

## Nonlinear Similarity Model of Financial Contagion: The Difference of the Subprime Crisis and the European Debt Crisis

<sup>1</sup>L. Liu, <sup>1</sup>X.F. Hui and <sup>2</sup>Y.F. Shao

<sup>1</sup>School of Management, Harbin Institute of Technology, Harbin, China

<sup>2</sup>State Key Laboratory of Nonlinear Mechanics (LNM), Institute of Mechanics,  
Chinese Academy of Sciences, Beijing, China

**Abstract:** This study develops a sign-modified nonlinear similarity model to study the contagion in financial crisis. We use the similarity index as an indicator to analyze the contagion of the United States and Greece to other six countries during the subprime crisis and the European debt crisis. Our results show that in the 2007-2009 subprime crisis the United States is the contagion source; and that in the 2010-2012 European debt crisis, the impacts of Greece to other countries are smaller than those of the United States except for Italy and Spain. We conclude that the harm of the financial crisis from a large economy is more severe than that from a small economy and the small economy's crisis mainly shocks the neighboring countries with similar economy.

**Key words:** Subprime crisis, European debt crisis, financial contagion, nonlinear similarity, similarity index

### INTRODUCTION

The breakout of the US subprime crisis in 2007 caused a global financial crisis in a wide range which impacted the world's major economies in different degrees. The European debt crisis has also threatened the world from 2010. Frequent outbreak of the international financial crisis makes the research on financial contagion realistic and important.

Scholars have done a lot of researches to test the existence of financial contagions. King and Wadhvani (1990) first used cross-market correlation coefficients to test contagion. Then other approaches such as probit model (Eichengreen *et al.*, 1996), extreme value theory (Longin and Solnik, 2001), GARCH model (Steeves, 2002), Vector Auto Regression (VAR) (Khalid and Kawai, 2003) and Copula model (Rodriguez, 2007) were all used to find contagion evidence.

After the global financial crisis in 2008, more methods are introduced into the research of financial contagion. Chiang and Wang (2011) adopted a smooth transition copula function to detect the volatility contagion between the stock markets of the G7 countries. Blancheton *et al.* (2012) built a structural break BEKK model to reveal the contagion mechanism between the United States, the United Kingdom and the Euro Zone. Gallegati (2012) and Madaleno and Pinho (2012) employed wavelet approaches to test whether contagion occurred in financial crisis.

However, most of the researches need to distinguish different periods or divide contagion stages before study.

The conclusions they get depend on the periods and stages they have divided. Nonlinear dynamical similarity (Schreiber and Schmitz, 1997) is a method to classify and analyze the time series data which is mainly used in natural science and engineering areas, especially in biology and neuroscience (Le Van Quyen *et al.*, 2001; Latchoumane and Jeong, 2011). The data from financial market are long time series data that have strong nonlinear characteristics. We regard different markets as different dynamical systems and apply the nonlinear similarity method to study contagion in financial markets. This model has two advantages: Firstly, it doesn't need to divide the financial time series which avoids the irrational conclusion due to subjective irrationality in period division. Secondly, nonlinear similarity doesn't need to summarize other characteristic numbers from the time series which avoid information losses in characteristic number selection (Schreiber and Schmitz, 1997).

In this study, we apply the nonlinear similarity model based on dynamic systems to analyze the similarity of stock-index time series data from different markets. Similarity index is used as an indicator to estimate the time and level of contagion. The purpose is to test and analyze the financial contagion from the US and Greece to other countries' markets in the US subprime crisis in 2007-2009 and the European debt crisis in 2010-2012.

### MATERIALS AND METHODS

**C-C method for reconstruction:** The primary step for dynamical research of nonlinear time series is phase space

reconstruction. It is also the basis of nonlinear time series data calculation and dynamics similarity analysis. The time delay method (Takens, 1981) laid the foundation for phase space reconstruction theory. For nonlinear time series  $x = \{x_i, i = 1, 2, \dots, N\}$ , the phase space reconstructed can be represented as:

$$X = \{X_i | X_i = [x_i, x_{i+\tau}, \dots, x_{i+(m-1)\tau}]\} \quad (1)$$

where,  $\tau$  denotes the index lag,  $m$  denotes the embedding dimension and  $N_1 = N - (m-1)\tau$  is the number of embedded points in phase space.

