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Advances in the understanding of nanomaterial–biomembrane interactions and their mathematical and numerical modeling

The widespread application of nanomaterials (NMs), which has accompanied advances in nanotechnology, has increased their chances of entering an organism, for example, via the respiratory system, skin absorption or intravenous injection. Although accumulating experimental evidence has indicated the important role of NM–biomembrane interaction in these processes, the underlying mechanisms remain unclear. Computational techniques, as an alternative to experimental efforts, are effective tools to simulate complicated biological behaviors. Computer simulations can investigate NM–biomembrane interactions at the nanoscale, providing fundamental insights into dynamic processes that are challenging to experimental observation. This paper reviews the current understanding of NM–biomembrane interactions, and existing mathematical and numerical modeling methods. We highlight the advantages and limitations of each method, and also discuss the future perspectives in this field. Better understanding of NM–biomembrane interactions can benefit various fields, including nanomedicine and diagnosis.

KEYWORDS: biomembrane interaction mathematical modeling nanomaterials

With the rapid development of nanotechnologies, nanomaterials (NMs) have become part of our daily lives. They are used in cosmetics, textiles, bioimaging, medicine and diagnosis [1–8]. The widespread applications of NMs increase their chances of entering the human body via a number of pathways, such as the respiratory system [9], skin absorption [10,11], intravenous injection [12] and implantation [13]. As a result, there is an urgent need to understand the potential physiological and pathological reactions after exposure to NMs [14,15]. NMs can induce a series of complicated reactions, but they first need to cross a biomembrane barrier (e.g., skin, lung surfactant, intestinal barrier or cell membrane) to reach target cellular compartments [16]. Therefore, it is of great importance to investigate NM–biomembrane interactions.

Significant efforts have been devoted to experimentally investigating NM–biomembrane interactions. Many important findings have been obtained from these efforts. For instance, it has been established that NMs can induce specific interactions with targeted cell membranes via specific ligands [17], modify the biconcave shape of erythrocytes [18], change cell membrane asymmetry and thickness [19,20], damage the integrity of cells through hole formation [21] and induce oxidative stress [18]. In addition, NMs can be rapidly taken up by cells; a characteristic that has been applied for cancer gene therapy [22,23], cancer imaging and phototherapy [24], and drug delivery [25,26]. However, despite all of these important applications, the mechanisms underlying NM–biomembrane interactions are poorly understood, as experimental tests and observations at the nanoscale for complex biological systems remain technically challenging.

Alternatively, it has been demonstrated that mathematical and numerical modeling can simulate the NM–biomembrane interaction at the nanoscale, as well as conditions that are challenging to realize experimentally, providing deep insight into the dynamic process of the system [27–29]. Moreover, the capability of numerical simulations has been significantly enhanced with the increase in computing power in the last few decades. In this review, we present the state-of-the-art advances in the understanding of NM–biomembrane interactions and their mathematical and numerical modeling. The ‘Pathway of cellular uptake of NMs’ and ‘Key factors for NM–biomembrane interactions’ sections describe the basic cellular uptake pathways of NMs and key factors associated with NM–biomembrane interactions, respectively. The ‘Mathematical & numerical modeling approaches for NM–biomembrane interactions’ section reviews the existing mathematical and numerical modeling approaches for NM–biomembrane interactions and discusses the advantages and disadvantages associated with each method. Finally, the ‘Discussion’ and ‘Conclusion & future perspective’ sections highlight the limitations of the
current modeling methods and future research topics for NM–biomembrane interactions.

Pathways of cellular uptake of NMs
The cellular uptake of NMs usually consists of two processes, adsorption of NMs on the membrane, and internalization of NMs by endocytic and nonendocytic pathways (Figure 1) [16,30–33].

Endocytic pathway
Endocytosis is an important pathway for cellular uptake because the most important substances, such as proteins, which are biomacromolecules, cannot directly penetrate the cell membrane. Endocytosis can be divided into phagocytosis and nonphagocytosis; the latter includes macropinocytosis, clathrin-mediated endocytosis (CME), caveolae-mediated endocytosis (CvME), and other clathrin- and caveolae-independent endocytosis pathways [34]. CME is the predominant receptor-mediated endocytosis pathway for most cells, and involves membrane curvature generation, clathrin lattice formation and clathrin-coated vesicle formation. CvME forms flask-shaped invaginations in the membrane that allow molecules to enter the cells through ligand–receptor binding [35]. Other types of clathrin- and caveolae-independent endocytic pathway have been discovered aside from CME and CvME, such as lipid raft-mediated and ARF6-mediated GEEC pathways [36].

