Studies on Thermocapillary Migration of Droplet

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Abstract *Thermocapillary migration of a planar droplet in a uniform temperature gradient at large Marangoni numbers is studied numerically by using the front tracking method. It is investigated that the thermocapillary motion of the planar droplet in the uniform temperature gradient is unsteady. For a fixed migration distance, the instantaneous thermocapillary droplet migration speed decreases as Marangoni number increases in the range of large Marangoni numbers. The result of above numerical simulation is qualitatively agreement with to those of experimental investigations. By using the asymptotic expansion method, a nonconservative integral thermal flux across the surface is identified in the steady thermocapillary droplet migration at large Marangoni numbers. This nonconservative flux may well result from the invalid assumption of quasi-steady state, which indicates that the thermocapillary droplet migration at large Marangoni numbers cannot reach steady state and is thus a unsteady process.*

Key Words: Droplet; Surface tension; Thermocapillary migration; Marangoni numbers; Microgravity

1. Introduction

The motion of a drop or bubble in microgravity environment embedded in an immiscible mother liquid at a uniform temperature gradient is termed thermocapillary migration of the drop or bubble, which is a very interesting topic on both fundamental theory and engineering application [1]. Young, Goldstein and Block (1959) carried out an initial study in this area and gave an analytical prediction for its migration speed at zero limit Reynolds(Re) and Marangoni(Ma) numbers, which is called as YGB model[2]. A series of theoretical analyses, numerical simulations and experimental investigations on this subject were carried out since then. For large Ma numbers, using thermal boundary layers, Balasubramanian and Subramanian (2000)[3] found that the migration speed of a drop increases with increase of Ma number, as is in qualitative agreement with the corresponding numerical simulation[4]. Both the theoretical analysis and numerical simulation are based on assumptions of the quasi-steady state and non-deformation of the drop. However, the experimental investigation carried out by Hadland et al(1999)[5] and Xie et al(2005)[6] gave results not in qualitative agreement with the above theoretical and numerical results, and it was shown that the drop migration speed non-dimensionalized by the YGB velocity decreases as Ma number increases. Therefore, the thermocapillary drop migration at large Ma numbers remains a topic to be studied with respect to its physical mechanism.

The planar or cylindrical drop/bubble as a simple model has been extensively used to study its dynamical mechanism. In this paper, theoretical and numerical studies on thermocapillary migration of droplet in a microgravity environment[7-9] are reviewed. Firstly, using the front-tracking method, we numerically study thermocapillary migration of a planar non-deformable drop in the liquid at large Ma numbers. Then, by using the asymptotic expansion method, a nonconservative integral thermal flux across the surface is identified in the steady thermocapillary droplet migration at large Ma numbers.

2. Physical models

Consider the thermocapillary migration of a planar droplet in a continuous phase fluid of infinite extent under a uniform temperature gradient G in Fig 1.Two-dimensional continuous, momentum and energy equations for the continuous phase fluid and the drop in a laboratory coordinate system are written as follows

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \mathbf{v}) = 0,$$
$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \bullet (\rho \mathbf{v}\mathbf{v}) = -\nabla p + \frac{1}{Re} \nabla \bullet \mu (\nabla \mathbf{v} + \nabla \mathbf{v}^T) + f_\sigma,$$
$$\frac{\partial T}{\partial t} + \nabla \bullet (\mathbf{v}T) = \frac{1}{Ma} \frac{\kappa}{k} \nabla \bullet (k \nabla T), \quad (1)$$

where **v** and T are velocity and temperature, respectively. f_σ is the surface tension acting on the interface. The solutions are satisfied boundary conditions at the infinity

$$\mathbf{v} = 0, \quad T = T_0 + Gz \quad (2)$$

and non-slip/periodic boundary conditions at the top and bottom walls/the horizontal boundaries.

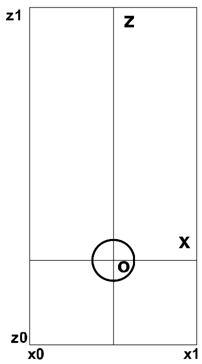

Fig. 1. Schematic of the computation domain for a planar droplet migration.