Following the C-C method Kim *et al.* (1999), we can optimize the delay time window  $\tau_w$  and the optimal delay time  $\tau_d$  by the conception of correlation integral:

$$C(m, N, \tau, t) = \frac{2}{N_1(N_1-1)} \sum_{i=1}^{N_1} \sum_{j=1}^{N_1} \Theta(r - \|X_i - X_j\|), r > 0 \quad (2)$$

where,  $r$  is a certain distance and  $\|\cdot\|$  denotes the Euclidean distance between two points.  $\Theta(x)$  is Heaviside function and  $\Theta(x) = 0$ , if  $x < 0$ ;  $\Theta(x) = 1$ , if  $x = 0$ .

We define the statistic:

$$S_1(m, N, \tau, t) = C(m, N, \tau, t) - C^m(1, N, \tau, t) \quad (3)$$

Then, subdivide  $N$  into  $t$  disjoint time series, that is:

$$S_2(m, N, \tau, t) = \frac{1}{t} \sum_{s=1}^t \left[ C_s\left(m, \frac{N}{t}, \tau, t\right) - C_s^m\left(1, \frac{N}{t}, \tau, t\right) \right] \quad (4)$$

and define:

$$\Delta S(m, t) = \max \{S(m, r_j, t)\} \quad (5)$$

According to the definition above, we calculate the following statistic:

$$\bar{s}(t) = \frac{1}{16} \sum_{a=2}^5 \sum_{j=1}^4 S(m, r_j, t) \quad (6)$$

where,  $r_j = j \times \sigma / 2$ ,  $\sigma$  is the standard deviation of time series:

$$\bar{\Delta S}(t) = \frac{1}{4} \sum_{a=2}^5 \Delta S(m, t) \quad (7)$$

$$\text{Scor}(t) = \Delta \bar{S}(t) + |\bar{S}(t)| \quad (8)$$

We estimate the optimal delay time  $\tau_d = \tau_s$  by index lag  $t$  at the first local minimum of  $\Delta(t)$  and estimate the delay time window  $\tau_w = \tau_s$  by index lag  $t$  at the minimum

of  $S_{\text{cor}}(t)$ , where  $\tau_s$  is the sampling interval. According to the relationship between  $\tau_w$  and  $\tau_d$  which was proposed by Kugiumtzis (1996), we can calculate  $m$ . Reconstruct phase space with the parameters  $\tau_d$  and  $m$ .

**Nonlinear similarity and similarity index:** The dynamical similarity theory of nonlinear time series bases on the following idea: in science and engineering, we can study systems through the characteristic time evolution of observable properties. For different dynamical systems, or different states of a single system, by analyzing the similarities and differences of the observable time series, we can obtain the variable characteristics in different systems or different stages of one system.

The calculation of correlation sum function is essential for dynamical similarity. Reconstruct two phase space matrices  $X, Y$  for two time series  $\{x\}, \{y\}$ , so the cross-correlation sum (Kantz, 1994) of  $X, Y$  is:

$$C_{XY} = \frac{1}{N_1 \times N_2} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} \Theta(\varepsilon - \|X_i - Y_j\|) \quad (9)$$

Accordingly, for phase space matrix  $X$ , the self-correlation sum is:

$$C_{XX} = \frac{1}{N_1^2} \sum_{i=1}^{N_1} \sum_{j=1}^{N_1} \Theta(\varepsilon - \|X_i - X_j\|) \quad (10)$$

where,  $N_1, N_2$  are the numbers of points in phase space  $X, Y$ , respectively.  $\varepsilon$  is the threshold whose value depends on the phase space.

In order to express the similarity of two dynamic systems more clearly, the similarity index (Schreiber and Schmitz, 1997) is defined as:

$$\gamma = \frac{C_{XY}}{\sqrt{C_{XX} \times C_{YY}}} \quad (11)$$

where,  $\gamma \in [0, 1]$ . The similarity index represents a measure of the similarity sensitivity between two dynamical systems. The higher the similarity of two dynamical systems is, the more  $\gamma$  approximates to 1. Conversely, the more significance the difference of two systems, the more  $\gamma$  decreases to 0.

**Sign of similarity index  $\gamma$ :** In multidisciplinary researches, the dynamical similarity index employs positive value from 0-1. However, it is very important to distinguish the rise and fall of financial time series in the financial study, especially in the financial crisis period.