A variety of NMs (e.g., carbon nanotubes [CNTs], metal NMs and semiconductor NMs) have been demonstrated to enter into the cells via endocytosis [37–44]. For instance, both single-walled nanotubes (SWNTs) and SWNT conjugates (SWNT–streptavidin, protein–SWNT and DNA–SWNT) can enter human promyelocytic leukemia (HL60) cells via endocytosis [39,40]. Several factors affect the endocytosis process [45], such as NM size, membrane elasticity and receptor–ligand interactions. For instance, most nanosized synthetic NMs are internalized through endocytosis [35], and the optimal size of NMs for endocytosis is 25–30 nm [46,47]. An increased membrane elasticity can reduce the membrane bending energy, enhancing the ability of the membrane to wrap around NMs [48]. In addition, specific receptor–ligand interactions are involved in the endocytosis process (Figure 2), which is the most effective promotive force for endocytosis [45].

Nonendocytic pathways
Although endocytosis is the main pathway for cellular uptake of NMs, nonendocytic pathways exist, where NMs directly penetrate the cells [35]. For instance, NMs with sufficiently small diameters (e.g., metal clusters [49]) and needle-shaped NMs (e.g., single CNTs [50]) can directly penetrate the membrane. Cationic NMs (e.g., PAMAM dendrimers [51,52], Au–NH$_2$ and SiO$_2$–NH$_2$ [17]) can pass through the membrane by forming pores in the membrane. Large NMs can also penetrate into the cells when attached to cell-penetrating peptides (CPPs) [33,53]. CPPs

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**Figure 1. Cellular uptake pathways of nanomaterials.** The cellular uptake pathways of nanomaterials can be simply divided into two kinds, namely, endocytic and nonendocytic. The endocytic pathway includes phagocytosis, macropinocytosis, clathrin-dependent endocytosis, caveolae-dependent endocytosis, and other mechanisms of clathrin- and caveolae-independent endocytosis. The nonendocytic pathway includes direct penetration and other pathways, which are not yet fully understood.
can effectively deliver nanomedicine to the intercellular target by passing through the plasma membrane [54]. Studies have found that peptide-encapsulated fluorescent silver nanoclusters [49] and polyarginine-conjugated iron oxide NMs loaded with siRNA can directly penetrate cell membranes [55]. Similar to CPPs, several synthetic NMs, such as synthetic amphipathic phospholipid polymers [56] and nanoparticles (NPs) coated with hydrophilic–hydrophobic striated ligand shell [57,58] can also penetrate the membrane in a similar manner to the CPPs without disrupting the membrane.

Table 1 summarizes the cellular uptake pathways of different NMs. Although numerous studies have been performed, as previously mentioned, the entry mechanism of NMs through the membrane is far from clear. For instance, no precise definition exists for different entry pathways of NMs into cells. More work is needed to better understand the NM–biomembrane interactions.

**Key factors for NM–biomembrane interactions**

- **Physicochemical properties of NMs**
  Key physicochemical properties of NMs which affect NM–biomembrane interactions include NM size, shape, surface roughness and rigidity, charge, hydrophobic properties, and protein corona.

  NM size can determine the entry pathways of the NMs into the cells and affect the rates of cellular uptake. For instance, silica NMs with a size of less than 1.2 nm can penetrate into a dimyristoylphosphatidylcholine lipid bilayer without damaging the membrane, while pores form in the lipid membrane for NMs with a size ranging from 1.2 to 22 nm, and the membrane envelopes NMs with a size over 22 nm (Figure 3) [59,60]. The modes of endocytosis depend on the NM size, which is typically 100 nm for CME and 50–80 nm for CvME, respectively [34]. Moreover, studies have found that the cellular uptake rate for gold (Au)-NMs with sizes of approximately 50 nm is faster than that of other sizes [37,61].

  NM shape is another important factor that affects the cellular uptake process. For instance, the polystyrene NM shape determines the formation of the actin structures that initiate phagocytosis [62]. The endocytosis rate of spherical Au-NMs (74 and 14 nm in diameter) was higher than that of rod-shaped Au-NMs of the same size in one study. Furthermore, the endocytosis rate of rod-shaped Au-NMs (74 × 14 nm) with a low aspect ratio (1:3) was greater than the high-aspect-ratio (1:5) NMs [37]. However, in another study, the rate of cellular uptake of
Table 1. Entry pathways of different nanomaterials into cells.

