

Characteristic Scales in Supersonic Combustion

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The characteristic length scales in supersonic combustion and the related influence factors were calculated and analyzed. In addition, using the hydrogen, ethylene and kerosene as fuels, the operating range of scramjet under the practical flight conditions indicated by the typical flight map was calculated. At last, the applicability of flamelet models for the numerical simulation of supersonic combustion was discussed.

Nomenclature

L	flow characteristic length scale
l_0	integral turbulence scale
l_λ	Taylor micro-scale
l_K	Kolmogorov scale
ρ	density
μ	dynamic viscosity
\bar{v}	supersonic inflow velocity
I	turbulence intensity
v'	eddy turnover velocity
S_L	laminar flame velocity
δ_L	laminar flame thickness
T_u	mixture temperature before burning
T_b	mixture temperature after burning
\bar{T}	averaged temperature
α	thermal diffusivity
Re	Reynolds number based upon the flow length scales
Re_{l_0}	turbulent Reynolds number based upon the integral turbulence scale
Da	Damkohler number
Φ	equivalence ratio
H	flight height
M_0	flight Mach number
Q_0	flight dynamic pressure
T_t	total temperature

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M_4	Mach number at the combustion chamber entrance
T_4	static temperature at the combustion chamber entrance
P_4	static pressure at the combustion chamber entrance

I. Introduction

SCRAMJET engines are promising candidates for future air-breathing systems because of high performance at large Mach number. Usually air entering the combustor must be supersonic at flight speeds beyond Mach 5, so the residence time of the air in a scramjet engine is on the order of 1 ms¹. The fuel must be injected, mixed with the air, and burned within such a short time. Therefore the interaction among shock wave, turbulence and chemical reaction²⁻⁴ makes the flow to be a complicated multi-scale phenomenon characterized by a wide range of length and time scales.

Prior work³⁻⁸ has already indicated that supersonic internal flows are characterized mainly by streamwise vorticity and thus by maximum helicity ($H = \vec{\omega} \cdot \vec{v}$). In the low speed incompressible flow, the Kolmogorov scale divides the flow into two ranges⁹: the inertial subrange and the viscous subrange. As to the supersonic internal flows, turbulent kinetic energy (TKE) decay is significantly affected by baroclinic and dilatational effects except for vortex stretching term due to high helicity, resulting in a larger scale of dissipative eddies compared to the Kolmogorov scale¹⁰⁻¹².

In supersonic combustion process, mixing is achieved by turbulent fluctuation and molecular diffusion. The gas mixture is compressed by the shock wave, and the temperature increases after combustion. Wherein, the flame surface is stretched and wrinkled by turbulent eddies, increasing the area of reaction sheets and the flame speed. Ingenito¹² indicated that the compressibility enhance the chemical reaction rate by a factor $1+2Ma_s^2$; accordingly, flame speed is increased by the factor $\sqrt{1+2Ma_s^2}$ and flame thickness is reduced by the same factor.

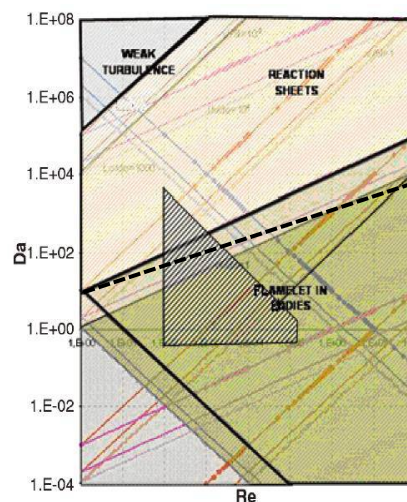


Figure 1. Most likely Supersonic combustion (SC) regimes by Ingenito¹²

As a summary, it can be expected that the ratio of the dissipative scale in turbulence to the flame thickness will increase in supersonic combustion with respect to subsonic combustion. In other words, the $l_K / \delta_L = 1.0$ line in flame regime will shift downward, as shown qualitatively in Figure 1 by the black dashed line. Therefore, it can be concluded that there are some new and different features in

supersonic combustion compared to low speed incompressible flow and combustion. Using some non-dimensional numbers based on the characteristic sales, Ingenito¹² estimated roughly the most likely supersonic combustion regimes shown in Figure 1. In order to understand supersonic combustion better and to derive and choose suitable turbulent combustion models describing turbulence chemistry interactions (TCI), the present work calculated and analyzed the characteristic scales including Kolmogorov scale, Taylor Scale and flame thickness in supersonic combustion and some non-dimensional numbers in details. The operating range of different fuels, such as hydrogen, ethylene and kerosene¹³ (chemical surrogated by a mixture of n-decane 80% and 1,2,4-trimethylbenzene 20% by weight), in a practical scramjet flight map¹⁴ was calculated. Based on the calculations, the applicability of flamelet models¹⁵ in the numerical simulation of supersonic combustion was also discussed.

