Optimal speed of hypersonic cruise flight

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(Received 8 November 2010; accepted 23 November 2010; published online 10 January 2011)

Abstract A coupling frame of speed gain and maintain was suggested to assess the flight performance of hypersonic cruise vehicles (HCV). The optimal cruise speed was obtained by analyzing the flight performance measured by the ratio of initial boost mass to generalized payload. The performance of HCVs based on rockets and air-breathing ramjets was studied and compared to that of a minimum-energy ballistic trajectory under a certain flight distance. It is concluded that rocket-based HCVs flying at the optimal speed are a very competitive choice at the current stage. [doi:10.1063/2.1101204]

Keywords hypersonic cruise vehicle, flight performance, optimal cruise speed

Hypersonic cruise vehicles (HCVs) can be regarded as a type of vehicles that combines technologies from both airplanes and aviation vehicles. Firstly, their speed range is between those of airplanes and long-range ballistic missiles. Secondly, rocket engines are a realistic way to obtain hypersonic cruise speed at the same value as the speed of ballistic missiles, whereas the mechanism for hypersonic cruise flights to resist the gravitational force is similar to that of airplanes by providing aerodynamic lift force.

The essential elements that flight vehicles have to consider include transported mass, distance and time. Given a flight distance, the flight time depends upon the speed. Either speed gain or maintain for a flight is usually achieved at the cost of fuel consumption. For example, ballistic missiles consume most of their fuel to gain a flight speed during the boost stage using a rocket engine and then fly under their own inertia, while business airplanes consume most of the fuel to maintain a cruise speed during nearly the whole distance range using jet engines to resist the aerodynamic drag.

Intuitively, HCV should have a performance between that of ballistic missiles and business airplanes. Given a flight distance, how do we choose the speed of HCV to make its total fuel consumption minimum? This requires a balance of fuel consumption between speed gain and maintain for a hypersonic flight, which is the focus of the present article.

The total initial mass of an aircraft system is the sum of the payload mass M_p, the fuel mass M_f, and the unloaded vehicle mass M_e, i.e.

$$M_\mathrm{B}^0 = M_\mathrm{p} + M_\mathrm{f} + M_\mathrm{e}. \tag{1}$$

As mentioned in Ref. [1], there are no strict criteria to distinguish the payload mass from the unloaded vehicle mass. Thus a generalized payload mass M_p^* is introduced and is defined as the sum of the payload and unloaded vehicle mass, i.e. $M_\mathrm{p}^* = M_\mathrm{p} + M_\mathrm{e}$. In addition, $M_\mathrm{B}^0/M_\mathrm{p}^*$ is suggested as an elementary indicator to evaluate the flight performance.

There are two essential requirements for a hypersonic cruise flight: balance between the aerodynamic lift and gravitational force, and balance between the aerodynamic drag and engine thrust, i.e.

$$F_\mathrm{L} = \left(M_\mathrm{p,c}^* + M_\mathrm{f,c}\right)\left(g - V_\mathrm{c}^2/r_\mathrm{E}\right), \tag{2}$$

$$F_\mathrm{jet} = F_\mathrm{D} = F_\mathrm{L}/C_\mathrm{L/D}, \tag{3}$$

where $M_\mathrm{p,c}^*$ is the generalized payload mass, $M_\mathrm{f,c}$ is the fuel mass during the cruise flight, V_c is the cruise speed, $C_\mathrm{L/D}$ is the lift-to-drag ratio of HCV, and $r_\mathrm{E} = 6371\,\mathrm{km}$ is the earth radius. Here r_E is used to evaluate the centrifugal force instead of $(r_\mathrm{E} + h)$ because the flight altitude of HCV, h, is usually around 30 km that is much smaller than r_E.

The engine thrust of a cruise flight is proportional to its fuel flow rate, i. e.

$$F_\mathrm{jet} = -\frac{\mathrm{d}M_\mathrm{f,c}}{\mathrm{d}t} I_\mathrm{c} g, \tag{4}$$

where I_c is the specific impulse.

Substitution of Eqs. (2) and (4) into Eq. (3) yields

$$\frac{\mathrm{d}M_\mathrm{f,c}}{M_\mathrm{p,c}^* + M_\mathrm{f,c}} = -\frac{1 - V_\mathrm{c}^2/(g r_\mathrm{E})}{C_\mathrm{L/D} I_\mathrm{c}}\mathrm{d}t. \tag{5}$$

If the fuel is assumed to be used up at the end of cruise flight, Eq. (5) can be integrated as

$$\frac{M_\mathrm{p,c}^* + M_\mathrm{f,c}}{M_\mathrm{p,c}^*} = \exp\left[\frac{1 - V_\mathrm{c}^2/(g r_\mathrm{E})}{C_\mathrm{L/D}} I_\mathrm{c} \Delta t_\mathrm{c}\right], \tag{6}$$

where Δt_c is the cruise time of the flight.

