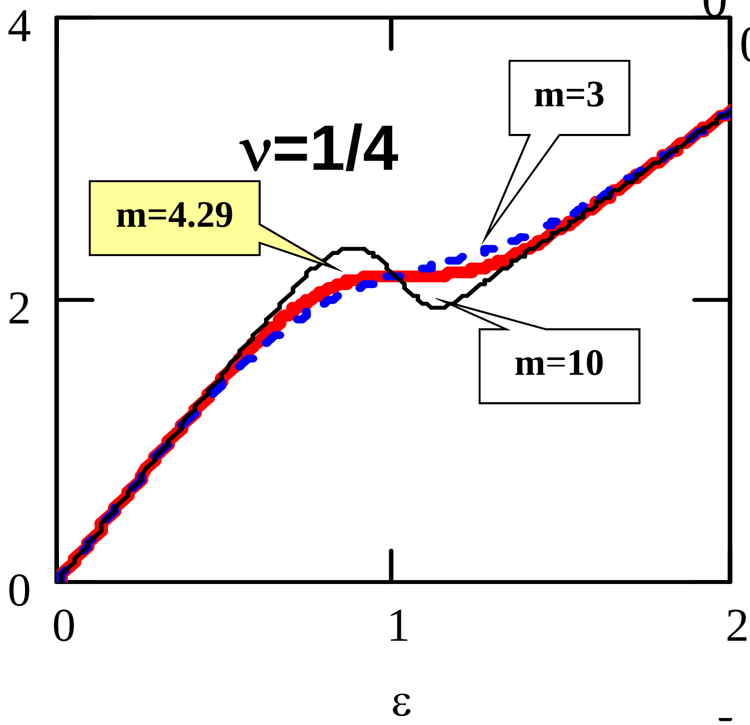
m>m_c CR

m<m_c Gradual failure

m_C=3.59

(k=1)
1D stress state

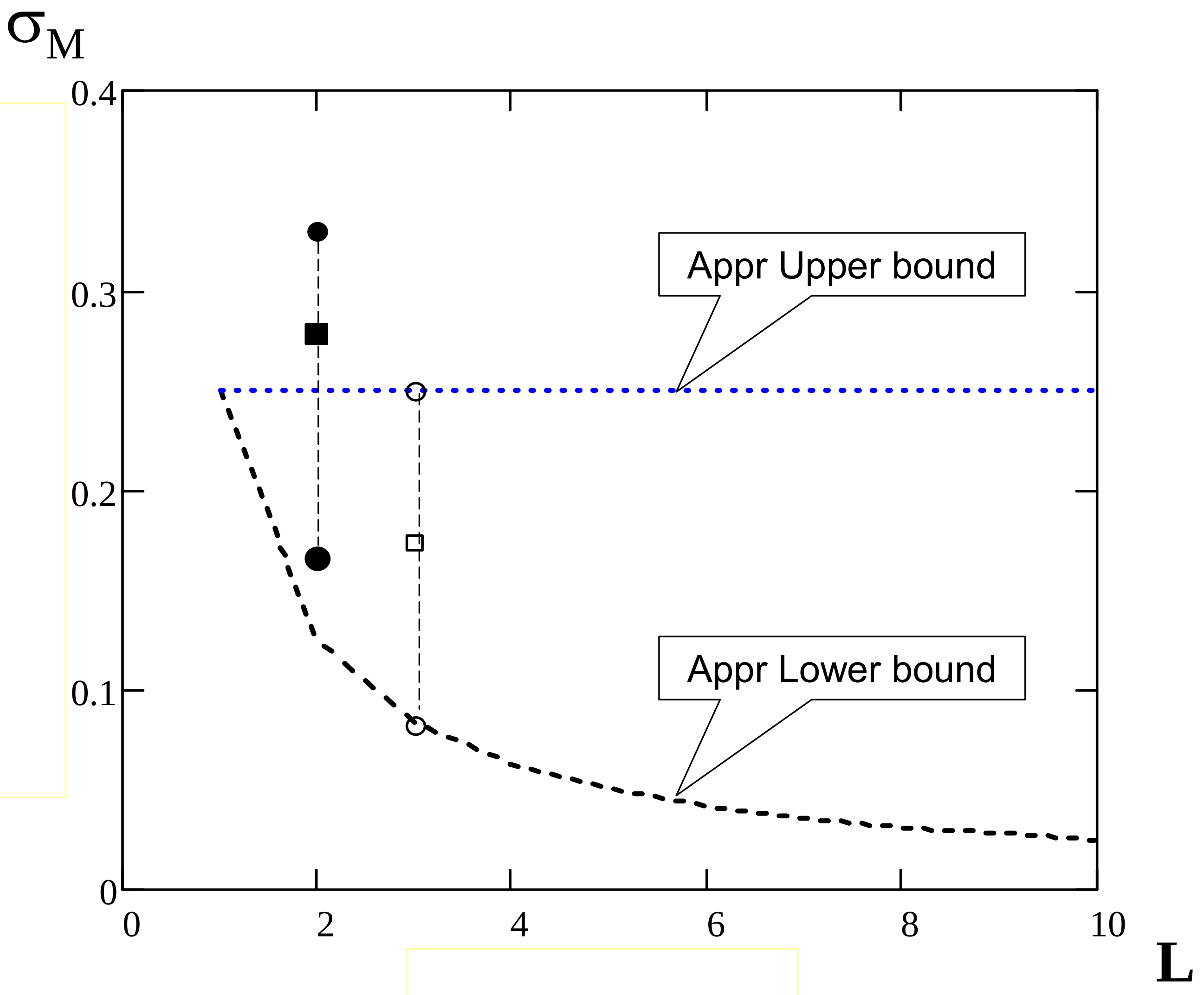


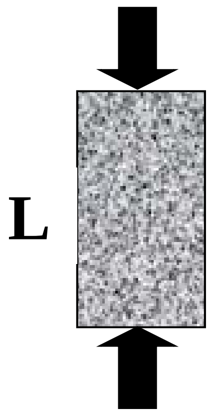


m_c=4.29

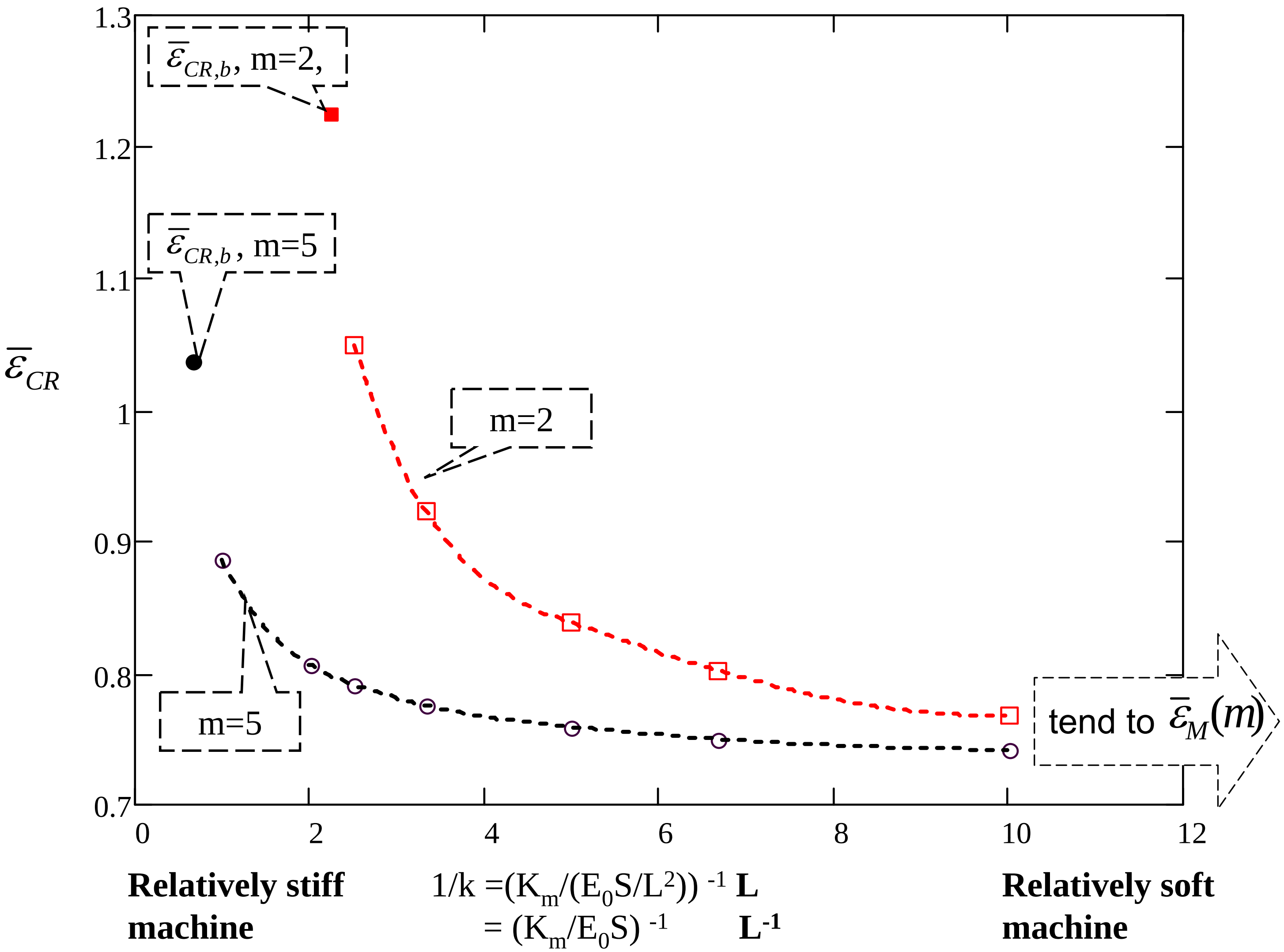
(ν=1/4)
1D strain state

Size effect – statistical interpretation

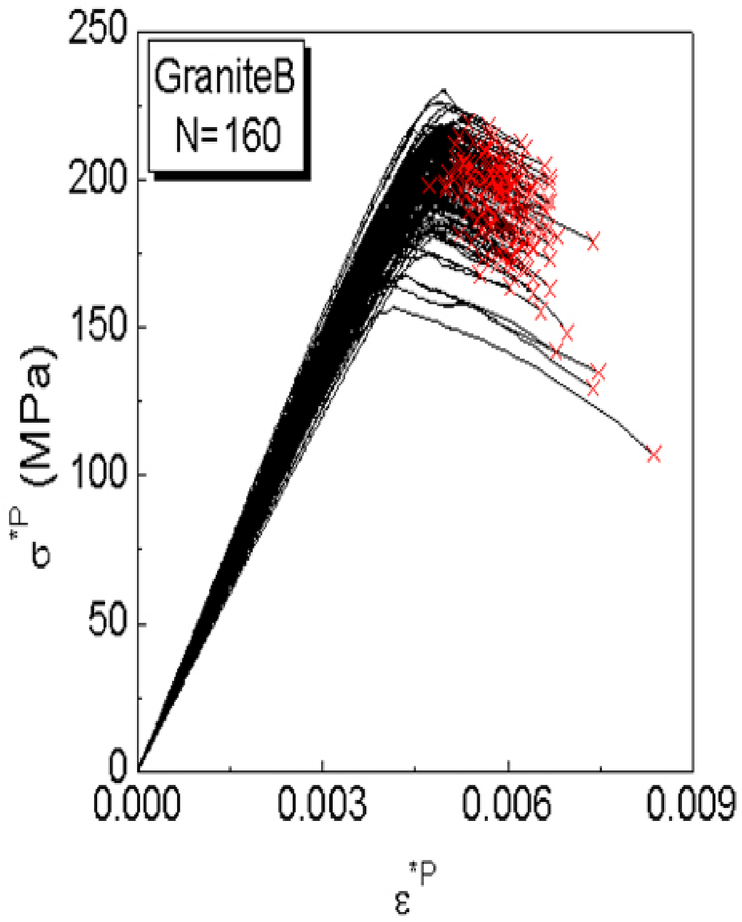




Size effect – statistical interpretation



CR
Global Mean Field (**GMF**) Approximation



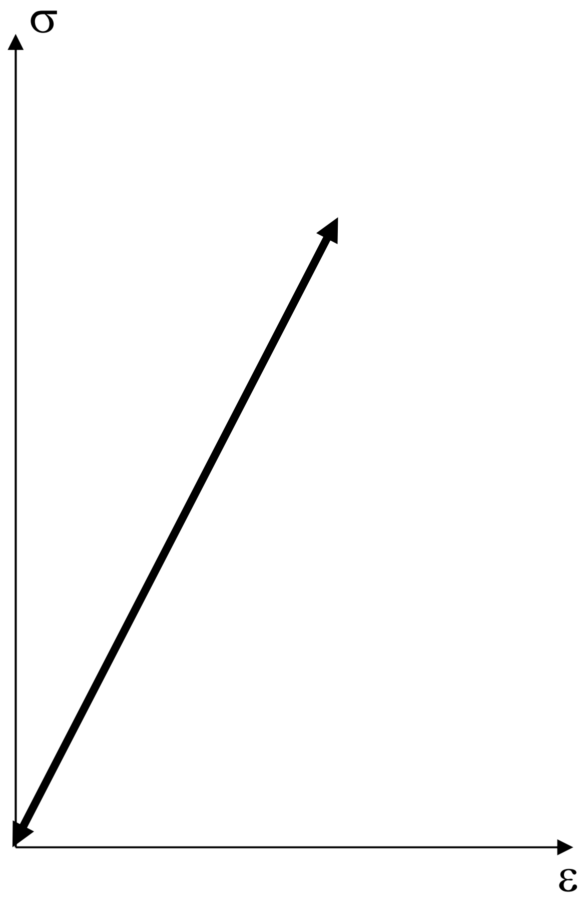
Exp strain at CR (Granite)	Cal strain at CR (with Localized zone γ)	**Cal strain at CR** (GMF)	Width of localized zone γ
0.00850	0.00816	**0.00879**	7.1 *mm*