II. Combustion Regimes

Balakrishnan and Williams¹⁶ declared the turbulent Reynolds number (Re_{l_0}) and Damkohler number (Da) are the most significant parameters for defining the supersonic combustion regime. The turbulent Reynolds number is defined:

$$Re_{l_0} = \frac{\rho v' l_0}{\mu} \quad (1)$$

where ρ is the density, v' is the eddy turnover velocity, l_0 is the integral turbulence scale, μ is the dynamic viscosity. The Damkohler number is the ratio of turbulence time scale (t_{flow}) to chemical time scale (t_{chem}):

$$Da = \frac{t_{flow}}{t_{chem}} \quad (2)$$

The other three important non-dimensional parameters used in combustion regimes are l_k / δ_L , l_0 / δ_L , and v' / S_L , where l_k is the dissipative scale (or called Kolmogorov scale), and δ_L is the laminar flame thickness. There is another important length scale in turbulence flow, the Taylor micro-scale (l_λ), which lies between the integral turbulence scale and the dissipative scale. The relationships between these turbulence scales are^{1,9}:

$$l_0 / l_k = Re_{l_0}^{3/4} \quad (3)$$

$$l_0 / l_\lambda = Re_{l_0}^{1/2} \quad (4)$$

Defining a reference kinematic viscosity $\nu = S_L \delta_L$, the turbulent Reynolds number and Damkohler number are:

$$Re_{l_0} = \frac{v' l_0}{S_L \delta_L} \quad (5)$$

$$Da = \frac{t_{flow}}{t_{chem}} = \frac{l_0 / v'}{\delta_L / S_L} = \left(\frac{l_0}{\delta_L} \right) \left(\frac{S_L}{v'} \right) \quad (6)$$

From the above Equation (5-6), the relation between Re_{l_0} and Da can be written as:

$$\frac{v'}{S_L} = Re_{l_0} \left(\frac{l_0}{\delta_L} \right)^{-1} = \frac{1}{Da} \left(\frac{l_0}{\delta_L} \right) \quad (7)$$

Plotting the logarithm of Da vs the logarithm of Re_{l_0} yields the Balakrishnan-Williams diagram¹⁶ for premixed combustion (see Figure 2). The lines $l_k/\delta_L=1$, $l_\lambda/\delta_L=1$ (equivalent to $Da=1$), and $l_0/\delta_L=1$ shown in Figure 2 define four different reaction regimes:

Regime A. Reaction sheets ($l_k/\delta_L > 1$). Above the unity Kolmogorov to chemical length scale ratio line, all turbulence length scales are larger than the chemical length scale, resulting in the reaction sheets regime. In this region, the turbulent eddies cannot enter the reaction zone, but only wrinkle it. What's more, the turbulent eddies can enter the preheat zone and cause weak TCIs. The flame still retains laminar like structure and $Da \gg 1$, which mean that kinetics are very fast compared with transport, and the assumption of fast chemistry (and that of a global mechanism) may be justified.

Regime B. Flamelet in eddies ($l_k/\delta_L < 1$ and $l_0/\delta_L > 1$). The region between the $l_k/\delta_L=1$ and $l_0/\delta_L=1$ lines can be described as the flamelet in eddies regime. In this region, the reaction zone may be wrapped around the large eddies, $v'/S_L > 1.0$, and the turnover velocity of the large eddies is faster than that of the flame front: large eddies can convolute the flame front enough to form multiple reaction sheets. What's more, the smallest eddies can enter into the flame, which means the flame structure may be perturbed by the turbulent fluctuations, such as thickening its structure. Hence the TCIs are limited in this regime. This region is further divided into 2 zones, and the region between $l_k/\delta_L=1$ and $Da=1$ lines is called broken flamelet regime. From the $l_k/\delta_L=1$ line, the degree to which turbulence structures can enter the reaction zone increases, until the $l_0/\delta_L=1$ line is reached.