**Table 1: Phase space parameters**

	2007-2009	2010-2012	
	DJI	DJI	ASE
$\tau_d$	8	14	6
$\tau_w$	29	29	19
$m$	5	4	5

**Table 2: Time structures of 2007-2009 and 2010-2012**

2007-2009			2010-2012					
DJI			ASE			DJI		
Start date	End date	Sign	Start date	End date	Sign	Start date	End date	Sign
2007-03-28	2007-10-09	-	2010-03-31	2010-09-03	+	2010-03-31	2010-04-26	-
2007-10-10	2008-03-07	+	2010-09-06	2011-02-09	-	2010-04-27	2010-07-02	+
2008-03-10	2008-05-02	-	2011-02-10	2011-07-19	+	2010-07-05	2011-04-28	-
2008-05-05	2009-03-05	+	2011-07-20	2011-12-23	-	2011-04-29	2011-09-09	+
2009-03-06	2009-12-30	-	2011-12-26	2012-05-30	+	2011-09-12	2012-04-02	-
2012-04-03	2012-05-30	+						

We assign similarity index  $\gamma$  plus when financial time series declines and  $\gamma$  min when financial time series rises. Therefore,  $\gamma$  value ranges from -1-1.

### Empirical analyses

**Data and pretreatment:** The following examples compare the global financial crisis initiated by US subprime crisis with the European debt crisis. We take two time periods: (1) Jan 4th, 2007-Dec 30th, 2009 and (2) Jan 4th, 2010 to May 30th, 2012. Daily closing prices of eight stock market indices, namely DJI (US), ASE (Greece), FTSE100 (UK), DAX (German), FTMIB (Italy), IGBM (Spain), KS11 (South Korea), SSI (China), are used. Stock price data were taken from Bloomberg.

We adjust the time series by piecewise linear interpolation when the opening dates in different stock markets are not the same. After interpolation, there are  $N_1 = 777$  observations and  $N_2 = 624$  observations in the two periods separately. All the time series in the samples are regularized by min-max standardization.

Then, the samples are taken into the C-C method in 2.1 to reconstruct the phase space. Setting the sampling interval  $\tau_s = 1$ , we calculate the optimal delay time  $\tau_d$  and the delay time window  $\tau_w$  and get the embedding dimension  $m$ . Because the US is the source of the subprime financial crisis and the world's largest economy in 2007-2009, we take the US as the reference country. In 2010-2012, Greece is the source of the European debt crisis whereas the US is the world's largest economy, so we take Greece and the US as the reference country, respectively. The calculation results in two periods are shown as Table 1.

In 2007-2009, we use the US's parameters  $\tau_d = 8$  and  $m = 5$  to reconstruct the phase space. As the sample length  $N_1 = 777$ , we get  $N_0 = 745$  embedded points for each market.

In 2010-2012, we select Greece  $\tau_{dG} = 6$ ,  $m_G = 5$  and the US  $\tau_{dA} = 8$ ,  $m_A = 5$  to reconstruct the phase space, respectively. We get  $N_{0G} = 600$  embedded points for the source Greece and  $N_{0A} = 592$  embedded points for the largest economy US.

**Similarity index in financial crisis:** Calculate the nonlinear similarity index of each country to the source and to the largest economy. We set the sliding window  $W = 60$ . The threshold  $\epsilon$  changes as the sliding window moving on, taking 30% of the cumulative neighborhood distribution of the reference country's time series (Navarro *et al.*, 2002).

In the 2007-2009 global financial crisis initiated by the US subprime crisis, there are  $N_1 = N_2 = 28$  embedded points in every sliding window. Calculating one similarity index from 28 points as the sliding window moving on, we obtain 718 positive similarity indices  $\gamma$  for every country. Derive the sign of similarity index  $\gamma$  from the trend of the source US DJI. After smoothing, we subdivide the index curve with the extremum points.

Similarly, during the European debt crisis in 2010-2012, there are  $N_{1G} = N_{2G} = 36$  embedded points in every sliding window when we take Greece as the reference country and  $N_{1A} = N_{2A} = 28$  embedded points when we take the US. We obtain 565 positive similarity indices  $\gamma$  for every country.

The time structures of the two periods are shown in Table 2.

Assigning the similarity index according to Table 2, we gain the similarity index  $\gamma$  of six countries. The curves of the similarity index in 2007-2009 are shown in Fig. 1 and the curves in 2010-2012 are shown in Fig. 2.

The mean value and the standard deviation of similarity index  $\gamma$  of six countries to Greece and the US are calculated respectively, as shown in Table 3-5.