0.00655	0.00694	**0.00839**	7.9 *mm*
0.00645	0.00674	**0.00810**	6.7*mm*
0.00636	0.00611	**0.00694**	7.9 *mm*
0.00634	0.00633	**0.00682**	6.3 *mm*
0.00515	0.00606	**0.00661**	6.9 *mm*

~(+32%)

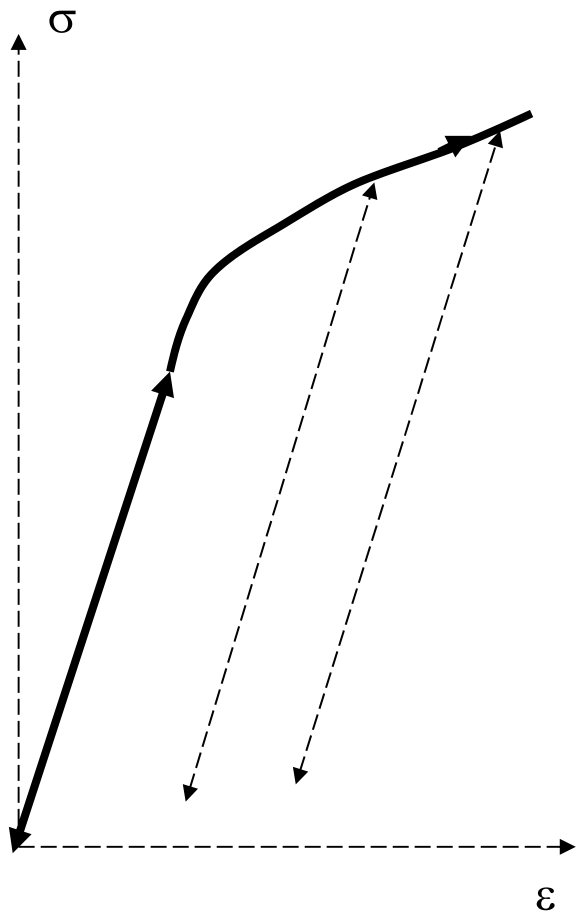
Too late
Why?

LNM

现状2: 固体力学理论中还缺什么

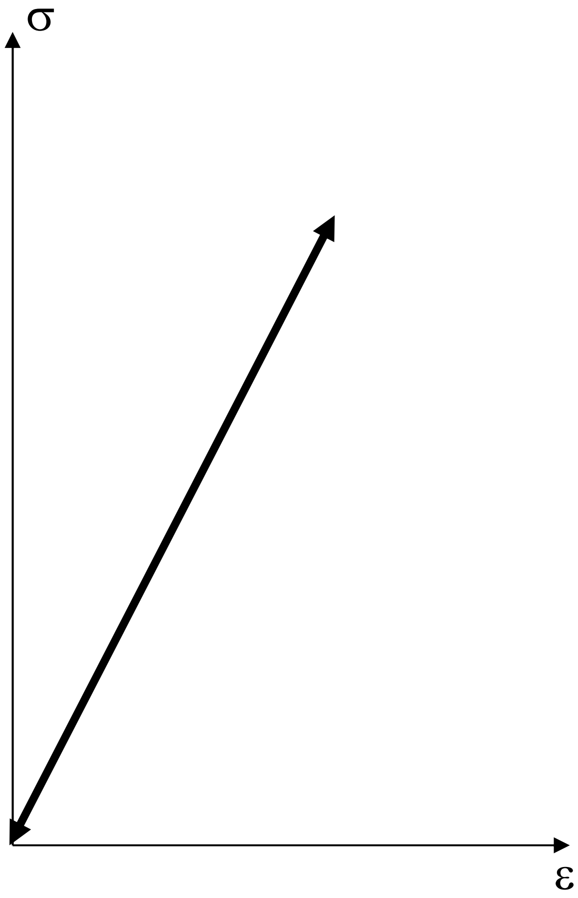


Elastic

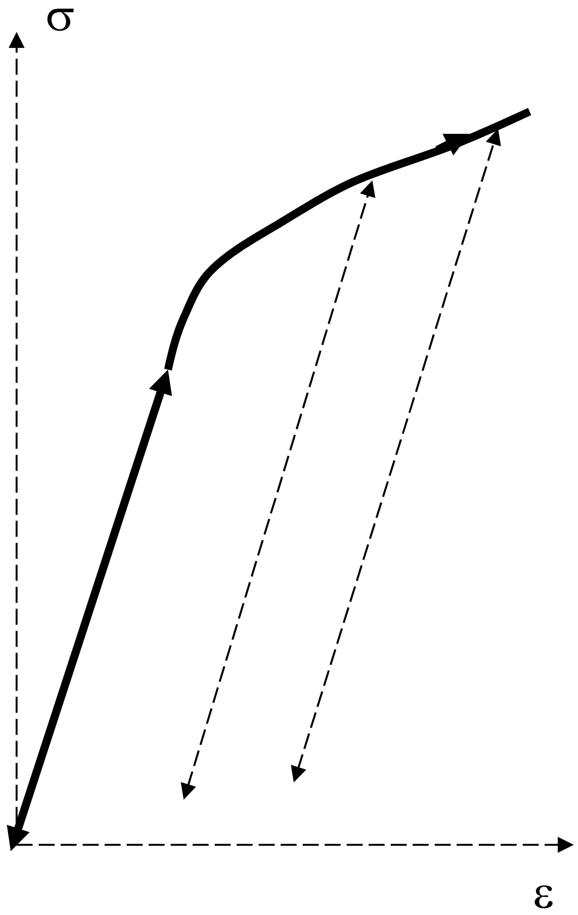


Elastic-Plastic

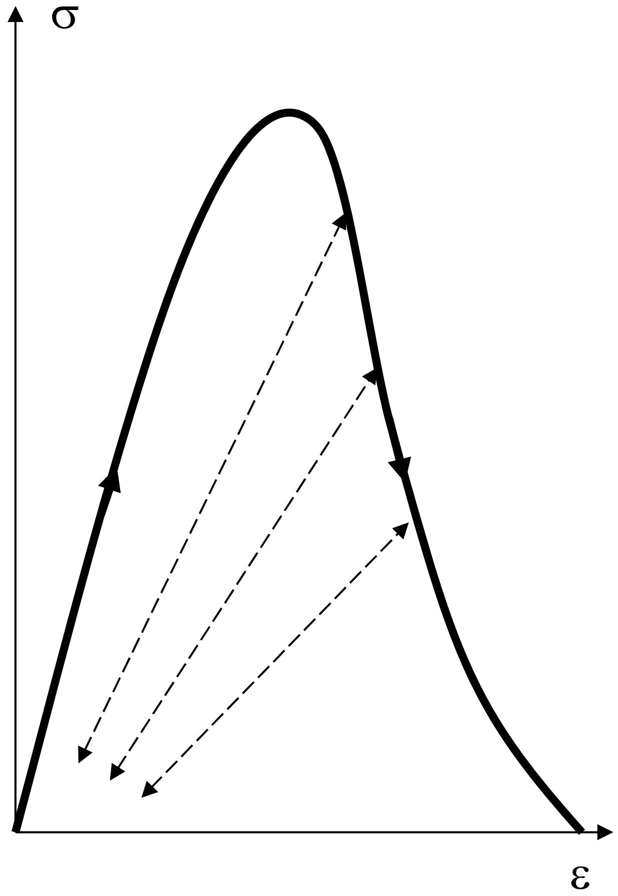
?



Elastic



Elastic-Plastic



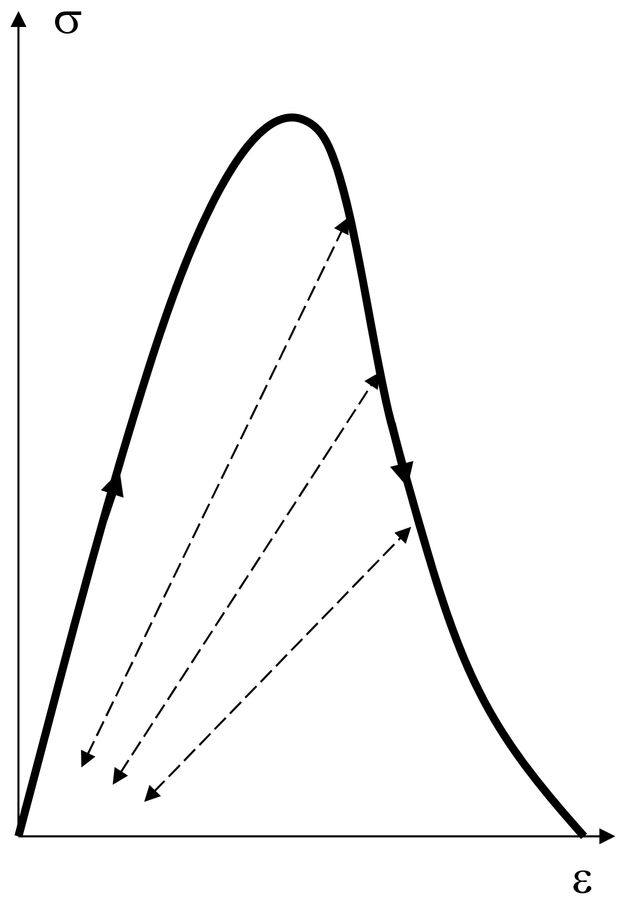
ESB

Q: Stiff machine >>>
Full stress- strain relation?

Theories available >>> YES
In Practice >>> usually not

QQ

Then, what happens?
why?