The flight of a hypersonic cruise missile can be divided into four phases: boost, cruise, glide and dive. Because the boost and dive distances are relatively short, the total flight distance L can be approximated as the sum of the cruise distance L_C and the glide distance L_G. Therefore we have

$$\Delta t_\mathrm{c} = \frac{L_\mathrm{C}}{V_\mathrm{C}} \approx \frac{L - L_\mathrm{G}}{V_\mathrm{C}}. \tag{7}$$

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During the glide phase when the cruise engine stops working, the missile speed will decrease gradually from V_c to V_d due to the aerodynamic drag, where V_d is the speed to start the dive phase. The glide distance can be estimated under a small angle of attack following Eggers, Allen & Neice[2,3]

$$\frac{L_\mathrm{G}}{r_\mathrm{E}} = 0.5 C_\mathrm{L/D} \ln \frac{1 - V_\mathrm{d}^2/(2gr_\mathrm{E})}{1 - V_\mathrm{c}^2/(2gr_\mathrm{E})}. \tag{8}$$

The lift-to-drag ratio of HCV depends upon its flight altitude and velocity. Its value will increase during the glide phase as both the altitude and velocity decrease continuously. For convenience, the same value of lift-to-drag ratio is assumed for both the cruise and dive phases. This is a conservative estimation for the performance evaluation of long-distance hypersonic cruise flight.

The right-hand-side of Eq. (8) can be approximated using the first-order Taylor expansion, which leads to

$$\ln \frac{1 - V_\mathrm{d}^2/(2gr_\mathrm{E})}{1 - V_\mathrm{c}^2/(2gr_\mathrm{E})} \approx \frac{V_\mathrm{c}^2 - V_\mathrm{d}^2}{2gr_\mathrm{E}}. \tag{9}$$

Substitution of Eqs. (7)∼(9) into Eq. (6) yields

$$\frac{M_\mathrm{p,c}^* + M_\mathrm{f,c}^*}{M_\mathrm{p,c}^*} = \exp \left[\frac{L - 0.25 C_\mathrm{L/D}\left(V_\mathrm{c}^2 - V_\mathrm{d}^2\right)/g}{C_\mathrm{L/D} I_\mathrm{c} V_\mathrm{c}} \left(1 - \frac{V_\mathrm{c}^2}{g r_\mathrm{E}}\right)\right]. \tag{10}$$

The cruise speed of a flight is usually achieved using chemical rockets. Then the ratio of the initial mass of a boost rocket to its payload, which is the total mass of HCV at the beginning of the cruise phase, can be written as[4]

$$\frac{M_\mathrm{B}^0}{M_\mathrm{p,c}^* + M_\mathrm{f,c}^*} = \prod_{i=1}^{N} \frac{1}{\exp\left(-\Delta v_i / c_{R,i}\right) - \alpha_{R,i}}, \tag{11}$$

where N is the total stage number of boost rocket, $c_{R,i}\,(= I_{R,i} \times g)$ is the jet velocity, and $\alpha_{R,i}$ is the structure mass ratio of the i-th stage.

The total velocity increment contributed by all N stages is mainly used to obtain the hypersonic cruise speed, and therefore

$$\sum_{i=1}^{N} \Delta v_i = V_\mathrm{c} + V_\mathrm{r}. \tag{12}$$

The value of V_r depends on its launch type. It is about 500 m/s for a ground launch, and about −270 m/s for an air launch. More details can be found in Ref. [5].

Combining Eqs. (10) and (11), we have

$$\frac{M_\mathrm{B}^0}{M_\mathrm{p,c}^*} = \exp\left[\frac{L - 0.25 C_\mathrm{L/D}(V_\mathrm{c}^2 - V_\mathrm{d}^2)/g}{C_\mathrm{L/D} I_\mathrm{c} V_\mathrm{c}} \left(1 - \frac{V_\mathrm{c}^2}{g r_\mathrm{E}}\right)\right] \Big/ \prod_{i=1}^{N} [\exp(-\Delta v_i / c_{\mathrm{R},i}) - \alpha_{\mathrm{R},i}]. \tag{13}$$