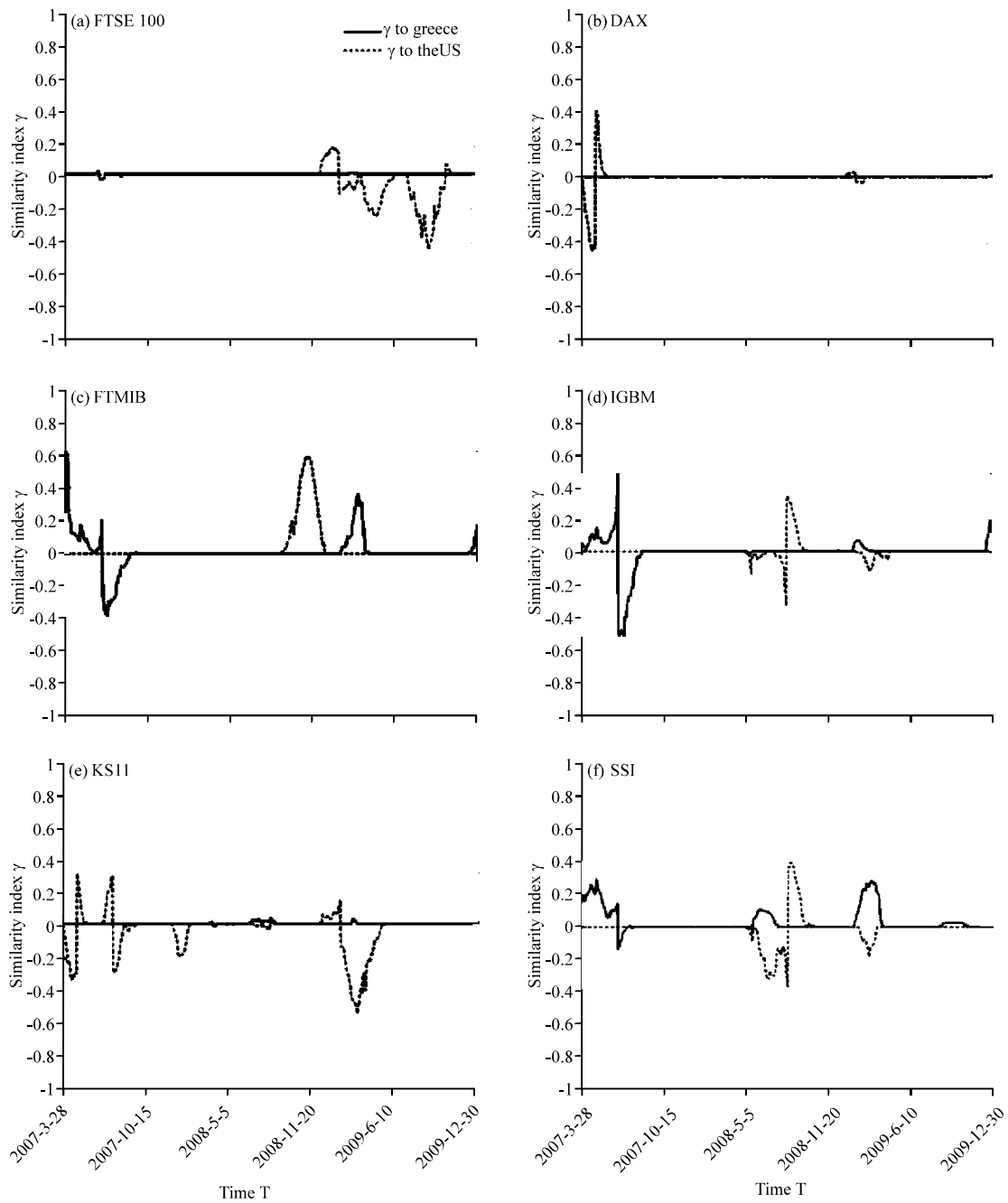


Fig. 1(a-f): Similarity index  $\gamma$  of six countries in 2007-2009

Table 3: Statistics of similarity index  $\gamma$  of six countries to US in 2007-2009

	FTSE100	DAX	FTMIB	IGBM	KS11	SSI
Mean	0.057983	-0.02067	-0.04253	-0.07217	0.032875	-0.04146
STD	0.418673	0.400179	0.222566	0.340636	0.119572	0.125482

Table 4: Statistics of similarity index  $\gamma$  of six countries to Greece in 2010-2012