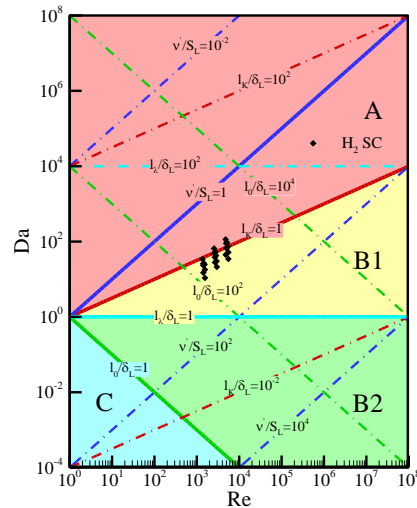


Figure 2. Regime diagram: Log-log plot of Damkohler number versus turbulent Reynolds number

Regime C. Distributed reactions ($l_0/\delta_L < 1$). All turbulence length scales are smaller than the chemical length scale beyond the $l_0/\delta_L=1$ line, causing distributed reactions to occur. In this region, the turbulent eddies can penetrate into both the preheat and the reaction zone. Due to this, TCIs are very predominant and laminar like structure can't be identified in this region.

Most of the practical applications fall under the reaction sheets or the flamelet in eddies regimes. Hence, most of the turbulence chemistry models are developed for this region. However, in a combustor, all the regions can simultaneously present at different locations and an ideal model should be the one that has applicability in all the regimes. Ingenito and Bruno¹² estimated the turbulent large-scale Reynolds number (2000~100000) and turbulent large-scale Damkohler number (9~2000)

from a large eddy simulation of the SCHOLAR scramjet test case using Hydrogen/air 1-step chemistry at flight Mach 7~9. And Figure 1 places the most likely supersonic combustion regimes proposed by Ingenito and Bruno¹² on the diagram described. Their studies suggested the broken flamelets regime is the most likely in supersonic combustion.

III. Calculation Conditions and Methods

The flight map of a hypersonic vehicle under the practical flight conditions was generated by Gruber¹⁴ using both the Ramjet Performance Analysis (RJPA) code's and the direct-connect test facility. The range of flight conditions ($15\text{km} < H < 35\text{km}$, $3.5 < M_0 < 7.0$, and $23.9\text{kPa} < Q_0 < 95.8\text{kPa}$) is presented in Figure 3. Yang and Yuan¹⁷⁻¹⁸ investigated the flow and combustion characteristics of supercritical kerosene jets transversely injected into Mach 2 and 3 crossflows using high speed pulse Schlieren system, and the combustor with a cavity they used can stabilize the flame well. So the flow length scale is set as the height of combustor, $L = 60\text{mm}$, in the present work. According to the results of experiments (see Figure 4), the integral turbulence scale which is equal to scale of the largest eddy in the turbulence flow is assumed to be $l_0 = 10\text{mm}$. The dynamic viscosity μ is calculated according to the NASA CEA database¹⁹⁻²⁰. The laminar flame velocity S_L is calculated from the CHEMKIN software using GRI3.0 mechanism²¹ (for H_2). Then the laminar flame thickness δ_L can be obtained from the thermal diffusivity α ⁹:

$$\delta_L = 2\alpha / S_L \quad (8)$$

where α is taken to be the data of air at $\bar{T} = 0.5(T_u + T_b)$ with pressure correction (the thermal diffusivity inversely proportional to the pressure).