If set $\partial(M_\mathrm{B}^0/M_\mathrm{p,c}^*)/\partial V_\mathrm{c} = 0$, then we obtain the governing equation for the optimal speed $V_\mathrm{c,o}$. For one-stage boost rocket, the governing equation is

$$\frac{3V_\mathrm{c,o}^2}{4gr_\mathrm{E}} + \frac{I_\mathrm{c}}{I_\mathrm{R,1}\left\{1 - \alpha_\mathrm{R,1} \exp\left[(V_\mathrm{c,o} + V_\mathrm{r})/c_\mathrm{R,1}\right]\right\}} = \left(\frac{gL}{C_\mathrm{L/D}} + \frac{V_\mathrm{d}^2}{4}\right)\left(\frac{1}{V_\mathrm{c,o}^2} + \frac{1}{gr_\mathrm{E}}\right) + \frac{1}{4}; \tag{14}$$

for two-stage boost rocket, it is

$$\begin{aligned}\frac{3V_\mathrm{c,o}^2}{4gr_\mathrm{E}} &+ I_\mathrm{c}[I_\mathrm{R,1} + I_\mathrm{R,2} - I_\mathrm{R,1}\alpha_\mathrm{R,1}\exp(\Delta v_1/c_\mathrm{R,1}) \\ &- I_\mathrm{R,2}\alpha_\mathrm{R,2}\exp(\delta v_2/c_\mathrm{R,2})]/\{2I_\mathrm{R,1}I_\mathrm{R,2} \\ &[1 - \alpha_\mathrm{R,1}\exp(\Delta v_1/c_\mathrm{R,1})][1 - \alpha_\mathrm{R,2}\exp(\Delta v_2/c_\mathrm{R,2})]\} \\ &= \left(\frac{gL}{C_\mathrm{L/D}} + \frac{V_\mathrm{d}^2}{4}\right)\left(\frac{1}{V_\mathrm{c,o}^2} + \frac{1}{gr_\mathrm{E}}\right) + \frac{1}{4}.\end{aligned} \tag{15}$$

Equations (14) and (15) can be easily solved numerically, which is illustrated in Fig. 1. The computational parameters are as follows: ground launch, rocket-based cruise flight, $I_\mathrm{R,1} = 250\,\mathrm{s}$, $I_\mathrm{R,2} = I_\mathrm{C} = 290\,\mathrm{s}$, $V_\mathrm{r} = 0.5\,\mathrm{km/s}$, $V_\mathrm{d} = 1.5\,\mathrm{km/s}$, and $\alpha_\mathrm{R,1} = \alpha_\mathrm{R,2} = 0.07$.

Figure 1 demonstrates two features clearly: for a certain lift-to-drag ratio, the optimal cruise speed increases significantly with the flight distance; for a certain flight distance, the optimal cruise speed decreases when the lift-to-drag ratio increases.

Equation (13) reveals quantitatively the effects of various factors on the performance of HCVs. For a given distance, the first important factor is the cruise speed V_c, the second is the lift-to-drag ratio $C_\mathrm{L/D}$, and the third is the specific impulse of the cruise engine I_c.

Figure 2 shows the effects of the cruise speed on the flight performance of a rocket-based HCV at distances of 6 000 km and 10 000 km, respectively. The computational parameters are the same as those for Fig. 1 except that the lift-to-drag ratio is 4. Clearly, the cruise speed has an obvious effect on $M_\mathrm{B}^0/M_\mathrm{p,c}^*$. Take the distance of 10 000 km as an example, $M_\mathrm{B}^0/M_\mathrm{p,c}^*$ equals 28.3 for the optimal cruise speed of about 5 km/s whereas its value increases to 50.6 for a cruise speed of 3 km/s that is arbitrarily selected.

Hypersonic flight can be realized by different means and the flight performances are compared in Fig. 3 for three types of vehicles: the rocket-based HCV, air-breathing based HCV and the minimum-energy ballistic missile. The computational parameters for the rocket-based HCV are the same as those for Fig. 1, while the air-breathing based HCV employs the flight parameters of X-51A that were widely reported.

Figure 3 shows that both the rocket-based and air-breathing based HCVs perform much better than the minimum-energy ballistic missile does. The relative performances of the rocket-based and air-breathing based HCVs are very interesting. The values of $M_\mathrm{B}^0/M_\mathrm{p,c}^*$ for these two types are close at a distance of 1 500 km, however the air-breathing based HCV has a value of 25% lower that that of the rocket-based HCV