ESB

Content

A. Heterogeneity

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Continuous Bifurcation,

Damage Localization

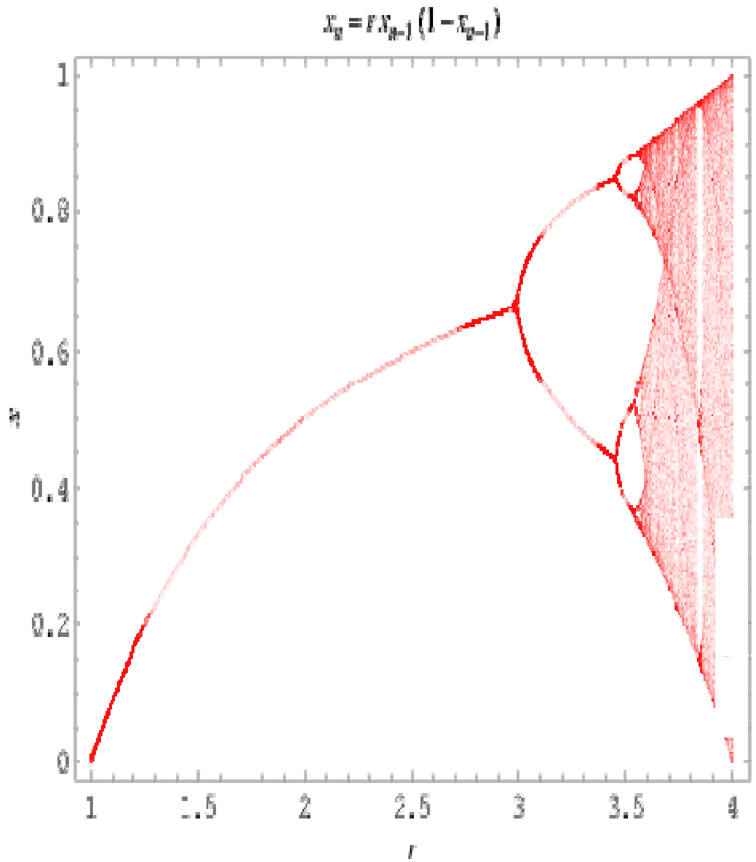
Catastrophic Rupture (CR)

C. Preliminary application

M9 earthquake, 2011, Japan

M8 earthquake, 2008, Wenchuan

B-1. Continuous Bifurcation

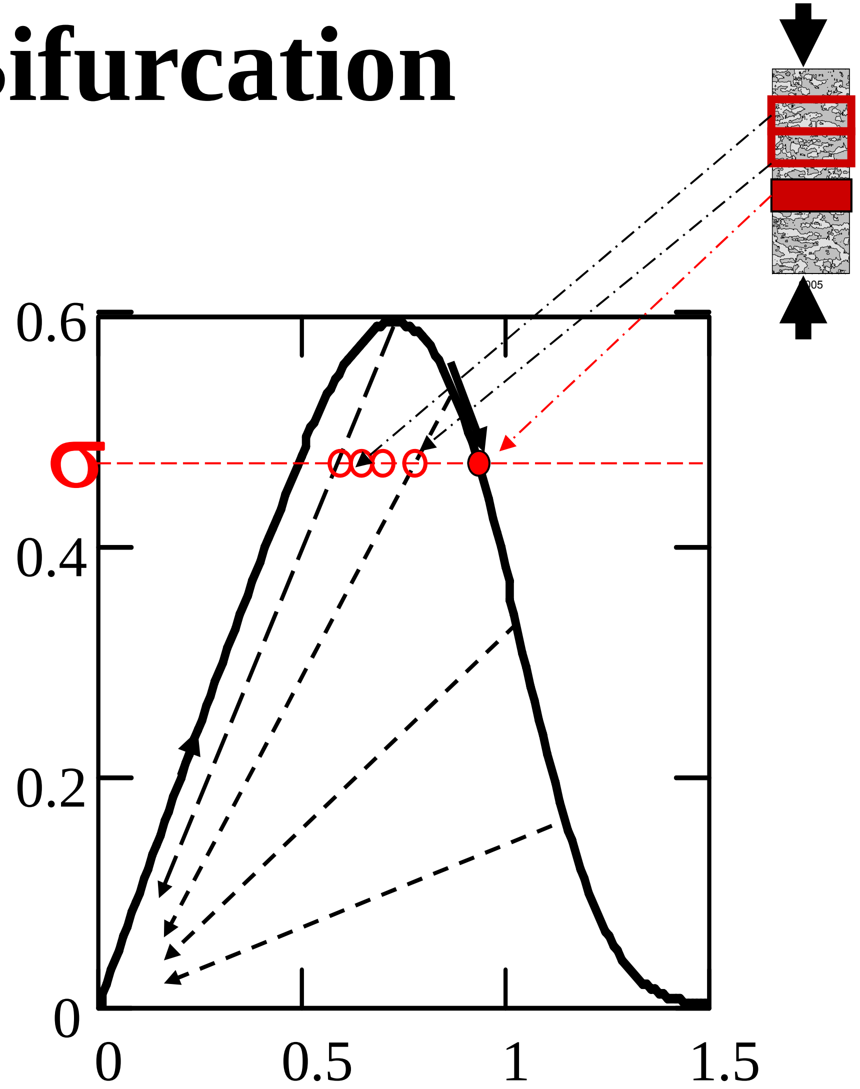


Logistic mapping (λ)

Multiple bifurcation

State: OR

$\frac{\sigma(\varepsilon)}{}$



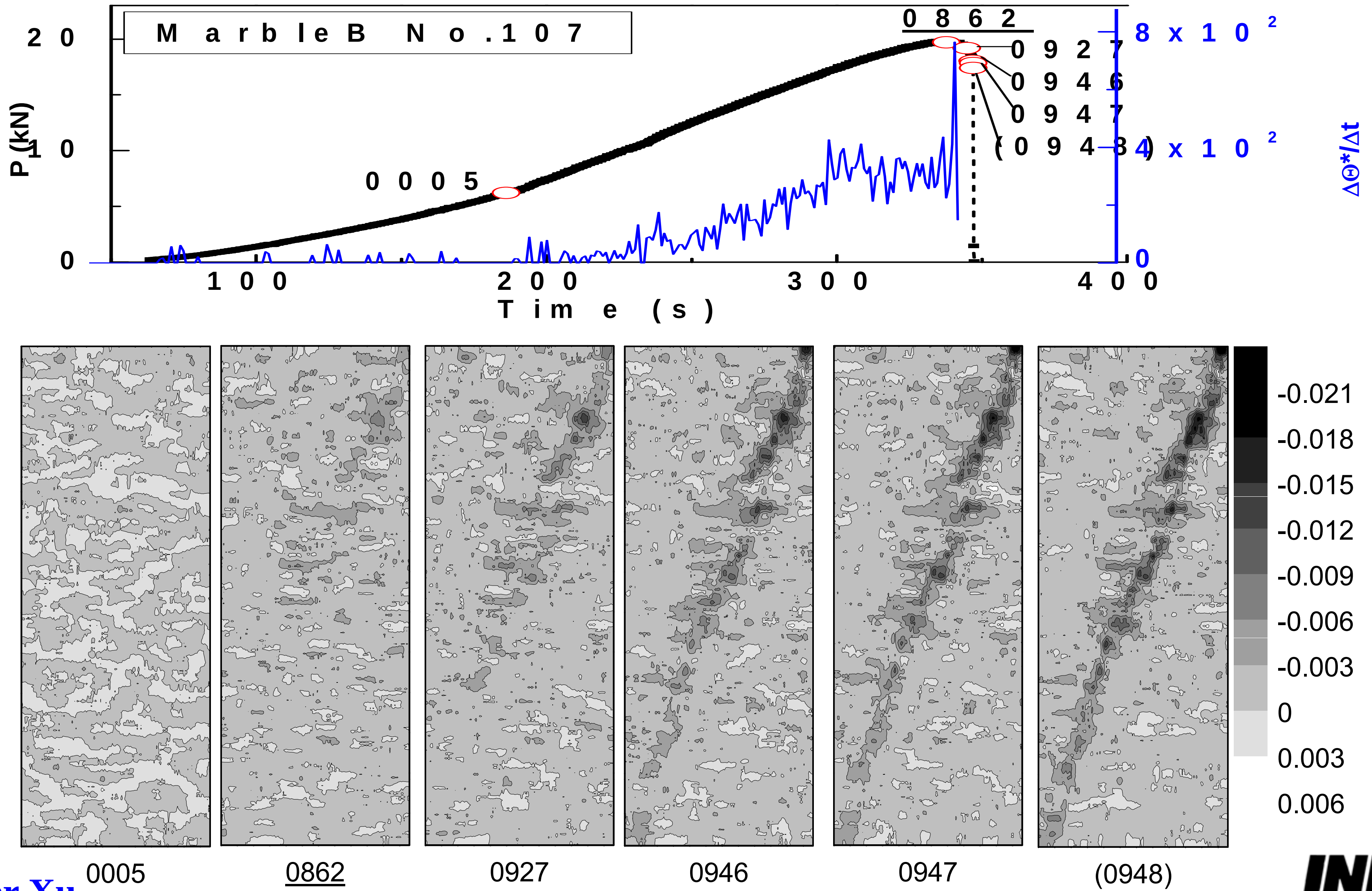
Softening ($d_\varepsilon \sigma < 0$ & $d\varepsilon > 0$