3. Numerical methods

In the computation, we use a fixed regular staggered MAC grid in the computational domain. To discretize Eqs. (1), we adopt a second-order central difference scheme for the spatial variables and an explicit predictor-corrector second-order scheme for time integration.

$$\frac{\rho^{n+1}\mathbf{v}^* - \rho^n \mathbf{v}^n}{\Delta t} = -\nabla \bullet (\rho^n \mathbf{v}^n \mathbf{v}^n) + \frac{1}{Re} \nabla \bullet \mu^n (\nabla \mathbf{v}^n + \nabla \mathbf{v}^{nT}) + \mathbf{f}_\sigma.$$

$$\frac{\rho^{n+1}\mathbf{v}^{n+1} - \rho^{n+1}\mathbf{v}^*}{\Delta t} = -\nabla p^{n+1},$$

$$\nabla \frac{1}{\rho^{n+1}} \bullet \nabla p^{n+1} = \frac{1}{\Delta t} \nabla \bullet \mathbf{v}^*.$$

$$\frac{T^{n+1} - T^n}{\Delta t} = -\nabla \bullet (\mathbf{v}^{n+1} T^n) + \frac{1}{Ma} \frac{\kappa^n}{k^n} \nabla \bullet (k^n \nabla T^n). \quad (3)$$

Since both fluids are assumed immiscible, all physical coefficients are discontinuous across the interface. The interface is captured and updated by the front-tracking method. A weighting function suggested by Peskin[10] is adopted as

$$d(r) = \begin{cases} (1/4\Delta r)[1 + \cos(\pi r/2h)], & |r| < 2\Delta r, \\ 0, & |r| \geq 2\Delta r, \end{cases} \quad (4)$$

and (x_p, z_p) is the interface node.

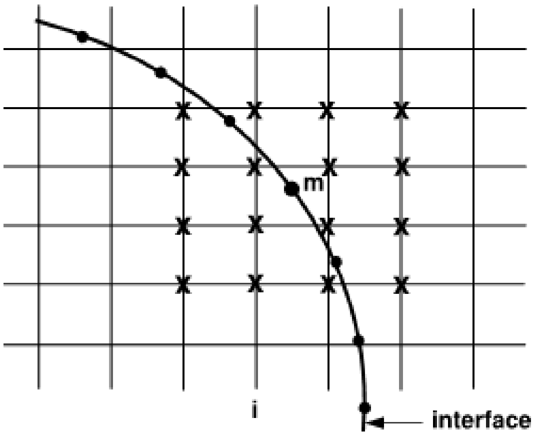

Fig. 2. Relation between nodes at the interface and mesh in the domain described by the Peskin wieghting function.

4. Numerical results

Fig. 3 displays the computed velocity fields at t=20 in both the laboratory coordinate frame and the reference frame moving with the droplet at Re=16.5(Ma=1118.1). In the laboratory coordinate frame, the streamlines for a moving droplet are closed and symmetric about the z-axis. In the reference frame, when the external streamlines go around the droplet, a pair of vortices is formed inside the droplet. Fig. 4 displays the time evolution of droplet migration velocities for five sets of non-dimensional coefficients. In the present range of Ma, the migration velocities versus time have complex behaviors, which can be classified into three types based on the curve characters. At Ma=44.7(Re=0.66), the initial migration velocity increases sharply before t=3, and then drops to approach a steady value.

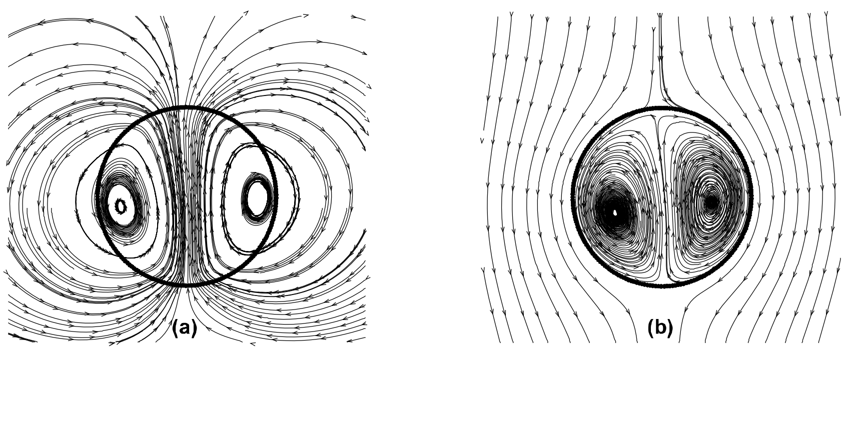

Fig. 3. Computed velocity fields at t=20 under R_0=0.25cm, Re=16.5, Ma=1118.1 in (a) the laboratory coordinate frame and (b) the reference frame moving with the droplet.