	FTSE100	DAX	FTMIB	IGBM	KS11	SSI
Mean	-0.00011	0	0.00851	-0.00463	0.001176	0.033694
Std	0.003705	0	0.09351	0.088532	0.004941	0.075946

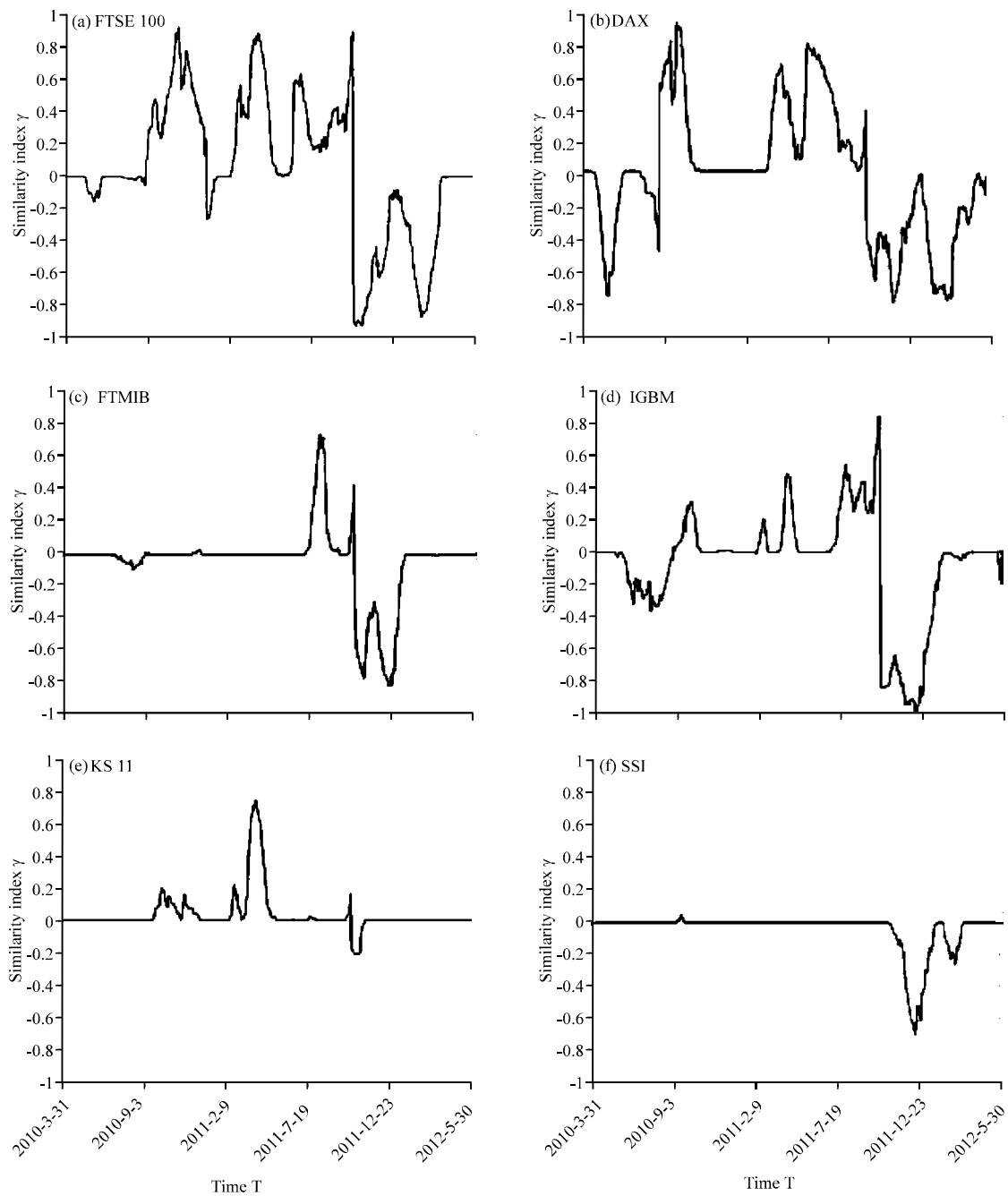


Fig. 2: Similarity index  $\gamma$  of six countries in 2010-2012

Table 5: Statistics of similarity index  $\gamma$  of six countries to US in 2010-2012

	FTSE100	DAX	FTMIB	IGBM	KS11	SSI
Mean	-0.02902	-0.00576	0.030252	0.000553	-0.03486	-0.01084
Std	0.088954	0.068286	0.109731	0.051825	0.113266	0.092312

**Comparison of the US subprime crisis with the European debt crisis:** We have used the nonlinear similarity index as a quantitative indicator to measure the linkage between different countries' financial markets. When the economy upturns, the similarity

index  $\gamma$  represents the level of common prosperity; whereas, during the financial crisis, it represents the level of contagion. So, the similarity index can be used to study the characters of financial contagion.