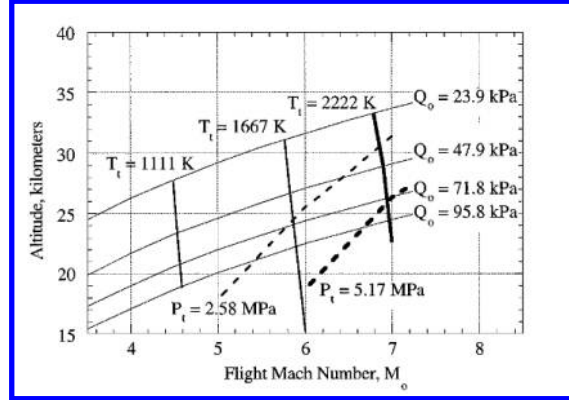


Figure 3. Flight map of a scramjet by Gruber¹⁴

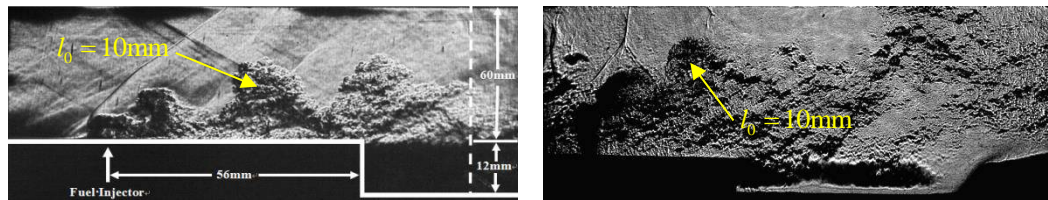


Figure 4. High speed schlieren image of supercritical kerosene jet in supersonic cross-flow (JISF) by Yang and Yuan¹⁷⁻¹⁸ (Left: No combustion; Right: Combustion)

The turbulence length scales is calculated according to equations (1)(3)(4), where turbulent fluctuation velocity is calculated from the supersonic inflow velocity and turbulence intensity as:

$$\mathbf{v}' = \mathbf{I} \cdot \bar{\mathbf{v}} \quad (9)$$

where the turbulence intensity is estimated from the fully developed turbulent flow as:

$$I = 0.16 \text{Re}^{-1/8} \quad (10)$$

where Re is the Reynolds number of supersonic inflow based upon flow length scales L .

IV. Results and Discussion

A. Characteristic Scales in Supersonic Combustion

Figure 5 shows the characteristic length scales in supersonic combustion of Air/H₂ when the equivalence ratio is fixed at $\Phi = 1.0$ under the practical flight conditions indicated by the flight map (Figure 3). The δ_L is larger than the l_k while smaller than the l_λ . The δ_L is close to the l_λ when the temperature and pressure of supersonic inflow are low, which means the reaction zone is mainly enclosed with the medium scale eddies. At the same time, the smallest eddies can grow and break in the reaction zone and the flame structure may be disturbed by the turbulent fluctuations, such as thickening its structure. With the increase of pressure and temperature, the δ_L approach to l_k gradually. At this point, the breaking of turbulent eddies and the chemical reactions occur in almost the same length scale, which means turbulent fluctuation will promote the chemical reactions and make the combustion in scramjet to transform to be fast chemical reactions.

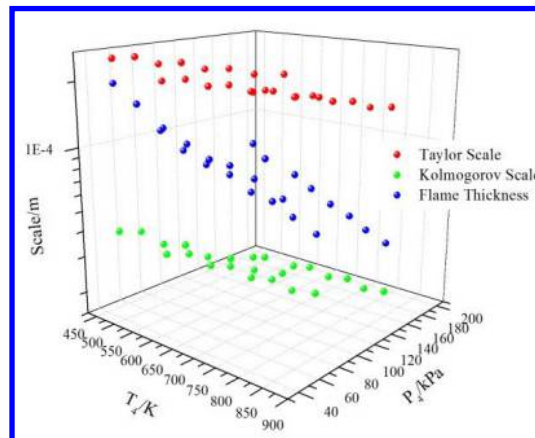


Figure 5. Characteristic scales in supersonic combustion in flight map (H₂)

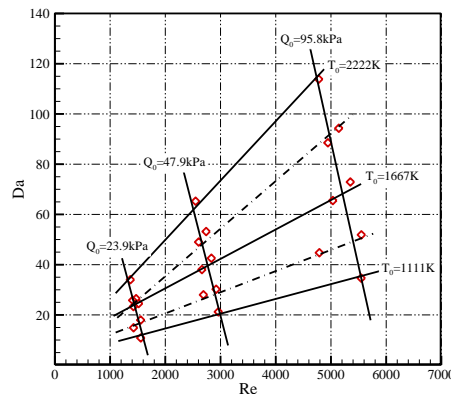


Figure 6. Re-Da relation in flight map (H₂)

The black diamond points in Figure 2 show the distribution of Re-Da relation in combustion regime for H_2 , and Figure 6 gives the details. The results show that the chemical reaction in a scramjet mainly takes place in B zone, namely flamelet in eddies regime, using H_2 as fuel. The combustion may take place in reaction sheets regime under certain conditions, such as increasing total temperature or dynamic pressure. Increasing the dynamic pressure will make the Re-Da curve to shift the right side, which is corresponding to the decrease of the Kolmogorov scale. However, increasing the total temperature will make the Re-Da curve translate upwards, which is corresponding to the decrease of flame thickness.