Continuous Bifurcation

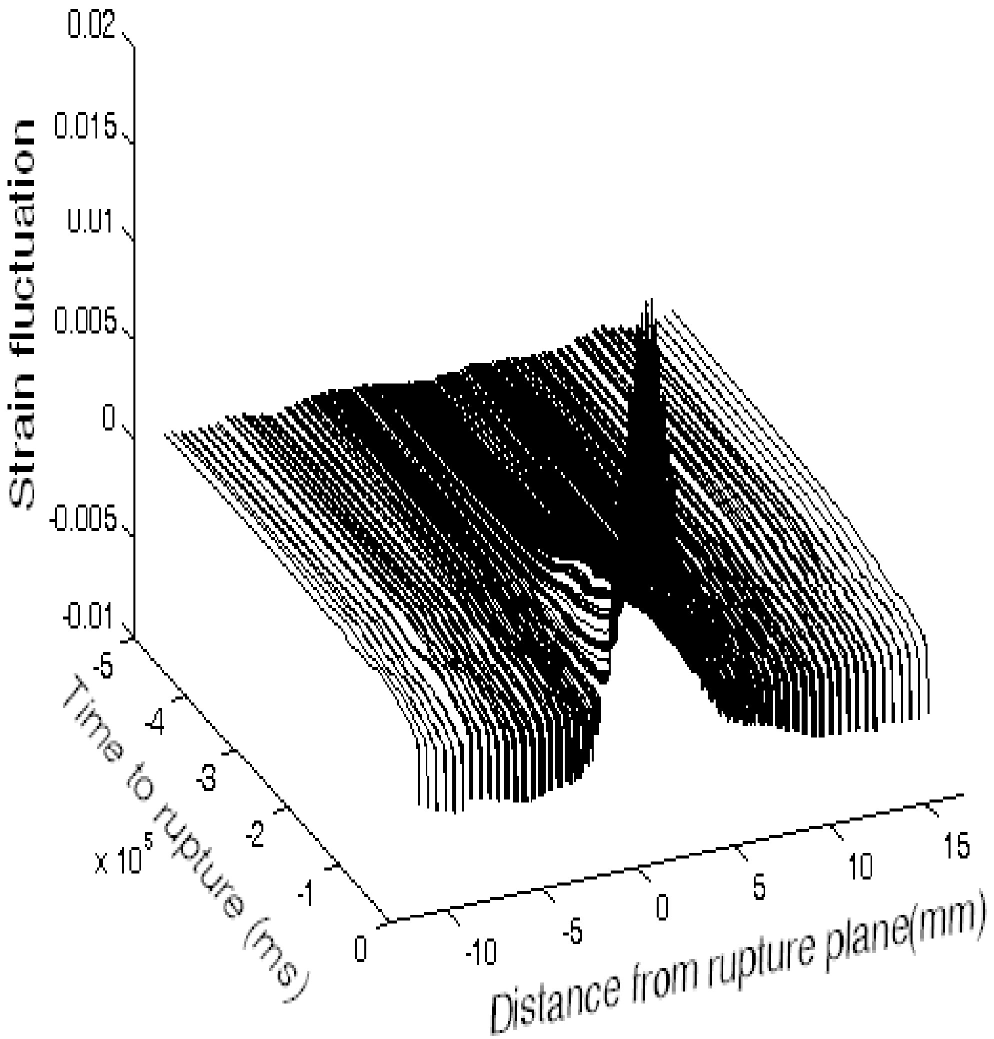
State : AND

Deterministic ⇒ Stochastic

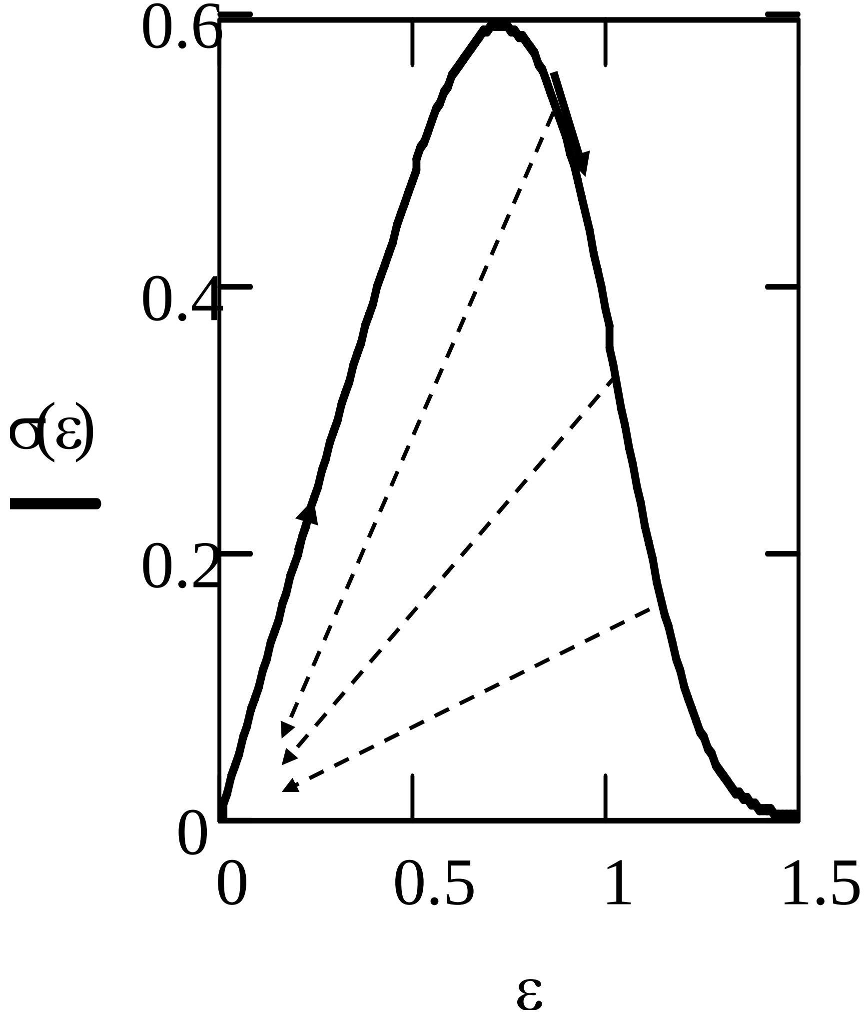
B-2 Localization



After Xu

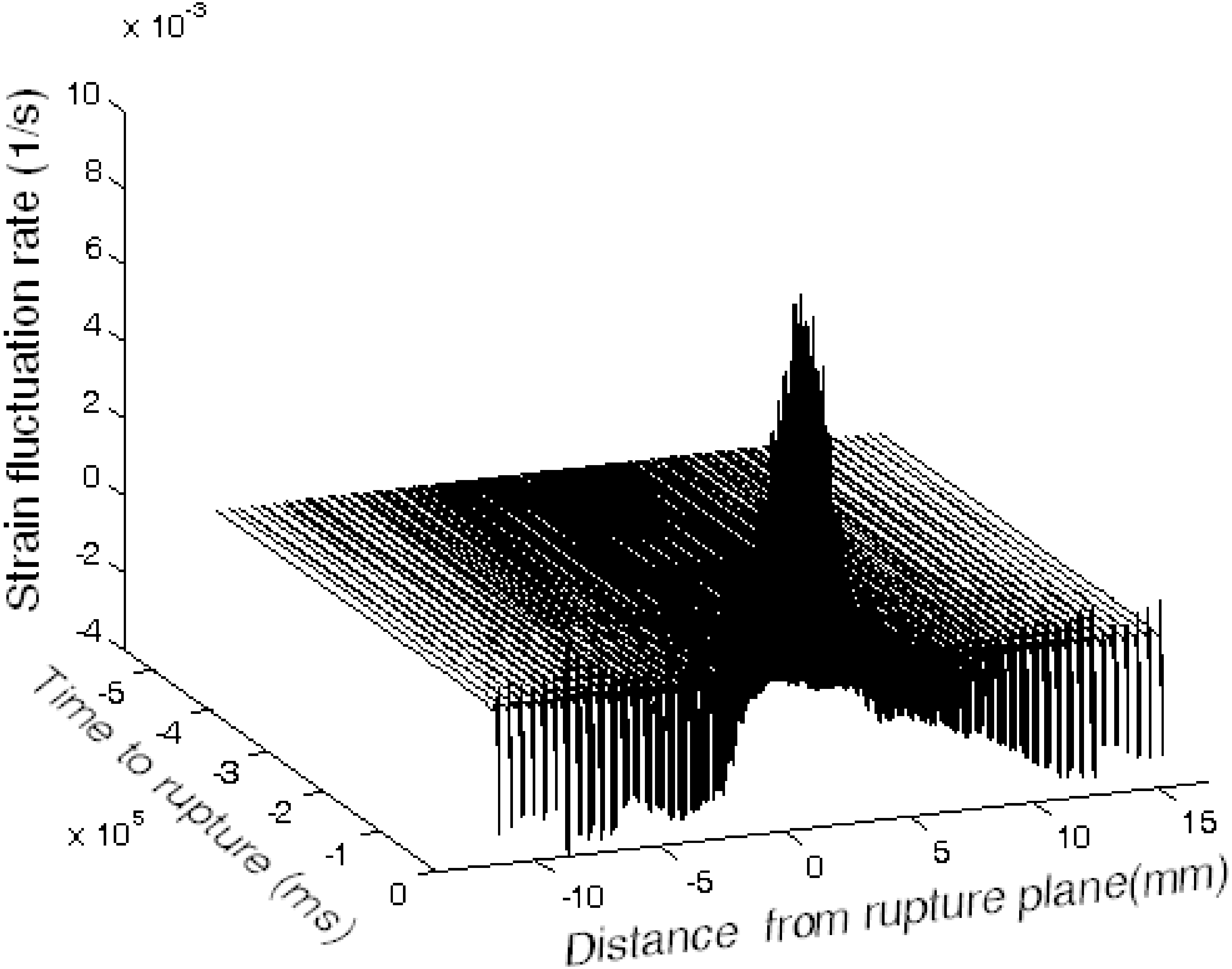


0.02
0.015
0.01
0.005
0
-0.005
-0.01
Strain fluctuation
-5
-4
-3
-2
-1
0
x 10
Time to rupture (ms)
-10
-5
0
5
10
15
Distance from rupture plane(mm)