Firstly, we compare the contagion effects of the two sources to other countries. The US and Greece are the sources of the subprime crisis and the European debt crisis, respectively. As Fig. 1 and the solid line in Fig. 2 show us, the similarity indices of the UK, Germany, Italy, Spain, South Korea to the US in 2007-2009, are significantly greater than the five countries to Greece in 2010-2012. In other words, the linkages between the five countries and the US in 2007-2009, are stronger than the linkages between them and Greece in 2010-2012. According to Forbes and Rigobon definition (2002), financial contagion is a significant increase in cross-market linkages after a shock to other countries. Therefore, it implies that the degree of contagion in the subprime crisis is greater than that in the European debt crisis. The statistics in Table 3 and 4 also show that, for these five countries, the contagion effect of the US is greater than Greece.

Secondly, we compare the effects of the source with the largest economy in the same period. In 2010-2012, Greece is the main source of the European debt crisis while the US is the world's largest economy. Comparing two lines in Fig. 2, the similarity indices of the UK, Germany and South Korea to Greece are nearly to zero, while the similarity indices of the three countries to the US have significantly positive and negative growth. This suggests that the three countries have no obvious linkage with the Greece stock market but do have a linkage with the US. The influence of the US is greater than Greece. For Italy and Spain, their similarity indices to Greece are greater than the similarity indices to the US which means that the linkages of the two countries with Greece are stronger than with the US. That is to say, the contagion effect of Greece is greater than that of the US for Italy and Spain. Data from Table 4 and 5 also show that, in 2010-2012, Britain, Germany and South Korea were influenced by the US financial markets, whereas Italy and Spain were impacted more by Greece than by the US. It is consistent with the conclusion of Madaleno and Pinho (2012) that geographically and economically closer markets exhibit higher correlation.

Finally, we compare the effects of the US as a source and as the largest economy in different periods. Comparing Fig. 1 and the pecked line in Fig. 2, it is clear that the US has influenced other countries both in the 2007-2009 subprime crisis caused by itself and in the 2010-2012 European debt crisis caused by another country. In 2007-2009, the similarity indices of the UK, Germany, Italy, Spain and South Korea to the US have soared many times and the absolute value is larger than zero for a long time in each picture. In 2010-2012, the

absolute values of these countries' similarity indices to the US are less than those in Fig. 1 and the time of them above zero is much shorter. This shows that the linkages of these five countries with the US in 2007-2009 global financial crisis are stronger than in 2010-2012 both in sustainability and amplitude. On one hand, it conforms with Forbes and Rigobon definition (2002) and verifies the US as the source in the 2007-2009 financial crisis. On the other hand, the comparison of six countries in Fig. 1 and 2, as well as Table 3 and 5, show that the US' influences to the five countries as a contagion source are bigger than as a non-source.

China is a special case in the sample. The similarity indices of China neither change distinctly to different sources, nor in different crisis periods. This means that Chinese financial market is not strong correlated with foreign countries. It is consistent with other scholars' conclusion (Yu *et al.*, 2009) that China's capital market is low connection with that of other countries.

## CONCLUSION

In this study, we have introduced a dynamical similarity method of nonlinear time series for financial contagion. Employing the similarity index as the indicator, we have examined the contagion for six countries in the subprime crisis and the European debt crisis. To gain further insight into the nature of financial contagion, the two financial crises are compared in three aspects.

The empirical result proves that the contagion of the US to other countries in the 2007-2009 subprime crisis is greater than that of Greece in 2010-2012 in scope and intensity. In other words, the contagion triggered by a large economy harms more than that by a small economy. This suggests us to prevent financial contagion from large economies, particularly from the US. Due to the special status of the US in the world's financial market, the financial contagion from the US impacts the world in the widest range.

During the European debt crisis, the influences of the US to most countries are still greater than the source Greece. It means that we need to pay attention to the influence of the US besides the source country during financial crisis. For the countries that connects with the world in economy and finance, the influence of the US is often bigger than the source.