B. Effect of the Inflow Temperature

Figure 7 shows the influence of static temperature T_4 (400 ~ 900K) to the characteristic scales of Air/ H_2 when the equivalence ratio is fixed at $\Phi = 1.0$ and the flight Mach number $M_0 = 6.0$, dynamic pressure $Q_0 = 95.8\text{kPa}$, which is corresponding to the $M_4 = 2.6$, $P_4 = 133.4\text{kPa}$ at the combustion chamber entrance.

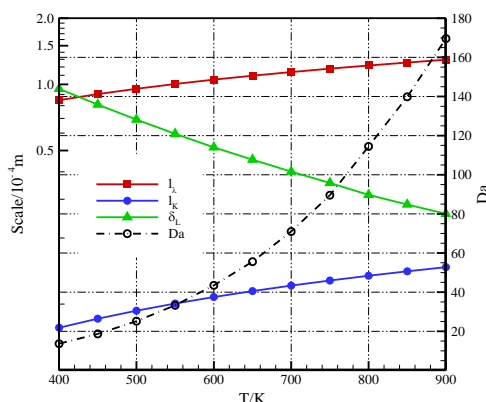


Figure 7. Influence of T_4 to the characteristic length scales

It is shown by Figure 7 that the l_λ and l_k enlarge, while the δ_L reduces with the increase of the static temperature T_4 . The δ_L is large, even larger than l_λ , when the inflow temperature is relatively low ($T_4 < 700\text{K}$). At this time, the medium scale eddies play an important role in combustion. The unburned mixtures spread to the reaction zone not only by the simple diffusion (mass diffusion), but also through the turbulent eddies' rewinding. What's more, the turbulent eddies take the combustion products and the heat away quickly, resulting in a relative slow chemistry. The combustion regime is the flamelet in eddies regime, which is to say that the combustion mainly takes place in B zone. Increasing the static temperature until $T_4 > 700\text{K}$, the δ_L reduces significantly and is close to the l_k gradually, when the smallest eddies begin to play key roles. At this point, the generating and breaking out of the turbulent eddies and the chemical reactions occur in almost the same length scale, which improve the mixing and combustion to a large extent and result in a relative fast chemistry. The combustion comes to take place in A zone.

C. Effect of the Inflow Pressure

Figure 8 shows the influence of static pressure P_4 (35 ~ 260kPa) to the characteristic scales of

Air/H₂ when the equivalence ratio is fixed at $\Phi = 1.0$ and the flight Mach number $M_0 = 6.0$, total temperature $T_t = 1667\text{K}$, which is corresponding to the $M_4 = 2.6$, $T_4 = 715\text{K}$ at the combustion chamber entrance.

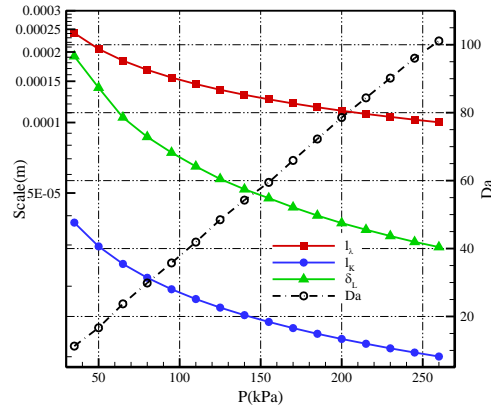


Figure 8. Influence of P_4 to the characteristic length scales

The influence of static pressure P_4 to the characteristic length scales is very similar to that of static temperature. When the inflow pressure is relative low ($P_4 < 150\text{kPa}$), the δ_L is closer to the l_λ , which means that medium scale eddies are dominant in the combustion. The chemistry is relative slow and mainly takes place in B zone. With the increase of the static pressure P_4 ($P_4 > 150\text{kPa}$), both the medium scale eddies and the smallest eddies affect the laminar flame thickness and the combustion rate at the same time, resulting in a relative fast chemistry. The combustion starts to take place in A zone gradually. What's more, it's worth noting that Da increases with the pressure linearly.