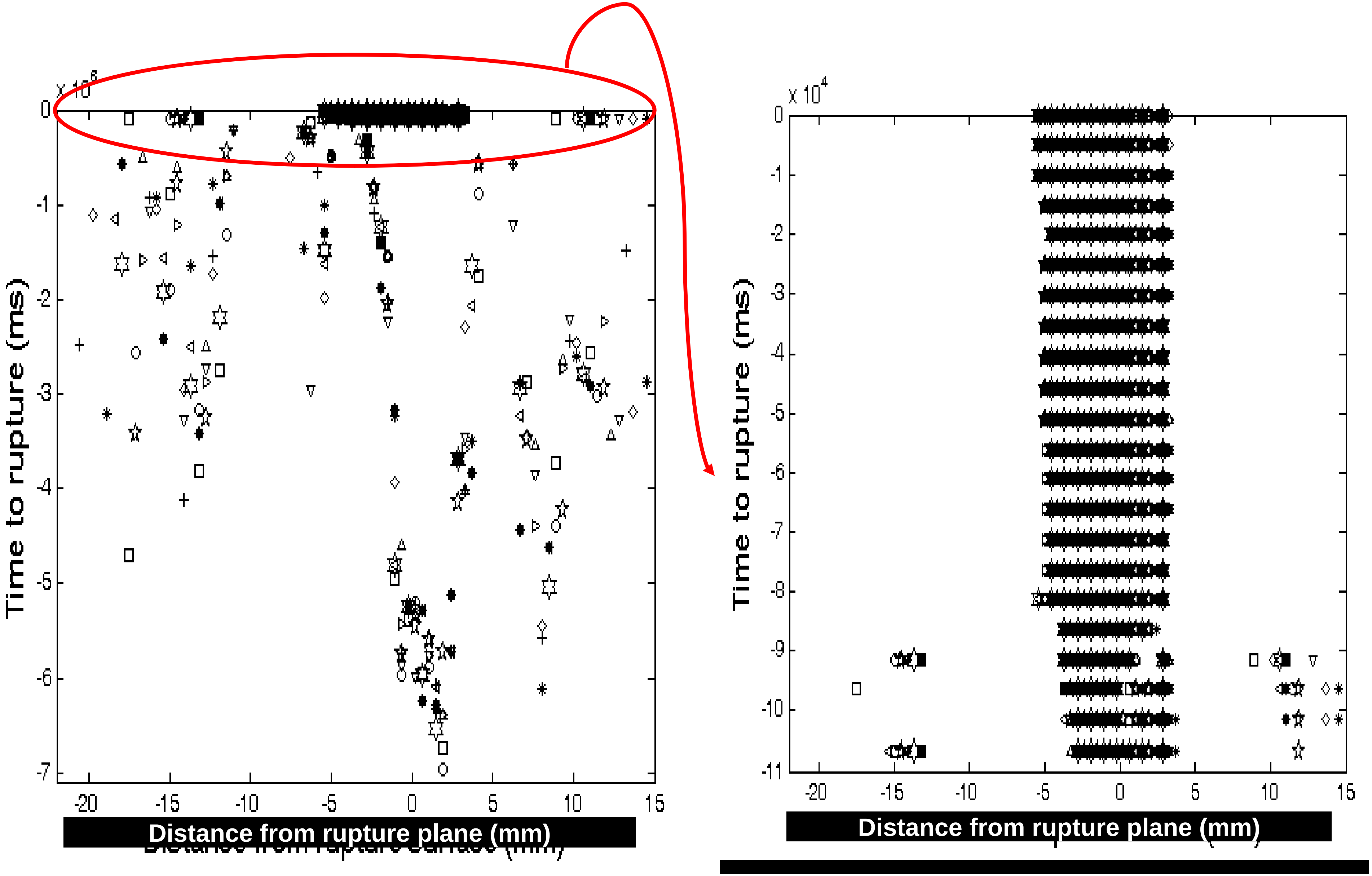


0.6
0.4
0.2
0
σ(ε)
0
0.5
1
1.5
ε

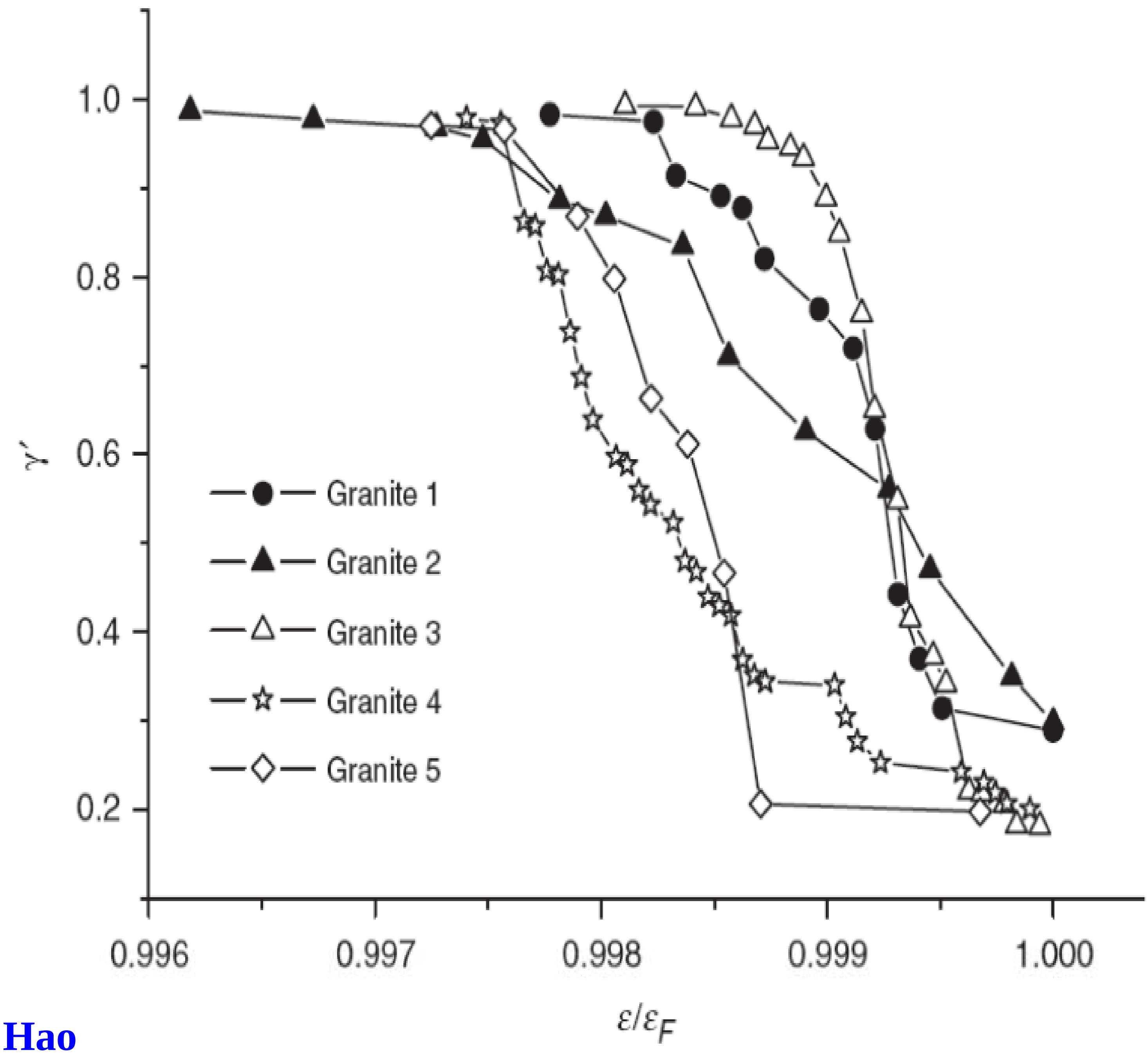
After Hao


x 10^-3
Strain fluctuation rate (1/s)
10
8
6
4
2
0
-2
-4
Time to rupture (ms)
-5
-4
-3
-2
-1
0
x 10^5
Distance from rupture plane(mm)
-10
-5
0
5
10
15

After Hao

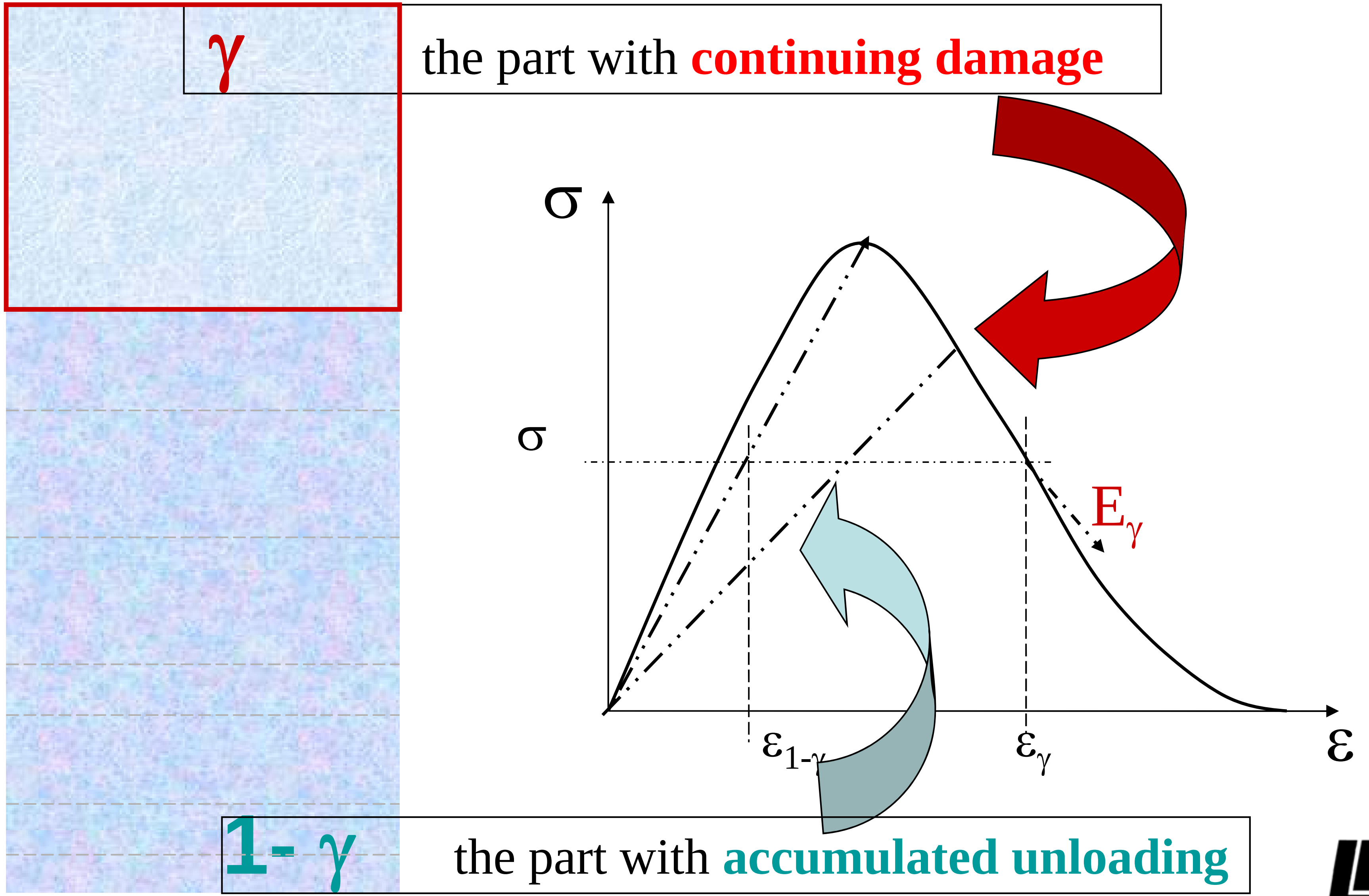

x 10^6
Time to rupture (ms)
Distance from rupture plane (mm)
x 10^4
Time to rupture (ms)
Distance from rupture plane (mm)

After Hao

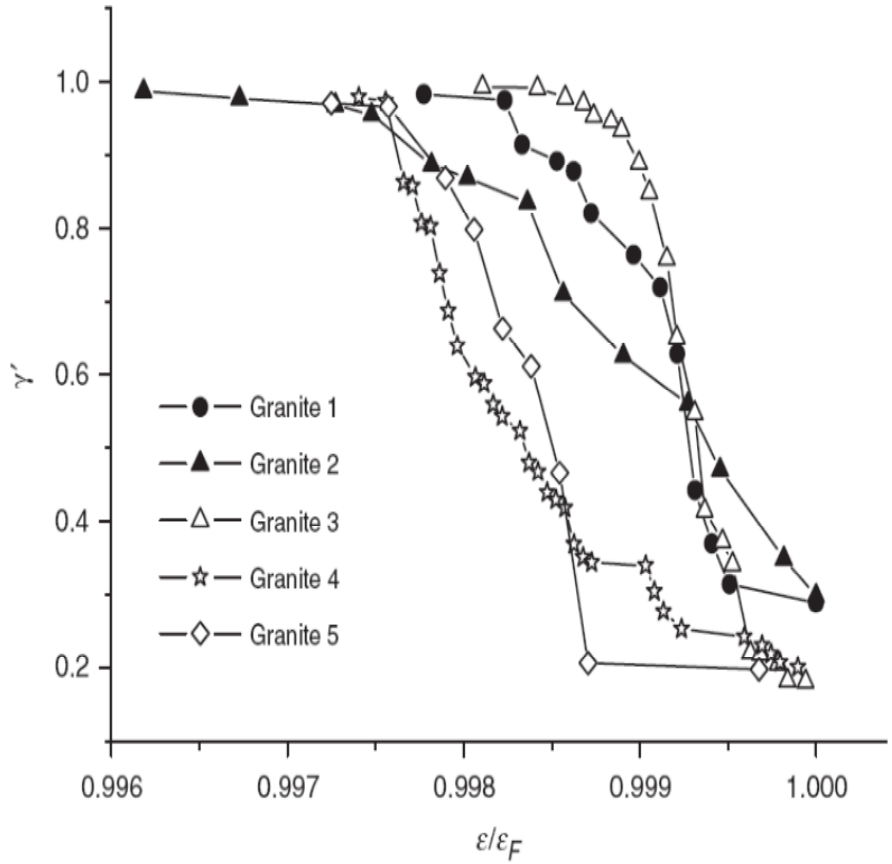

1.0
0.8
0.6
0.4
0.2
γ'
Granite 1
Granite 2
Granite 3
Granite 4
Granite 5
0.996
0.997
0.998
0.999
1.000
ε/ε_F