<table>
<thead>
<tr>
<th>Study (year)</th>
<th>Nanomaterials</th>
<th>Properties</th>
<th>Size</th>
<th>Cell/biomembrane</th>
<th>Mechanism</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Kam et al. (2006)</td>
<td>CGMD; DPD</td>
<td>Protein–SWNT DNA–SWNT</td>
<td>Length: 200 nm</td>
<td>HL60 cells</td>
<td>Endocytosis</td>
<td>[40]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Length: 200 nm–2 μm Diameter: 3–15 nm</td>
<td>HeLa cells</td>
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<tr>
<td>Petri-Fink et al.</td>
<td>Metal NMs</td>
<td>Functionalized SPION</td>
<td>Diameter: 9 nm</td>
<td>Human melanoma cells</td>
<td>Endocytosis</td>
<td>[112]</td>
</tr>
<tr>
<td>(2005)</td>
<td></td>
<td></td>
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<tr>
<td>Chithrani and Chan</td>
<td>Metal NMs</td>
<td>Transferrin-coated spherical and</td>
<td>14, 50, 74 or 100 nm</td>
<td>Mouse embryonic fibroblast line (STO) HeLa cells</td>
<td>Endocytosis</td>
<td>[38]</td>
</tr>
<tr>
<td>(2007)</td>
<td></td>
<td>rod-shaped Au-NM</td>
<td></td>
<td>Human glioblastoma cell line (SNB19 cells)</td>
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<td>Zhang and</td>
<td>QDs</td>
<td>QD655 with a carboxylic acid</td>
<td>Ellipsoid core with diameters of 6 or 12 nm (minor or major axis, respectively)</td>
<td>Human epidermal keratinocytes</td>
<td>Endocytosis (lipid</td>
<td>[42]</td>
</tr>
<tr>
<td>Monteiro-Riviere</td>
<td></td>
<td>surface coating</td>
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<td>rafts)</td>
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<td>(2009)</td>
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<tr>
<td>Qaddoumi et al.</td>
<td>Polymer</td>
<td>PLGA</td>
<td>100 nm, 800 nm or 10 μm</td>
<td>Rabbit conjunctival epithelial cells</td>
<td>Endocytosis (CME)</td>
<td>[113]</td>
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<tr>
<td>(2004)</td>
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<tr>
<td>Harush-Frenkel et</td>
<td>Polymer</td>
<td>Polymers</td>
<td>73–97 nm</td>
<td>HeLa cells</td>
<td>Endocytosis (CME)</td>
<td>[114]</td>
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<td>al. (2007)</td>
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<tr>
<td>Dausend et al.</td>
<td>Polymer</td>
<td>Functionalized polystyrene NMs</td>
<td>Diameter: 113 or 121 nm</td>
<td>HeLa cells</td>
<td>Endocytosis</td>
<td>[115]</td>
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<tr>
<td>(2008)</td>
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<tr>
<td>Sahay et al.</td>
<td>Polymer</td>
<td>Polymeric micelles</td>
<td>110–200 nm</td>
<td>Epithelial cancer cells</td>
<td>Endocytosis (CvME)</td>
<td>[116]</td>
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<tr>
<td>(2010)</td>
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<tr>
<td>Huang et al.</td>
<td>SNTs</td>
<td>SNTs with different shapes</td>
<td>Length: 100, 240 or 450 nm</td>
<td>A357 human melanoma cells</td>
<td>Endocytosis</td>
<td>[117]</td>
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<tr>
<td>(2010)</td>
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<tr>
<td>Nan et al. (2008)</td>
<td>SNTs</td>
<td>Bare SNTs</td>
<td>Length: 200–500 nm</td>
<td>Epithelial breast cancer cell line, umbilical vein endothelial cell line</td>
<td>Endocytosis (CME)</td>
<td>[118]</td>
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<tr>
<td></td>
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<td>Positively charged aminosilane-functionalized SNT</td>
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<tr>
<td>Brown et al.</td>
<td>Carbon nanotubes</td>
<td>Fluorescently labeled ammonium-functionalized SWNT</td>
<td>Diameter: 20–100 nm</td>
<td>3T6 3T3</td>
<td>Nonendocytic (penetration)</td>
<td>[119]</td>
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<tr>
<td>(2007)</td>
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<tr>
<td>Yu et al. (2007)</td>
<td>Metal</td>
<td>Peptide-encapsulated fluorescent silver nanoclusters</td>
<td></td>
<td>NIH/3T3</td>
<td>Nonendocytic (penetration)</td>
<td>[49]</td>
</tr>
<tr>
<td>Veiseh et al.</td>
<td>Metal</td>
<td>Iron oxide NMs pArg-coated NMs</td>
<td>10–12 nm</td>
<td>Tumor cell lines of the brain, breast and prostate</td>
<td>Nonendocytic (penetration)</td>
<td>[55]</td>
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<tr>
<td>(2011)</td>
<td></td>
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<tr>
<td>Verma et al.</td>
<td>Metal</td>
<td>Monolayer-protected 66–34 OT Au-NMs</td>
<td>4–5 nm</td>
<td>Dendritic cells</td>
<td>Nonendocytic (penetration)</td>
<td>[57]</td>
</tr>
<tr>
<td>(2008)</td>
<td></td>
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<tr>
<td>Goda et al. (2010)</td>
<td>Polymer</td>
<td>Synthetic amphiphatic phospholipid polymers</td>
<td>9–12 nm</td>
<td>Human caucasian hepatocyte carcinoma (HepG2)</td>
<td>Nonendocytic (penetration)</td>
<td>[56]</td>
</tr>
</tbody>
</table>