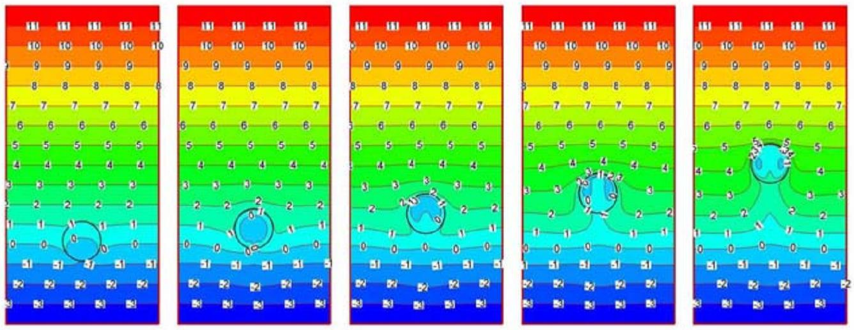

Fig. 4 Isotherms in a laboratory cooredinate frame are selected from the computation of the droplet migration under Re=5.93 and Ma=402.5.

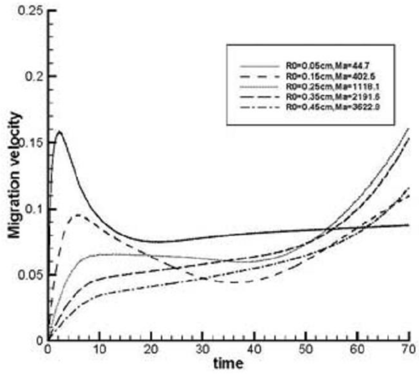

Fig. 5 Droplet migration velocity in a flow field with the temperature gradient *G*=12K/cm versus non-dimensional time at Ma=44.7, 402.5, 1118.1, 2191.6 and 3622.8.

For Ma=402.5-1118.1(Re=5.93-16.5), the initial accelerating process has smaller peak value as Ma increases. After the increasing-decreasing oscillation process, the terminal droplet migration velocity increases with time, i.e., the droplet migration is in an accelerating state. The slope of the curve increases as Ma increases. For Ma=2191.6-3622.8(Re=32.3-53.4), the droplet migration velocity increases monotonously with time and decreases with increasing Ma. We can thus conclude that in the time frame under investigation the thermocapillary droplet migration is steady at moderate Ma numbers, but becomes unsteady at large Ma numbers. In the two space experiments, Figs. 4 of [5] and [6] showed that the whole migration processes were unsteady and didn't reach any steady state. Even a plateau appears in the curve of migration velocity vs migration distance, the migration process seems to be an accelerating one after the slow varying period.

5. Theoretical analysis

The overall steady-state energy balance with two phases in a flow domain requires that the change in energy of the domain is equal to the difference between the total energy entering the domain and that leaving the domain. From the condition, the integral thermal flux across the surface is studied for a steady thermocapillary drop migration in a flow field with uniform temperature gradient at large Ma(Re) numbers. Under quasi-steady state assumption, Eqs.(1) can be formulated as

$$\nabla \bullet \mathbf{u}_i = 0,$$

$$\mathbf{u}_i \bullet \nabla \mathbf{u}_i = -\nabla p_i + \frac{\nu_i/\nu_1}{Re}\Delta \mathbf{u}_i,$$

$$V_\infty + \mathbf{u}_i \bullet \nabla T_i = \frac{\kappa_i/\kappa_1}{Ma}\Delta T_i. \quad (5)$$

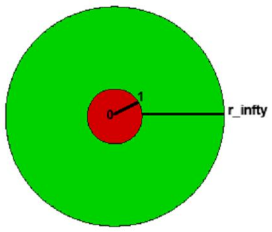

Fig.6 Two flow domains for steady thermocapillary migration of droplet

In two flow domains for the steady droplet migration, the energy equations in Eqs. (5) can be written in the following dimensionless form

$$1 + u_1\partial T_1/\partial r + v_1/r\, \partial T_1/\partial \theta = \varepsilon^2 \Delta T_1, \quad (6)$$

$$1 + u_2\partial T_2/\partial r + v_2/r\, \partial T_2/\partial \theta = \lambda\, \varepsilon^2 \Delta T_2, \quad (7)$$