D. Effect of the Inflow Mach Number

Figure 9 shows the influence of M_4 ($0.1 \sim 3.0$) to the characteristic scales of Air/H₂ when the equivalence ratio is fixed at $\Phi = 1.0$ and the flight Mach number $M_0 = 3.5$, total temperature $T_t = 1111\text{K}$, which is corresponding to the $P_4 = 132.4\text{kPa}$, $T_4 = 468\text{K}$ at the combustion chamber entrance.

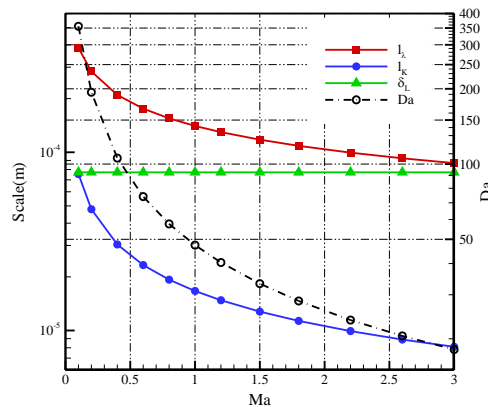


Figure 9. Influence of M_4 to the characteristic scales

It is shown by Figure 9 that the laminar flame thickness hardly changes with M_4 . When $M_4 < 0.4$, the δ_L is close to the l_k , which means the smallest eddies play a key role in combustion. At this point, the mixtures burn well and the chemistry is relative fast and the combustion mainly takes place in A zone, which correspond to subsonic combustion. But it's should be noticed that the Da decrease from 400 to about 100 rapidly with the increase of M_4 . When $0.4 < M_4 < 0.6$, both the medium scale eddies and the smallest eddies affect the laminar flame thickness and the combustion rate at the same time, and the combustion still takes place in A zone. When $M_4 > 0.6$, the effects of medium scale eddies to the combustion are more significant and the combustion starts to take place in B zone. When $M_4 > 1.0$, the combustion in a scramjet is controlled almost entirely by the medium scale eddies, and the chemical reaction rate is further reduced, resulting in a slow chemistry.

Based on the requirements of fast and efficient fuel-air mixing, as well as the need to sustain and stabilize a flame, the injector designs in the scramjet are required to generate good penetration of the fuel into the free stream and enough recirculation zones of hot products. Usually, the flow is subsonic in the recirculation zones ($M < 1.0$). And it can be concluded from the above that the existence of these subsonic flow regions will improve the combustion efficiency of the scramjet.

E. Effect of the Inflow Equivalence Ratio

There are substantial differences in the local equivalence ratios in different locations because of the poor fuel-air mixing efficiency in the scramjet. For example, the mixture is fuel lean in the corner of the cavity combustor, while the mixture is fuel rich in the center of the shear layer. The distribution of the local equivalence ratios affects the combustion efficiency, heat release, and the overall thrust of the engine.

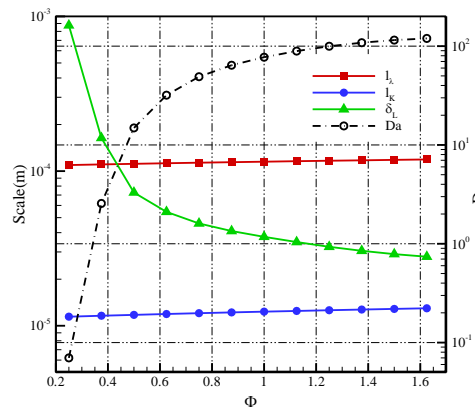


Figure 10. Influence of Φ to the characteristic scales

Figure 10 shows the influence of the inflow equivalence ratio Φ (0.25~1.625) to the characteristic scales of Air/ H_2 when the flight Mach number $M_0 = 6.0$, dynamic pressure $Q_0 = 95.8\text{kPa}$, total temperature $T_t = 1667\text{K}$, which is corresponding to the $M_4 = 2.6$, $P_4 = 133.4\text{kPa}$, $T_4 = 715\text{K}$ at the combustion chamber entrance. It can be seen that the δ_L is large, even an order of magnitude larger than the l_λ , when the $\Phi < 0.4$. At this point, the turbulent mixing is closely coupled with the chemical reaction. Both the mass diffusion and the turbulent fluctuation control the molecular transport. Particularly, the large scale eddies can convolute and wrinkle the flame