CGMD: Coarse-grained molecular dynamic; CME: Clathrin-mediated endocytosis; CvME: Caveolae-mediated endocytosis; DMPC: Dimyristoylphosphatidylcholine; DPD: Dissipative particle dynamics; NM: Nanomaterial; OT: 1-octanethiol; PAMAM: Polyamidoamine; PLGA: Poly(lactic-co-glycolic acid); QD: Quantum dot; SNT: Silica nanotube; SPION: Superparamagnetic iron oxide nanoparticle; SWNT: Single-walled nanotube.
high-aspect-ratio cylindrical hydrogel particles (diameter: 150 nm; height: 450 nm) into HeLa cells was four-times faster than that of low-aspect-ratio cylindrical NMs (diameter: 200 nm; height: 200 nm) with a similar volume. One possible explanation is that the high-aspect-ratio cylindrical particles have larger surface areas to contact the membrane [63]. Numerical simulations have also revealed that the minimum driving forces for NMs to penetrate the membrane depend on the NM shape (e.g., sphere, ellipsoids, rods, discs and pushpin-like) [64]. Shi et al. simulated the entry mechanism of 1D CNTs with different tips (hemispherical cap, enlarged cap, CNT bundle tip and open tip) into cells and found that different tips resulted in different cell entry modes [65].

Surface roughness and rigidity of NMs also affect the NM–biomembrane interactions. For instance, silica NMs with a smooth surface (22 nm in diameter) were completely enveloped by the lipid membrane, whereas the bumpy NMs with the same size were only partially enveloped, as shown in Figure 3 [60]. In the study of Shi et al., soft NMs were more difficult to be fully wrapped by the membrane than rigid NMs, because a larger adhesion energy was needed for the former [66].

Electrostatic interactions and hydrophobic interactions are the two main nonspecific interaction types in NM–biomembrane interactions. Negatively charged cell plasma membrane results in a greater internalization potential of positively charged NMs relative to neutral and negatively charged NMs [41,67], because the electrostatic attraction promotes the adsorption of positively charged NMs on the membrane. Both experimental studies and computer simulations indicated that cationic Au-NMs induce more disruption of the cell membrane than anionic Au-NMs [68,69], as it depends on the charge density. For the hydrophobic interactions, a simulation study showed that hydrophobic NMs (diameter: 10 nm) could be embedded into the cell membrane, whereas semihydrophilic NMs only adhered to the membrane [70]. 11-mercapto-1-undecanesulfonate Au-NPs that only had hydrophilic sulfonate ligands were endocytosed by cells, whereas the 66–34 1-octanethiol Au-NPs (Au-NPs coated with a 2:1 molar mixture of 11-mercapto-1-undecanesulfonate and 1-octanethiol) with a hydrophilic–hydrophobic striated ligand shell directly penetrated the cell membrane owing to ligand rearrangement [57].

Before the NMs get close to the membrane surface, such as in the blood, plasma

<table>
<thead>
<tr>
<th>Study (year)</th>
<th>NMs</th>
<th>Properties</th>
<th>Cell/biomembrane</th>
<th>Mechanism</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roiter et al. (2008)</td>
<td>Silica NMs</td>
<td>1.2–22 nm</td>
<td>DMPC-supported lipid membrane</td>
<td>Nonendocytic (pore formation)</td>
<td>[59]</td>
</tr>
<tr>
<td>Leroueil et al. (2008)</td>
<td>Silica NMs</td>
<td>5–6 nm</td>
<td>DMPC-supported lipid membrane</td>
<td>Nonendocytic (pore formation)</td>
<td>[59]</td>
</tr>
<tr>
<td>Hong et al. (2006 and 2004)</td>
<td>Dendrimers</td>
<td>10–100 nm</td>
<td>DMPC-supported lipid membrane</td>
<td>Nonendocytic (pore formation)</td>
<td>[51,52]</td>
</tr>
<tr>
<td>Matsuzaki et al. (1998)</td>
<td>Peptide</td>
<td>Magainin 2</td>
<td>DMPC-supported lipid membrane</td>
<td>Nonendocytic (pore formation)</td>
<td>[120]</td>
</tr>
</tbody>
</table>

CGMMD: Coarse-grained molecular dynamic; CME, Clathrin-mediated endocytosis; CME, Caveolae-mediated endocytosis; DMPC, Dimyristoylphosphatidylcholine; DPD, Dissipative particle dynamics; NM, Nanomaterial; OT: 1-octanethiol; PAMAM: Polyamidoamine; PLGA, Poly(lactic-co-glycolic acid); QD, Quantum dot; SNT, Silica nanotubes; SPION, Superparamagnetic iron oxide nanoparticle; SWNT, Single-walled nanotube.
and interstitial fluid, where there are a lot of proteins, the protein corona of NMs will be formed, which may affect the cellular uptake of NMs [45,71]. For instance, polymeric carrier NMs can be bound by opsonin proteins in the blood serum, recognized and phagocytozed by macrophages and then deliver drugs to their destination [72]. Researchers also use the protein corona of NMs to target specific cell types, such as apolipoproteins–quantum dots, which complex with murine macrophages, human dermal fibroblasts and primary murine glial cells [73]. In some cases, the formation of protein corona of NMs can also reduce cellular uptake. For instance, serum proteins adsorbed on CNPs can reduce the cellular uptake of HeLa cells [74].