After Hao

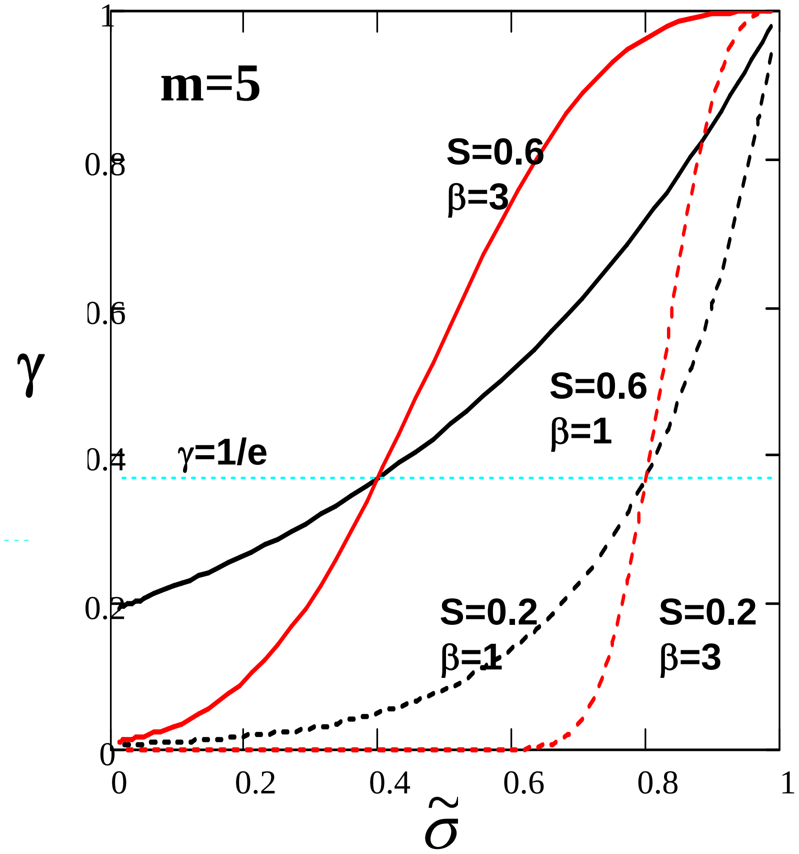
Damage Localization



Evolution of localized damage zone γ



Granite



$$\gamma(\tilde{\sigma}) = \exp[-(\frac{1-\tilde{\sigma}}{S})^{\beta}]$$

Continuous Bifurcation
Damage Localization

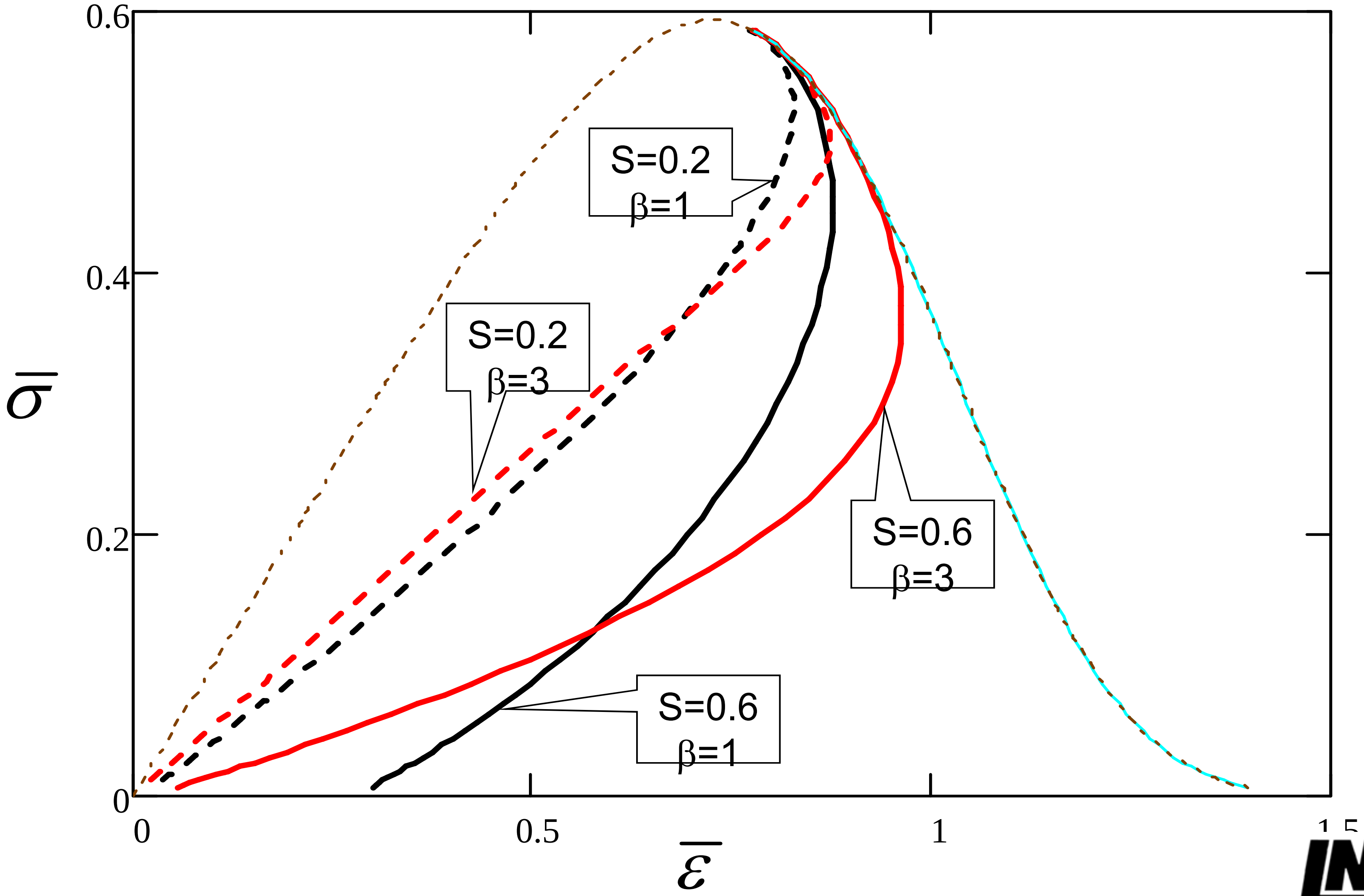
Any localized zone γ satisfies all conservation laws
(mass, force and energy)

σ= σ (ε, ∫γ)

$$\bar{\varepsilon}_{nominal}(\bar{\sigma}) = \frac{\bar{\sigma}}{\bar{E}_u(\bar{\sigma}_M)} - \bar{\sigma}\int_{\bar{\sigma}_M}^{\bar{\sigma}} \gamma(\bar{\sigma}_u)\frac{\bar{E}_u'(\bar{\sigma}_u)}{\bar{E}_u^{\,2}(\bar{\sigma}_u)} \cdot d\bar{\sigma}_u$$

LNM

Examples of σ (ε, ∫γ)



Catastrophic Rupture (CR)

With localization

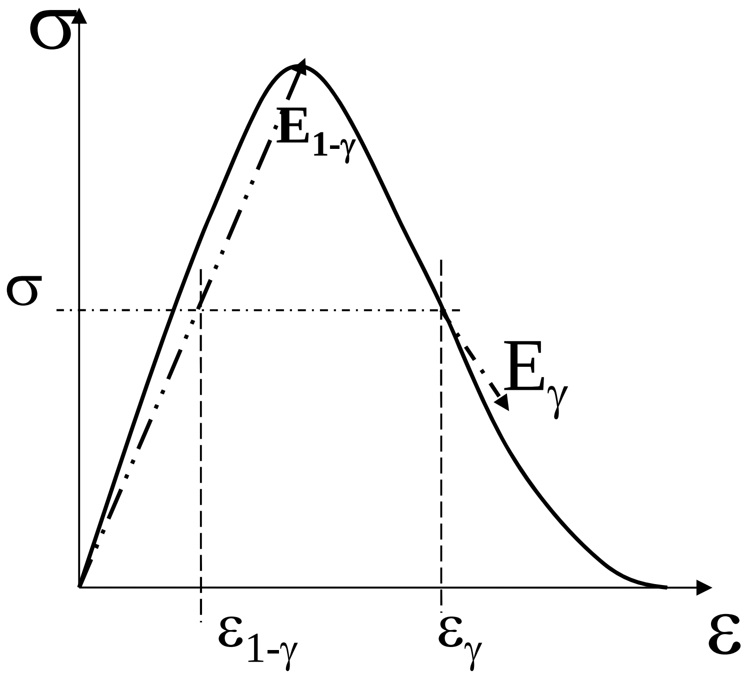
- Two modes:

$$d\bar{W} > 0 \quad GS$$

$$d\bar{W} \approx 0 \quad CR$$

$$\frac{\bar{\sigma}\bar{E}_u'(\bar{\sigma})}{\bar{E}_u^2(\bar{\sigma})}\gamma(\bar{\sigma}) - \int_{\bar{\sigma}}^{\bar{\sigma}_M} \frac{\bar{E}_u'(\bar{\sigma}_u)}{\bar{E}_u^2(\bar{\sigma}_u)}\gamma(\bar{\sigma}_u)\,d\bar{\sigma}_u \begin{matrix} > \\ \approx \end{matrix} \frac{1}{\bar{E}_u(\bar{\sigma}_M)}, \qquad \begin{matrix} GS \\ CR \end{matrix}$$

CR with simplified damage localization



Exp strain at CR	Cal strain at CR **(with Localized zone γ)**	Cal strain at CR (GMF)	Width of localized zone γ
0.00850	**0.00816**	0.00879	7.1 *mm*
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0.00645	**0.00674**	0.00810	6.7*mm*
0.00636	**0.00611**	0.00694	7.9 *mm*
0.00634	**0.00633**	0.00682	6.3 *mm*
0.00515	**0.00606**	0.00661	6.9 *mm*

±6%

after Hao et al 2007

LNM

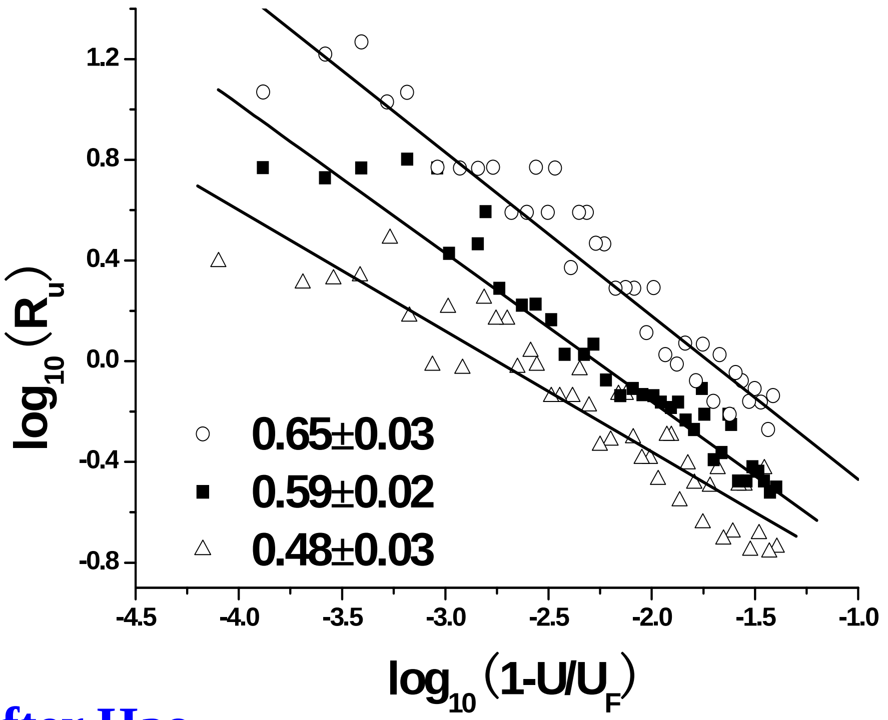
B-4. Power law singularity

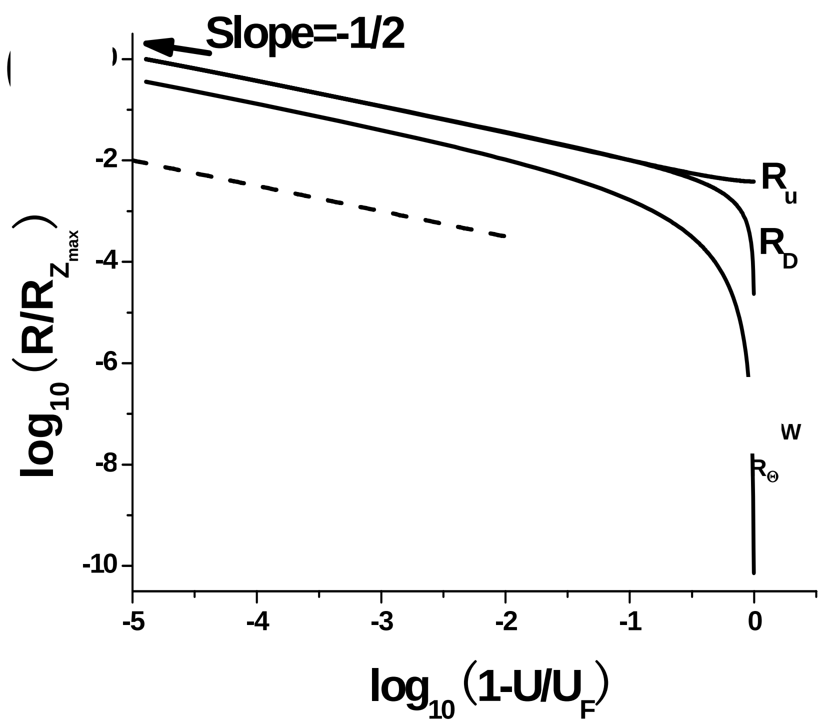
Critical point U_{CR} is sensitive to damage localization