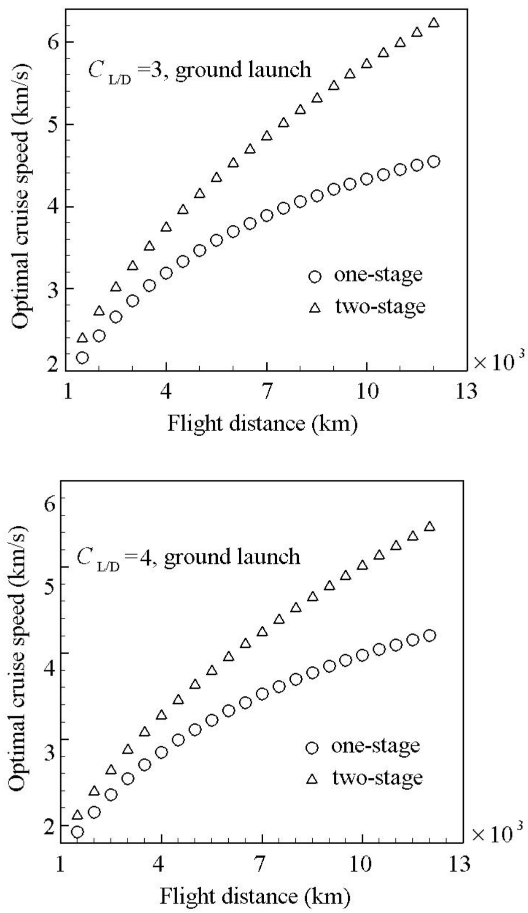

Fig. 1. Optimal cruise speed of a rocket-based HCV versus its flight distance at different lift-to-drag ratios.

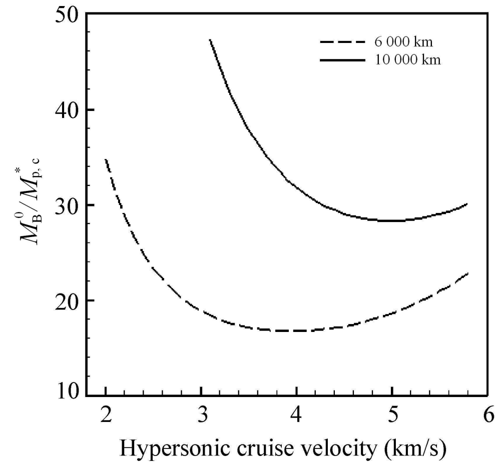

Fig. 2. Flight performance of a rocket-based HCV versus the cruise speed.

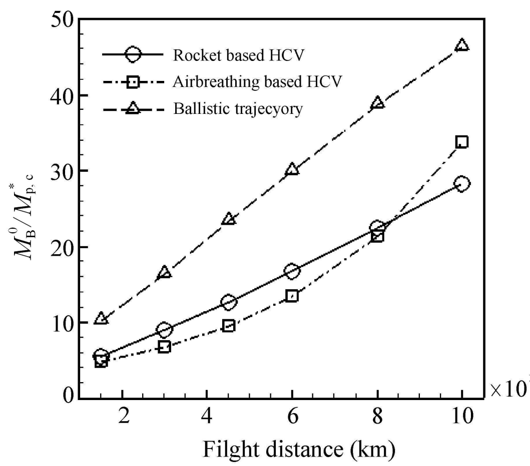

Fig. 3. Comparison of hypersonic flight performances.

at a distance of 4 000 km, and the two types of HCVs reach the same value again at a distance of 8 000 km. Finally the rocket-based HCV shows a smaller value of $M_{\rm B}^0/M_{\rm p,c}^*$ when the distance is 10 000 km.

Roughly speaking, the performances of the rocket-based and air-breathing based HCVs are at the same level, which is not as commonly expected since the air-breathing engine of X-51A has a specific impulse of 800-1000 s whereas the specific impulse of a chemical rocket is only about 250-290 s as used in this article. There are two reasons for this finding. Firstly, an air-breathing ramjet may have a large specific impulse, but the lift-to-drag ratio is usually not large (it is about 2.2 for X-51A). Secondly, an air-breathing ramjet has a strict limitation on the cruise speed, for example, the cruise Mach number of X-51A is between 5 and 7. When the Mach number increases a number of new issues appear, such as accumulated carbon in coolant channels, reduced combustor performance, deteriorated thermal environment, and even a lack of capable ground test facilities.

Hypersonic cruise vehicles (HCVs) are a focus of international aerospace technologies in the 21^{st} century. Rocket-based HCVs have relative good flight performance at the current stage. A rocket-based HCV having a realistic lift-to-drag ratio can have much better performance than a minimum-energy ballistic trajectory missile over a wide distance range (1 500 km to 10 000 km). Furthermore, its performance is close to that of an air-breathing based HCV, because air-breathing based HCVs usually have small value of lift-to-drag ratio and their cruise speed is strictly limited due to current technological levels. In the light of the mature technology of chemical rockets, rocket-based HCVs flying at the optimal speed are currently a very competitive choice.

The work was supported by the National Natural Science Foundation of China (Grant No. 10921062).

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