Properties of the biomembrane
Phospholipids are the main lipids in the cellular membrane and can be divided into phosphatidylcholine, phosphatidylserine and phosphatidylethanolamine depending on their hydrophilic head groups. Phosphatidylcholine and phosphatidylethanolamine are neutrally charged, while phosphatidylserine is negatively charged. In general, the cell membrane is electronegative and will repel negatively charged and attract positively charged NMs [75]. The membrane structure (e.g., phase transition and heterogeneity of the membrane) is also a key factor. For instance, the polycationic polymer NMs disrupt the lipid membrane in a membrane structure-dependent manner, with only the liquid–crystalline phase found to induce disruption [76]. With proteins, sugar and cholesterol on it, the membrane always presents heterogeneity. The interaction of NPs and the heterogeneous membrane with an adhesive raft on the membrane was simulated in one study, where the raft drove the entry process of membrane-wrapped NPs into the vesicle [77].

The properties of biomembranes are significantly dependent on environmental factors, such as temperature, pH and salt concentration [78–80]. For instance, the membrane fluidity decreases and changes from the liquid–crystalline to gel phase with decreasing temperature, which inhibits NM–biomembrane interactions [76,80]. The environmental pH and salt concentration can alter the mechanical and electrical properties of the membrane. The compressive modulus of the PC membrane was reduced by 30% at pH 2 compared with at pH 3 or 9 [78]. Therefore, a decrease in membrane elasticity can reduce the endocytic ability of the cell. As for salt concentration, NaCl and CaCl$_2$ can significantly increase the order of lipid molecules, leading to reduced membrane elasticity [79]. The strong interactions between PC membranes and Na$^+$ ions caused notable membrane compression, whereas the interaction of K$^+$ ions with the membrane was much weaker [81].

**Mathematical & numerical modeling approaches for NM–biomembrane interactions**
Computer simulation can describe the trajectories of molecules and the dynamic process of the NM–biomembrane interactions, providing deep insights into the understanding of NM–biomembrane interactions at the nanoscale. Two types of dynamic models exist in the computer simulation model of NM–biomembrane interactions, namely the atomistic and coarse-grained models. The main difference between the two models is the simulation units that are adopted. Atomistic models employ
simulation units of a single atom, providing a detailed description of atom–atom interactions, whereas the simulation units of coarse-grained models include a cluster of atoms, molecules or chemical groups. This section reviews the recent advances in these modeling methods, and focuses on the advantages and disadvantages of each method (Table 2).

**Molecular dynamics method**

The molecular dynamics (MD) method is based on the classical Newton's law of motion. A trajectory that describes the change in an atom's position with time can be obtained by solving the differential equations of Newton's second law, with the potential energy of the atoms represented by the force field. The force field contains bond interactions that represent both the chemical bond (i.e., bond stretching, angle bending and torsion angle potentials) and non-bonded interactions (i.e., Van der Waals and electrostatic interactions). The potential energy of the system may be given as follows:

\[
V(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N) = \sum_{i=1}^{N} \Phi(\mathbf{r}_i)
\]

where \( \mathbf{r}_i \) is the location of atom \( i \) and \( \Phi(\mathbf{r}_i) \) is the interaction potential between atom \( i \) and atom \( j \). The control equations are:

\[
F_i = -\nabla \Phi(\mathbf{r}_i)
\]

\[
\mathbf{m}_i \ddot{\mathbf{r}}_i = \mathbf{F}_i = \sum_{j \neq i} \mathbf{F}_{ij}
\]

Numerous studies involving the MD method have been performed to determine the interactions between the biomembrane and C\textsubscript{60} NMs with different properties (i.e., morphology and concentration). The permeability coefficient had a decreasing trend as follows: nanoC\textsubscript{60}, C\textsubscript{60} and open-C\textsubscript{60} [82]. The interactions of nine C\textsubscript{60} molecules with the lipid bilayer were simulated to investigate the effect of a C\textsubscript{60} concentrated solution. The results show that C\textsubscript{60} molecules preferred to spread homogeneously near the membrane center when inserted into the membrane [83]. The preferred position, which minimized the potential of mean force, of a C\textsubscript{60} inside a liquid crystalline lipid bilayer was approximately 6–7 Å from the center of the bilayer [84]. For the more ordered gel-phase lipid bilayers, the position was approximately 18 Å from the bilayer center, because of the energy penalty [85]. The permeability of C\textsubscript{60} through the lipid bilayer correlated with its dispersion interaction with the surrounding medium [86]. The MD method was also used to investigate small molecules (e.g., benzocaine, ethanol, urea and caffeine) that penetrate across the lipid bilayer [75,87,88].