But -1/2 power law singularity emerges ahead of CR,

no matter damage localization

$$R_u = \frac{\mathrm{d}u}{\mathrm{d}U} \propto (U_{CR} - U)^{-\frac{1}{2}}$$





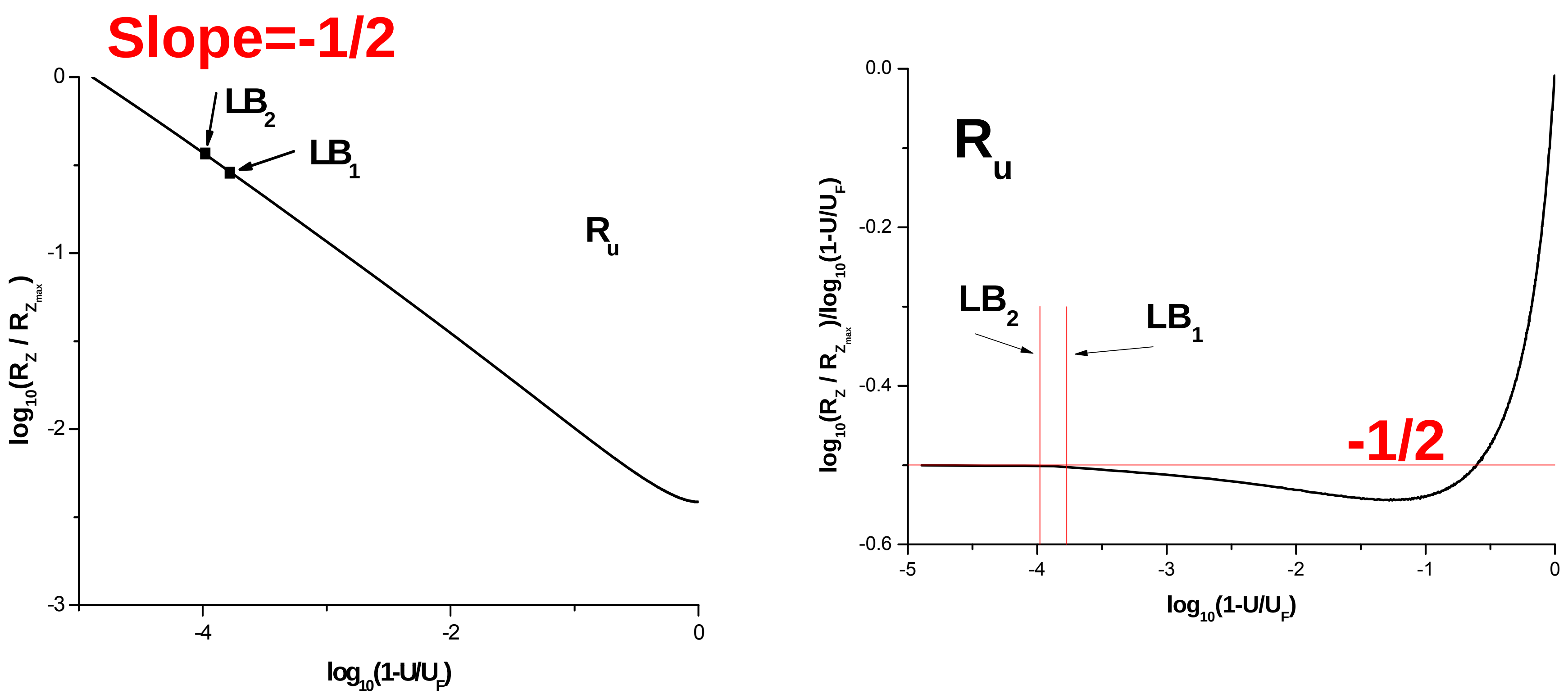
After Hao

Exp observations

ESB model

Power law singularity

$$R_u = \frac{\mathrm{d}u}{\mathrm{d}U} \propto \left(U_{\mathrm{CR}} - U\right)^{-\frac{1}{2}}$$



K=0.2, m=2 ESB model

LNM

Content

- 3 distinct features

continuous bifurcation,

damage localization

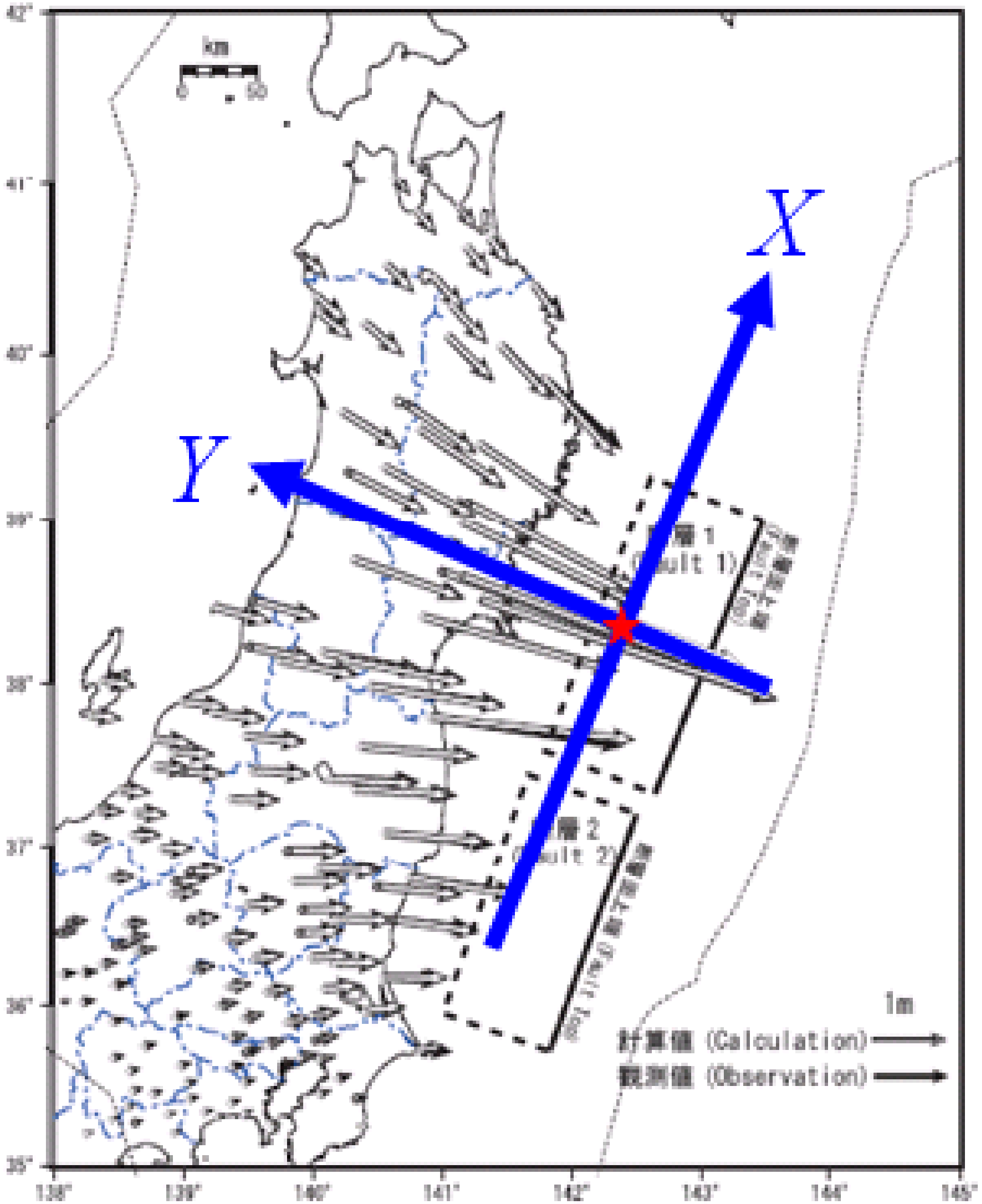
catastrophic rupture

- Preliminary applications

M9 earthquake, 2011, Japan

M8 earthquake, 2008, Wenchuan

Co-seismic $\Delta\varepsilon_Y$ M9 Japan



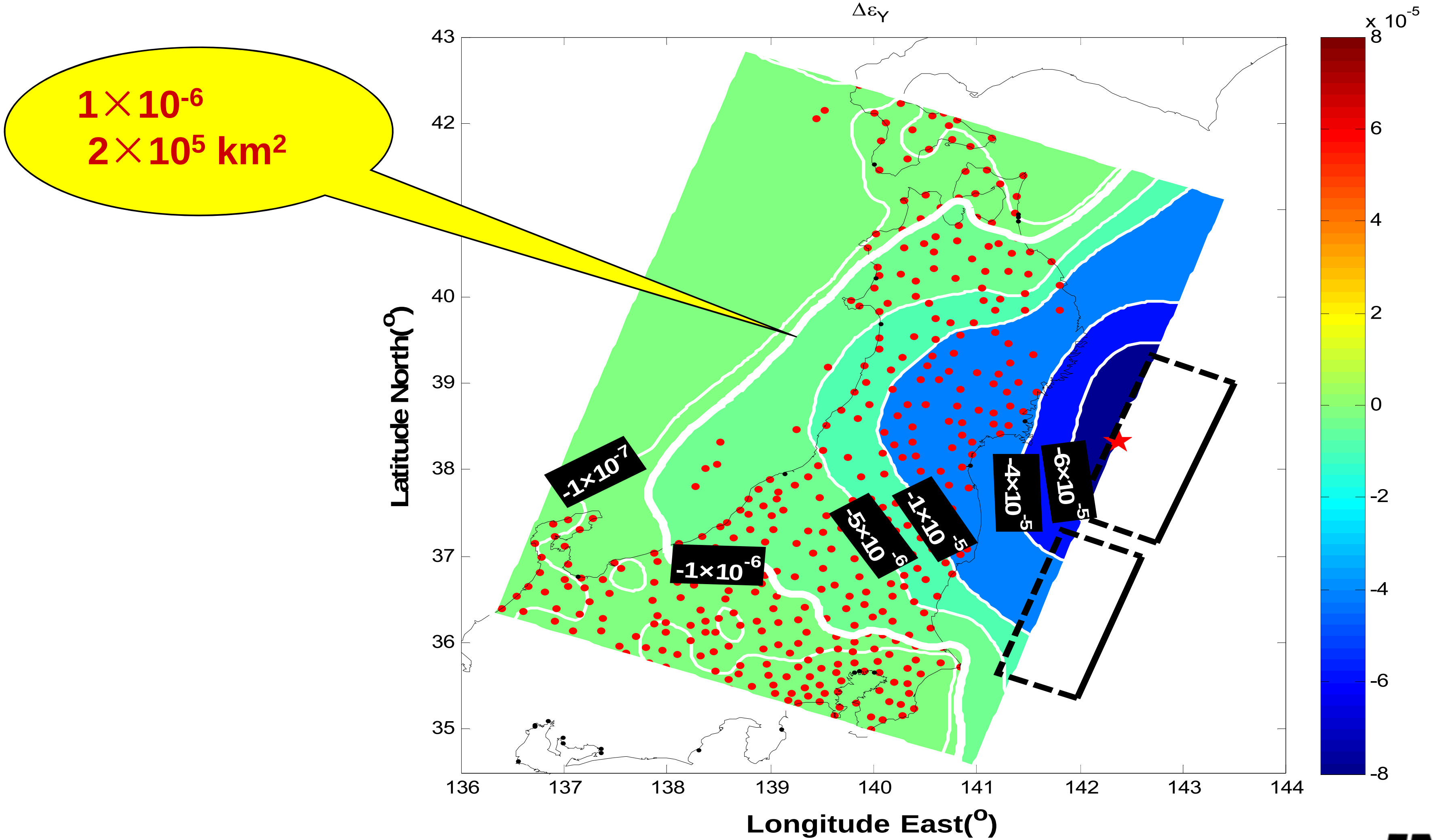
GPS
observations
M9 Japan
http://www.gsi.go.jp/

3 key parameters:

8×10^{18} J : **released energy**
2×10^{5} km^2 : **related area**
1×10^{-6} : **rebounded Δstrain**