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**Table 1. Properties comparison between H₂ and hydrocarbon fuels^{1,8}**10

hydrogen, and the combustion in the scramjet using hydrocarbon fuels mainly take place in B zone. Wherein, the Taylor micro-scale plays a very important role.

G. Applicability of the Flamelet Models in Supersonic Combustion

As mentioned previously, the strong nonlinear interactions between turbulence and chemistry (TCI) complicate the supersonic combustion. Many combustion models are developed for the accurate prediction of supersonic combustion, including the flamelet models, the linear eddy model (LEM), the fractal flame model (FFM), the probability density function (PDF) methods, the eddy dissipation concept (EDC) model, the thickened flame model (TFM) and so on. The choice of the combustion model forms the crux of most of the numerical studies on practical combustor studies. Wherein, the flamelet model proposed first by Peters¹⁵ and then extended to a family of modeling approaches (c/z equation flamelet model, G/z equation flamelet model, flame surface density model and so on) by some other researchers has been widely applied²⁴⁻²⁷ to the simulation of supersonic combustion.

In the flamelet models, the flame is considered thin compared to the length scales of the flow, even thinner than the Kolmogorov scale, and is thus an interface between fuel and oxidizer (for nonpremixed combustion) or between reactants and products (for premixed combustion)²⁸. In other words, the flamelet models consider that the combustion mainly takes place in reaction sheets regime, including A zone and part of B1 zone (Just as discussed previously, the $l_k / \delta_L = 1.0$ line in flame regime will shift downward, as shown qualitatively in Figure 1 by the black dashed line, which means the area of the reaction sheets regime will enlarge and encroach onto part of B1 zone). Balakrishnan and Williams¹⁶ considered that it was reasonable to employ the flamelet models in the simulation of the supersonic hydrogen/air diffusion flame, while Eifler²⁹ thought it was very difficult to use the flamelet models in the simulation of the supersonic combustion.

It can be seen from Figure 11 and Figure 13 that it is reasonable to employ the flamelet models in the simulation of the supersonic combustion for hydrogen/air under certain conditions, such as in high flight Mach number conditions case or in the recirculation zones of a combustor. However, it is almost completely unable to use the flamelet models in the simulation of the supersonic combustion when taking ethylene and kerosene as fuels. Due to the difficulties in the development of combustion models, as well as the low costs²⁴ of the flamelet models, many researchers^{25-27, 30} are still employing the flamelet models in the simulation of supersonic combustion, which is based upon the following fact: the numerical solutions according to the flamelet models even in the regimes where the flamelet assumptions are not valid are better than that of the quasi laminar chemistry model (QLC).

V. Conclusions

The characteristic length scales in supersonic combustion and some important dimensionless parameters were calculated in details, and the related factors were investigated. What's more, the operating range for different fuels of a scramjet under the practical flight conditions was calculated, and the applicability of flamelet models in the numerical simulation of supersonic combustion was also discussed. The following conclusions were drawn.

- 1) In the supersonic combustion, the $l_k / \delta_L = 1.0$ line in flame regime shift downward because of the baroclinic effects and the compressibility, which enlarged the area of the reaction sheets regime.
- 2) The flamelet in eddies regime played the main role in the combustion of a scramjet under the

practical flight conditions. The static temperature, static pressure, the Mach number, and the equivalence ratio at the combustion chamber entrance did have an important influence on the characteristic length scales. Wherein, the Da increased with the static pressure linearly.

3) The scramjet's operating range for hydrogen, ethylene and kerosene decreased successively and Da varied broadly in the order of magnitude, in which the Taylor micro-scale played an important part.

4) It was reasonable to employ the flamelet models in the simulation of the supersonic combustion for hydrogen/air under certain conditions, while it was almost completely unable to use the flamelet models when taking ethylene and kerosene as fuels.

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