The MD method is capable of capturing the details of atomic interactions. With the MD method, researchers can investigate structure dynamics changes, both for NMs and lipid molecules (e.g., the interactions between atoms in individual lipid molecules and different atom sizes of G3-PAMAM dendrimer) [89]. The atomic interactions can help us to understand interactions from the microscopic point of view, which is challenging for experimental observation. However, with present computing power, the MD method is limited by the small length and time scale (nanometer and nanosecond); hence, it is hard to simulate the endocytosis processes and large NM–biomembrane systems, which contain thousands of lipid molecules (Table 2), using this method.

**Coarse-grained MD method**

Compared with the MD method, the atoms are grouped together into clusters as simulation units in the coarse-grained MD (CGMD) method which will reduce computation consumption greatly. The MARTINI force field developed by Marrink et al. is a typical coarse-grained force field which is in agreement with experimental results [90]. It is widely used in biosystem simulation.

The CGMD method has been widely employed to simulate the effects of NM properties, environments and membrane properties on NM–biomembrane interactions. For example, Gu et al. simulated the effect of the hydrophobic property of NMs and the effect of charges on NM–biomembrane systems [70,91]. Hydrophobic interaction and electrostatic attraction improved the interaction process. Energy-independent endocytosis of NMs with varying shapes and ligand coverage (Figure 4) was simulated by Vácha et al. who indicated that endocytosis efficiency was higher for spherocylindrical NMs than for spheres [92]. As for the effect of pH, PAMAM dendrimers can be absorbed onto the negatively charged membrane at a low pH and can even induce hole formation in the membrane when the corresponding protonation levels are employed in the simulation to mimic the different pH conditions (e.g., pH 10, 7 and 5) [93,94]. Aside from pH, the formation of holes induced by Au-NMs and PAMAM dendrimers was also
Table 2. Comparison of different mathematical modeling methods for nanomaterial–cell interaction.

<table>
<thead>
<tr>
<th>Study (year)</th>
<th>NMs</th>
<th>Interactions</th>
<th>Force field and package</th>
<th>Molecular number (lipid/water)†</th>
<th>Time/length scale</th>
<th>Characteristics</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MD</strong></td>
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<tr>
<td>Chang and Lee (2010)</td>
<td>Fullerenes: $C_{60}$</td>
<td>Inside or outside the membrane</td>
<td>DL-POLY package</td>
<td>64/2000</td>
<td>Several nanoseconds</td>
<td>Microlevel</td>
<td>[83]</td>
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<tr>
<td>Bedrov et al. (2008)</td>
<td>Fullerenes: $C_{60}$</td>
<td>Passive transport</td>
<td>CHARMM27 Lucretius package</td>
<td>52/1800</td>
<td>Several nanoseconds</td>
<td>Microlevel</td>
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<tr>
<td>Li et al. (2007)</td>
<td>Fullerenes: $C_{60}$</td>
<td>Inside the DMPC membrane and the alkane melt</td>
<td>CHARMM27 Lucretius package</td>
<td>64/1600</td>
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<td>Microlevel</td>
<td>[84]</td>
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<td>Fiedler and Violi (2010)</td>
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<td>NM morphology and passive permeability rate</td>
<td>UA-OPLS-based parameterization [122,123] GROMACS package</td>
<td>48/1372</td>
<td>Several nanoseconds</td>
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<td>MacCallum and Tieleman (2005)</td>
<td>Hexane</td>
<td>Interacts with the DOPC membrane</td>
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<td>Localization within the asymmetric biomembrane</td>
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<td>Several nanoseconds</td>
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<td>PAMAM</td>
<td>Binds on the DMPC lipid bilayer</td>
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<td>512 lipid</td>
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<td>Microlevel</td>
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<td>Lin et al. (2010 and 2011)</td>
<td>Au-NM</td>
<td>Penetration, nanoscale hole formation</td>
<td>MARTINI GROMACS package</td>
<td>1152/50,300; 1152/60,000</td>
<td>Micro- to milli-seconds</td>
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<td>Micro- to milli-seconds</td>
<td>Mesoscopic level</td>
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</tbody>
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†Forward slashes separate lipid number and water number and semicolons separate different reference.

CGMD: Coarse-grained molecular dynamic; CNT: Carbon nanotube; DMPC: Dimyristoylphosphatidylcholine; DOPC: Dioleoylphosphatidylcholine; DPD: Dissipative particle dynamics; DPPC: Dipalmitoylphosphatidylcholine; MD: Molecular dynamics; NM: Nanomaterial; PAMAM: Polyamidoamine.
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<td>Peptide</td>
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<td>Micro- to milli-seconds</td>
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<tr>
<td>Smith et al. (2007)</td>
<td>NM: vesicles</td>
<td>Synthetic vesicles engulf NMs</td>
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<td>NM: ligands; shapes</td>
<td>Membrane response, endocytosis</td>
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<td>Yan and Yu (2009)</td>
<td>PAMAM</td>
<td>Permeability of charged NMs across tensed membrane</td>
<td>–</td>
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</tr>
<tr>
<td>Yan and Yu (2011)</td>
<td>PAMAM</td>
<td>Dendrimer and vesicle complex</td>
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<td>[103]</td>
</tr>
<tr>
<td>Höfinger et al. (2011)</td>
<td>CNT</td>
<td>Insertion of CNTs</td>
<td>CULGi V4.0.0 program</td>
<td>–</td>
<td>Micro- to milli-seconds</td>
<td>Mesoscopic level</td>
<td>[130]</td>
</tr>
<tr>
<td>Shi et al. (2011)</td>
<td>CNT</td>
<td>Enters into the DPPC membrane through the tip first</td>
<td>ESPResSo package</td>
<td>–</td>
<td>Micro- to milli-seconds</td>
<td>Mesoscopic level</td>
<td>[65]</td>
</tr>
</tbody>
</table>