Co-seismic incremental strain $\Delta\varepsilon_Y$

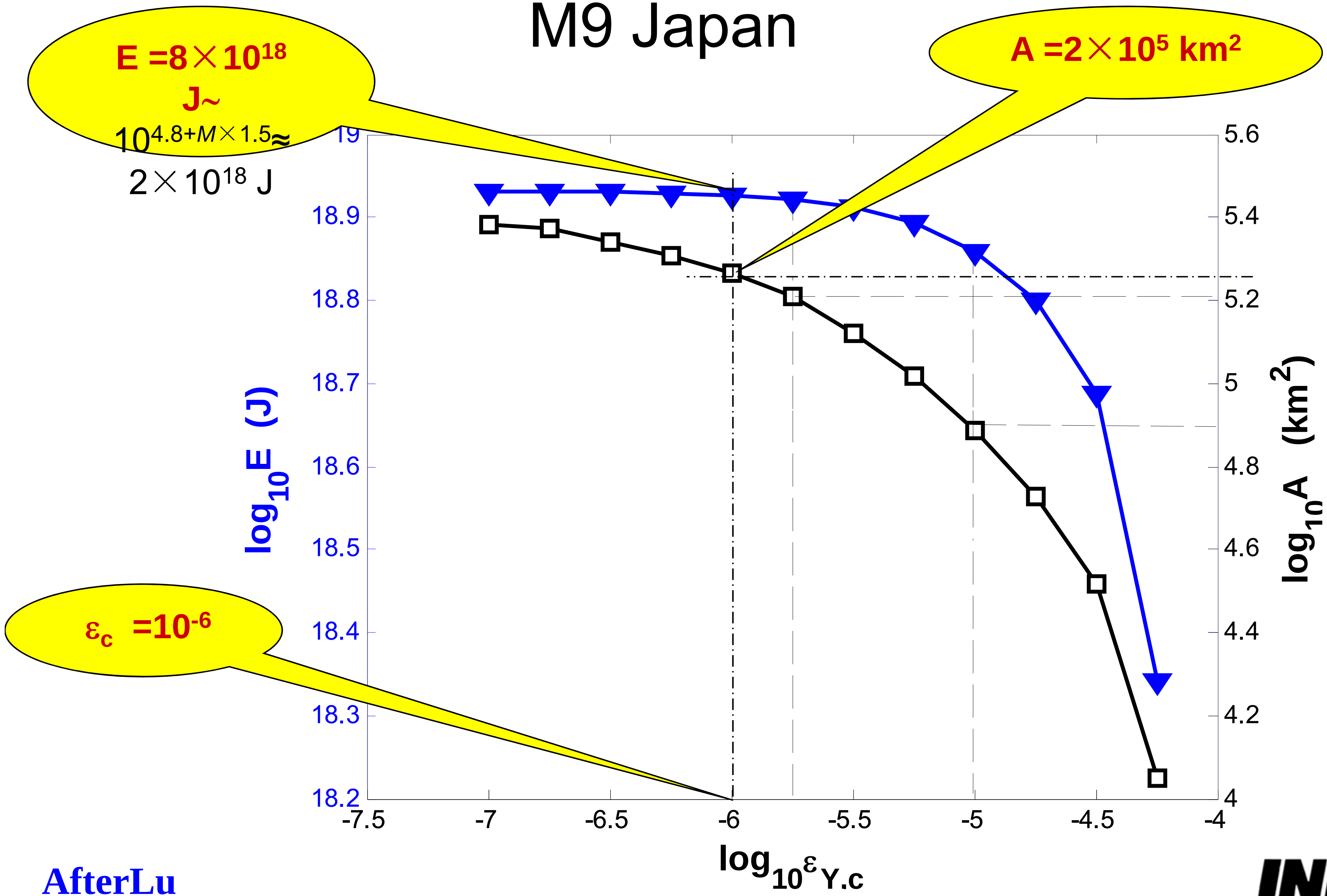
M9 Japan



Lu MF et al (2011) Sci China D

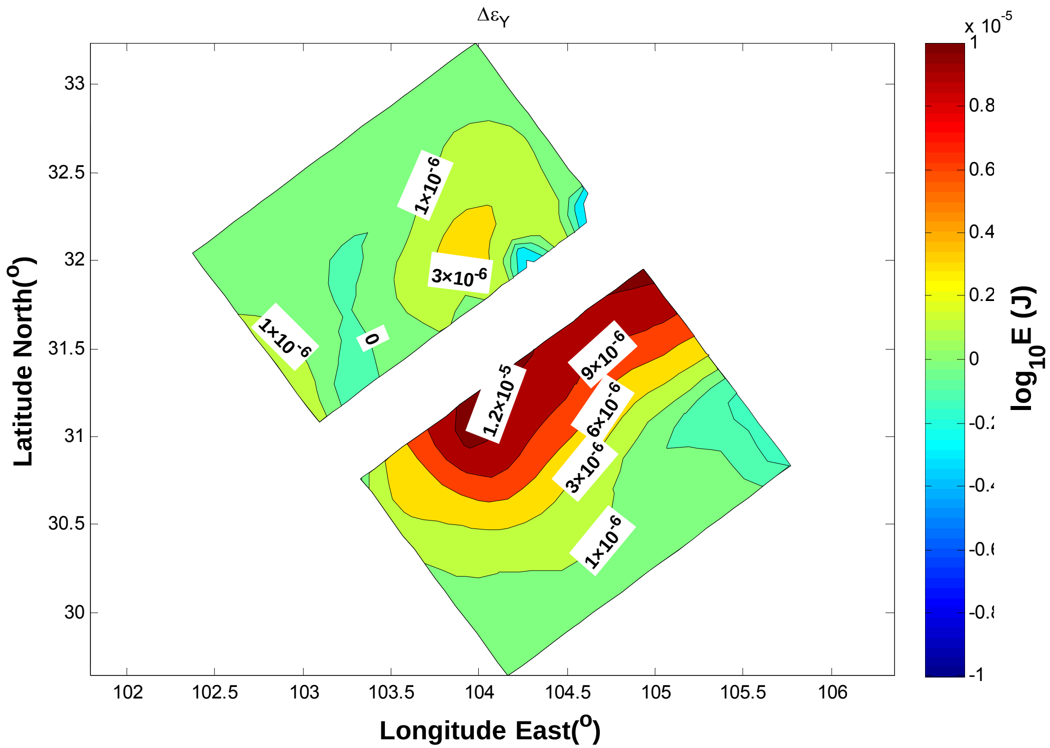
Relation btw E, A and ε_c

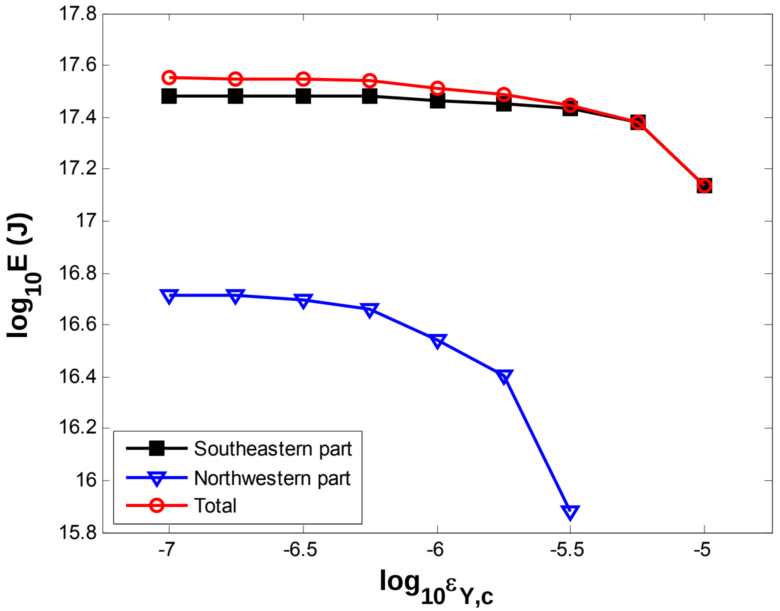
M9 Japan



AfterLu

Coseismic, M8 Wenchuan quake



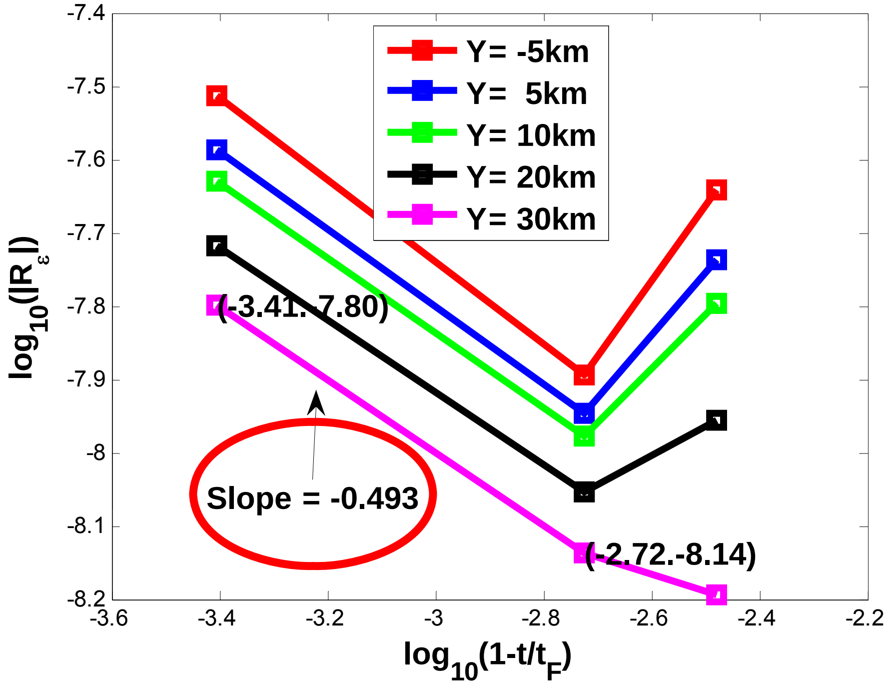


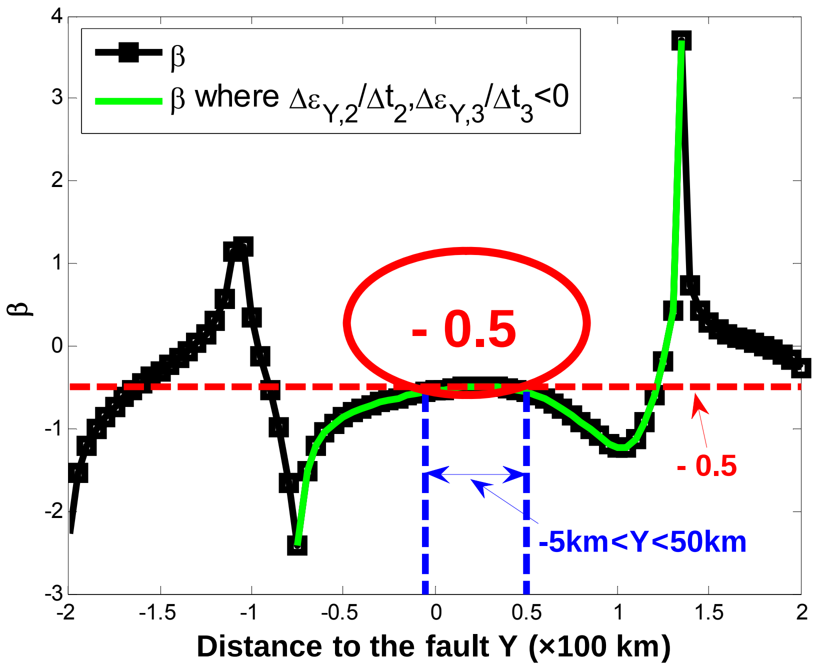
After Lu

Pre-seismic (2004.6~2007.6) of M8 Wenchuan quake(2008.5)

-1/2 power law emerges

ahead of quake in a region around quake rupture





AfterLu

Time

Space

Discussions

- **Heterogeneity** plays a key role in CR: $\mathbf{m_c}$
- Beyond Drucker's proposition, Continuous Bifurcation and Damage Localization **leads to "stochastic"** Catastrophic Rupture (CR)
- **-1/2 power law singularity emerges** before CR
- Application of the ideas to 2 quakes seems encouraging

LNM

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- HAO SW, et al, **Int J Rock Mechanics and Mining Sciences**, (accepted),


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Thanks
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