Meanwhile, we find the contagion effect from a small economy to large economies is limited. But the financial monitor still need keep an eye on the economic conditions of the neighboring countries. Even a small economy is able to influence the countries that are close in geography and economy.

The irregularity of the two financial crisis's influence on China is mainly because of not-fully open of Chinese capital market. Therefore we give the suggestion to open the capital market following the prudence principle, especially when the country's financial system is imperfect.

#### ACKNOWLEDGMENT

This study was supported by the National Natural Science Foundation of China (No. 71173060, Nonlinear dynamic research on multi-market financial contagion).

#### REFERENCES

- Blancheton, B., C. Bordes, S. Maveyraud and P. Rous, 2012. Risk of liquidity and contagion of the crisis on the United States, United Kingdom and euro zone money markets. *Int. J. Finan. Econ.*, 17: 124-146.
- Chiang, M.H. and L.M. Wang, 2011. Volatility contagion: A range-based volatility approach. *J. Econometrics*, 165: 175-189.
- Eichengreen, B., A. Rose and C. Wyplosz, 1996. Contagious currency crises: First tests. *The Scandinavian J. Econ.*, 98: 463-484.
- Forbes, K.J. and R. Rigobon, 2002. No Contagion, only interdependence: Measuring stock markets comovements. *The J. Finan.*, 57: 2223-2261.
- Gallegati, M., 2012. A wavelet-based approach to test for financial market contagion. *Computational Statist. Data Anal.*, 56: 3491-3497.
- Kantz, H., 1994. Quantifying the closeness of fractal measures. *Phys. Rev. E. Stat. Phys. Plasmas Fluids Relat. Interdiscip. Topics*, 49: 5091-5097.
- Khalid, A.M. and M. Kawai, 2003. Was financial market contagion the source of economic crisis in Asia?: Evidence using a multivariate VAR model. *J. Asian Econ.*, 14: 131-156.
- Kim, H.S., R. Eykholt and J.D. Salas, 1999. Nonlinear dynamics, delay times and embedding windows. *Physica D: Nonlinear Phenomena*, 127: 48-60.
- King, M.A. and S. Wadhvani, 1990. Transmission of volatility between stockmarkets. *Rev. Finan. Stud.*, 3: 5-33.
- Kugiurmtzis, D., 1996. State space reconstruction parameters in the analysis of chaotic times series-the role of the time window length. *Physica D: Nonlinear Phenomena*, 95: 13-28.
- Latchoumane, C.F.V. and J. Jeong, 2011. Quantification of brain macrostates using dynamical nonstationarity of physiological time series. *IEEE Trans. Biomed. Engin.*, 58: 1084-1093.
- Le Van Quyen, M., J. Martinerie, V. Navarro, P. Boon and M. D'Have *et al.*, 2001. Anticipation of epileptic seizures from standard EEG recordings. *Lancet*, 357: 183-188.
- Longin, F. and B. Solnik, 2001. Extreme correlation of international equity markets. *J. Finance*, 56: 649-676.
- Madaleno, M. and C. Pinho, 2012. International stock market indices comovements: A new look. *Int. J. Finan. Econ.*, 17: 89-102.
- Navarro, V., J. Martinerie, M.L.V. Quyen, S. Clemenceau, C. Adam, M. Baulac and F. Varela, 2002. Seizure anticipation in human neocortical partial epilepsy. *Brain*, 125: 640-655.
- Rodriguez, J.C., 2007. Measuring financial contagion: A copula approach. *J. Empirical Finan.*, 14: 401-423.
- Schreiber, T. and A. Schmitz, 1997. Classification of time series data with nonlinear similarity measures. *Phys. Rev. Lett.*, 79: 1475-1478.
- Steeves, G.M., 2002. Political shocks and abnormal returns during the Taiwan crisis: An event study analysis. Master's Thesis, Colorado University, Boulder.
- Takens, F., 1981. Detecting Strange Attractors in Turbulence. In: *Dynamical Systems and Turbulence: Lecture Notes in Mathematics*, Rand, D.A. and L.S. Young (Eds.). Vol. 899, Springer, Berlin, Heidelberg, pp: 366-381.
- Yu, H.X., B.L. Qiu and Y.L. Xiao, 2009. The empirical analysis of the contagion effect of financial crisis based on VAR model-The financial crisis in the United States. *China Bus.*, 24: 51-103.