†Forward slashes separate lipid number and water number and semicolons separate different references. CGMD: Coarse-grained molecular dynamic; CNT: Carbon nanotube; DMPC: Dimyristoylphosphatidylcholine; DOPC: Dioleoylphosphatidylcholine; DPD: Dissipative particle dynamics; DPPC: Dipalmitoylphosphatidylcholine; MD: Molecular dynamics; NM: Nanomaterial; PAMAM: Polyamidoamine.
affected by surface charge signs and density of NMs, membrane surface tension and temperature [69,95,96].

Compared with the MD method, the CGMD method has a longer time (millisecond) and length scale (micrometer), as shown in Table 2. Therefore, more experimental phenomena can be simulated by this method, such as the whole dynamic process of direct penetration and endocytosis. For instance, Figure 5 illustrates a single fullerene or a cluster of ten fullerenes penetrating the dioleoylphosphatidylcholine membrane [97]. The results are consistent with the MD simulations [83], in which the interactions were divided into two segments: the fullerenes lying inside or outside of the membrane. The permeation process of fullerene clusters into the lipid bilayer occurs on a microsecond timescale, which is too large for MD simulations to capture. Although the CGMD method loses the atomic details, acceptable simulation results can be achieved because the atom description has limited influence on the mesoscopic system [27].

### Dissipative particle dynamics method

The dissipative particle dynamics (DPD) method is also a coarse-grained model where individual molecules or atoms are lumped together into a ‘bead’. The motion of the bead obeys Newton’s equation, but each DPD bead is subjected to three types of forces, namely, conservative–repulsive forces, dissipative forces and random forces [98]. The dissipative forces and random forces are coupled through the fluctuation–dissipation theorem, and the system momentum is locally conserved.

Many complicated NM–biomembrane systems can be simulated using the DPD method. For ligand-coated NMs, a study has designed novel NMs with ligand-decorated surfaces through dynamic bonds and simulated its penetrating process through the membrane using the kinetic theory [99]:

\[
A + B \rightleftharpoons \kappa_{off} \rightarrow AB
\]

where \(A\) represents NMs, \(B\) represents free ligands, and \(AB\) stands for the NM–ligand...
complex. The bond is broken when the bond length between the NMs and the ligand head is longer than its initial length. The bond forms again if the length is shorter than its initial length. The study provided a new way of translocating NMs and designing ligand-coated NMs. Another study investigated the effect of ligand properties (e.g., ligand density, length and rigidity) in receptor-mediated endocytosis, and found that the engulfment degree (partial or full engulfment) was strongly affected by ligand properties [100].

Coulombic interactions are needed to simulate the interaction of charged NMs with the biomembrane. Groot proposed a method based on the local lattice grid to calculate the electrostatic force [101]. Based on this method, many studies have investigated the interaction between the charged dendrimer and the membranes of different structures, such as the vesicle membrane [102,103]. Figure 6 shows snapshots of hole formation induced by the charged dendrimers in the tensed bilayer membrane. The figure shows that the permeability of the...
charged dendrimers could be enhanced in the tensed membrane.

To simulate different membrane structures or surface tension, the bilayer membranes can be stretched by changing the lateral dimensions of the simulation box, in other words:

\[(x, y, z) \rightarrow \left(\frac{\partial x}{\partial t}, \frac{\partial y}{\partial t}, \frac{\partial z}{\partial t}\right)\]

to form pores or change surface tension in the membrane [102,104]. The N-varied DPD simulation with an adjusted bead number can be applied to release the tension induced by the membrane deformation, as well as to preset membrane tension and simulate the effect on the cytoskeleton and actin. This method has been employed to investigate the membrane response to the adsorption of ligand-coated NMs [105] and the endocytosis kinetics of ligand-coated NMs with different shapes [106].

Similar to the CGMD method, the DPD method has a time (millisecond) and length (micrometer) scale that allows simulation of the whole direct penetration process, the endocytosis process and other more complicated systems, such as NM–heterogeneous membrane interactions [77]. Furthermore, the DPD method allows larger time steps than the CGMD method, which greatly reduces computational efforts. In addition, the DPD method is able to describe the hydrodynamic effects because the momentum involved is conserved [107]. In addition, as with the CGMD method, the DPD method loses atomic details that cannot be used for structure analysis in the microscopic scale.