where $\varepsilon=1/(MaV_\infty)^{1/2}$ and $\lambda = \kappa_2/\kappa_1$. Integrating Eq. (6) and Eq. (7) in the continuous phase domain ($r\in[1,r_\infty]$, $\theta \in[0, 2\pi]$) and within the drop region ($r\in[0,1]$, $\theta \in[0, 2\pi]$) and using the zero normal velocity boundary condition at the interface, we can derive

$$\beta \oint \partial T_2/\partial r|_1 ds - \oint \partial T_1/\partial r|_1 ds = \pi(1+\beta/\lambda)/\varepsilon^2 = \pi(1+\beta/\lambda)V_\infty Ma \quad (8)$$

where $\beta=k_2/k_1$. Since both β and λ are positive, we have

$$\beta \oint \partial T_2/\partial r|_1 ds - \oint \partial T_1/\partial r|_1 ds >> 0 \quad (9)$$

at large Ma numbers. From the thermal flux boundary condition at the interface, we obtain

$$\oint \partial T_1/\partial r|_1 ds = \beta \oint \partial T_2/\partial r|_1 ds. \quad (10)$$

So, if the overall steady-state energy with two phases in the flow domain under integral boundary conditions is balanced, Eq. (9) should be reduced to Eq. (10), which seems impossible. It is termed as a nonconservative integral thermal flux across the surface for the steady thermocapillary drop migration at large Ma (Re) numbers. This implies the overall steady-state energy unbalance of two phases in the flow domain in the co-moving frame of reference and indicates that the thermocapillary drop migration at large Ma (Re) numbers cannot reach steady state. Thus, it is clear that the invalid assumption of quasi-steady state for the thermocapillary drop migration process is a reasonable explanation for the nonconservative integral thermal flux across the drop surface.

6. Conclusion

By using the frant tracking method, it is observed that the thermocapillary migration of a planar non-deformed droplet with an uniform temperature gradients is steady at moderate Ma numbers, but unsteady at large Ma numbers. The numerical results at large Ma numbers qualitatively agree with of those of experimental investigations. From the overal steady-state energy balance in the flow domain, a non-conservative integral thermal flux across the surface for a steady thermocapillary droplet migration at large Ma numbers is found by using the asymptotic analysis. It presents that the thermocapillary droplet migration at large Ma numbers cannot reach any steady states and is thus an unsteady process.

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References

[1]R. S. Subramanian and R. Balasubramaniam (2001), *The motion of bubbles and drops in reduced gravity*, Cambridge, England: Cambridge University Press.

[2]N. O. Young, J. S. Goldstein and M. J. Block (1959), The motion of bubbles in a vertical temperature gradient, *J. Fluid Mech*. **6**, 350.

[3]R. Balasubramaniam and R. S. Subramanian (2000), The migration of a drop in a uniform temperature gradient at large Marangoni numbers, *Phys. Fluids* **12**, 733.

[4]X. Ma, R. Balasubramiam and R. S. Subramanian (1999), Numerical simulation of thermocapillary drop motion with internal circulation, *Numer. Heat Transfer*, A **35**, 291.

[5]P. H. Hadland, R. Balasubramaniam, and G. Wozniak (1999), Thermocapillary migration of bubbles and drops at moderate to large Marangoni number and moderate Reynolds number in reduced gravity, *Exper. Fluid*, **26**, 240.

[6]J. C. Xie, H. Lin, P. Zhang, F. Liu, W. R. Hu (2005), Experimental investigation on thermocapillary drop migration at large Marangoni number in reduced gravity, *J. Colloid Interface Sci*. **285**, 737.

[7]Z. Yin, Z.-B. Wu and W. R Hu (2009), Thermacapillary migration of drops and bubbles, *Advances in Microgravity Sciences*, ed. W. R. Hu, Transworld Research Network, India 35-52.

[8]Z.-B. Wu and W. R. Hu (2012), Thermocapillary migration of a planar droplet at moderate and large Marangoni numbers, *Acta Mechanica*, **223**, 609-626.

[9]Z.-B. Wu and W. R. Hu (2013), Effects of Marangoni numbers on thermocapillary drop migration: Constant for quasi-steady state? *Journal of Mathematical Physics*, **54**, 023102(1-10).

[10]C. S. Peskin (1977), Numerical analysis of blood flow in the heart, *J. Comput. Phys*. **25**, 220.

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