**Other methods**

Aside from the three methods stated above, other methods have also been used to simulate NM–biomembrane interactions, such as the Brownian dynamics method, the Monte Carlo method, thermodynamic models and mathematical models.

The Brownian dynamics method is based on the Langevin equation, where explicit solvent molecules are replaced by stochastic forces [27]. This method can simulate the phenomena with longer length (micrometer) and time scales (seconds) than the CGMD and DPD methods. For
instance, the Brownian dynamics method was used to simulate the interactions between the lipid bilayer vesicles and adhesive NMs in which the NMs could induce vesicle budding, connection and fission [108]. The Monte Carlo method relies on repeated random sampling to obtain numerical results and it is used to model phenomena with significant uncertainty regarding inputs. For example, the Monte Carlo method was used to model NM surface rearrangements, which is important in nonspecific interactions [109]. In addition, a thermodynamic model has also been developed to investigate the condition in which the NMs could be adsorbed onto or disrupt the membrane. In this model, only the thermodynamic aspect information can be obtained without the dynamic process [110]. In contrast to molecular modeling simulations, Lunov et al. proposed a mathematic model to investigate the receptor-mediated endocytosis of engineered NPs by macrophages, which showed key parameters of this process (e.g., rate of uptake, the number of NPs per cell in saturation and the mean uptake time) [111]. They presented the understanding of NM–cell interactions in a more macroscopic way than the dynamic method.

Discussion
Despite extensive research on the mathematical and numerical simulations of NM–biomembrane interactions, a significant discrepancy exists between the experimental observation and simulation. For instance, in a nanotube–mouse liver cell interaction experiment, the cells were exposed to CNTs for 2 h before observation [65]. However, in the simulation, the CNT–lipid bilayer interaction lasted for a few microseconds. The reason for this discrepancy is that it takes a long time for the CNTs to get close to the membrane surface, while we placed the CNTs on the lipid bilayer surface in the simulation. In addition, in the process of NMs getting close to the membrane surface, the protein corona and environment can also affect the NMs’ properties, as discussed in the ‘Key factors for NM–biomembrane interactions’ section. It is challenging to observe the dynamic process of NM–biomembrane interactions in the appropriate time (microseconds) and length (several nanometers) scales. The invention of real-time monitoring microscopy with high resolution will greatly improve experimental studies. Meanwhile, for the simulation, it is difficult to take these factors into account because the simulation system within recent calculation capacity is only a few nanometers cubed, and only the lipid molecules, simplified proteins and solvents are simulated. Therefore, specific assumptions of the models are not realistic and further improvements are needed. Figure 7 shows the length and time scale for experiments and simulations. Table 2 summarizes the advantages and disadvantages of the main existing dynamic methods for simulating NM–cell interactions.

Conclusion & future perspective
This article mainly reviews the cellular uptake pathways of NMs, the key factors of NM–biomembrane interactions, and the mathematical and numerical methods for NM–biomembrane interactions. The advantages and limitations for each method in terms of time and length scales were highlighted, and which proper models can be chosen to simulate different phenomena were determined. Despite the extensive achievements in this important and fast-growing field, many challenges remain. The major topics for further studies are, therefore, identified as follows:

- The membrane structure is complex in vivo because numerous proteins, sugars and cholesterol are attached to the membrane. However, the existing simulations only give a simple description of the lipid membrane skeleton, which consists of lipid molecules and simplified proteins. Simulation of more realistic lipid membranes to interact with NMs is needed.

Figure 7. The length and time scale for experiments and simulations.
CGMD: Coarse-grained molecular dynamics; DPD: Dissipative particle dynamics; MD: Molecular dynamics.
The microenvironments for various cells are different, which affects the NM–biomembrane interactions. More work is needed to match the physiological situation more closely;

- The potential functions that describe the forces between simulation units are not precise. Searching for more precise and simpler potential functions is also a challenge for simulating NM–biomembrane simulations;

- Improvement of the computer calculation capacity is needed. Simulation of large and complicated NM–biomembrane systems with a long simulated time that matches the real experiment system more closely is another challenge.

### References

Papers of special note have been highlighted as:

- of interest


Advances in the understanding of nanomaterial–biomembrane interactions

27 Shilloch JC. Insight or illusion? Seeing inside the cell with mesoscopic simulations. HFSP J. 2(1), 1–6 (2008).


*Used the dissipative particle dynamics method to investigate nanoparticles with different shapes interacting with the lipid bilayer. Provides a practical guide to geometry considerations when designing nanoscale cargo carriers through simulation.*


*Investigates cell entry of 1D nanomaterials using the coarse-grained molecular dynamics method, which is in agreement with the experimental results of the coarse-grained molecular dynamic method in nanomaterial–biomembrane interaction simulation.*


Advances in the understanding of nanomaterial–biomembrane interactions


