SEVERAL PROBLEMS IN HYDRO-ELASTO-PLASTIC DYNAMICS

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I. INTRODUCTION

Hydro-elasto-plastic dynamics is that branch of continuum mechanics where a body is subjected to such intense loading and deformation conditions that to describe its motion and deformation properly, it has to be regarded as behaving both like a fluid and like a solid possessing elastic and plastic properties. These conditions usually prevail when the body is subject to strong explosion or high velocity impact.

At the Institute of Mechanics such problems first arose in the sixties in connection with the study of underground explosion, when it was found necessary to devise a constitutive relation which automatically permits the rock mass to flow like a fluid when subjected to a pressure far in excess of its strength and deform like an elasto-plastic solid otherwise[1]. More specifically, it was assumed (1) that to count for the pressure p/specific volume v relation, equations of state of a specific form exist, i.e.

$$p = f(V, T)$$
$$e = g(p, V) \tag{1}$$

where T is the temperature and e the specific internal energy, (2) the material behaves elastically when the shear stress is sufficiently low. The elastic constants are permitted to vary in such a way that certain thermodynamic requirements are met, (3) when the shear stress exceeds a certain value dependent on the instantaneous pressure, then in addition to elastic deformation, the material flows according to the Prandtl-Reuss flow rule. The flow stress σ is assumed to take the following form

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$$\sigma = \sigma_0 + \beta_1 \tanh \beta_2 p \quad (2)$$

where σ_0, β_1, β_2 are material constants. In a later calculation, allowance was made to permit the flow stress to decrease with cumulated plastic deformation.

In the early seventies, this study was extended to (1) systematic studies of the effect of varying the various material constants [2]; (2) asymptotic study of spherical elastic waves propagating in a bilinear loading and unloading medium [3]; (3) numerical simulation of surface burst and cratering [4]. It was shown analytically [5] why the index of shock wave decay is insensitive to the equation of state for a dense medium.

More recent studies at our institute and elsewhere consist in improvement in the numerical scheme and improvement in the constitutive relation. In some of the calculation, the cap model originally developed for soil was used. Numerical schemes, analogous to or modified after the HEMP and HELP codes have been used to study axially symmetric high speed impacts as well as near surface bursts.

With these brief remarks, this paper shall focus on some of the problems that have been treated analytically or semi-analytically rather than numerically at the Institute of Mechanics. We wish to show that even for such complicated problems of hydro-elasto-plastic dynamics, simple similarity or semi-analytical arguments can often lead to basic understanding of important physical phenomena.

II. STABILITY OF METAL JETS PRODUCED BY SHAPED CHARGE

It is well known that jets produced by shaped charges eventually break into droplets. Fig. 1 shows a typical X-ray radiograph of this phenomenon. Based on analytical tools, a rather complete theory of the breaking up of such jets was given by [6] in 1980. In particular, it was shown that the breaking

Fig.1

up of the low speed portion is due to necking and can be predicted in a simple manner.

We note the ratio of stress σ at a given section in the Euler coordinate system and the flux of momentum passing through that section $\rho_j u_j^2$ is very small where ρ_j is the density of the jet and u_j the particle velocity. For this reason the motion of the jet can be regarded as one of hydro-elasto-plastic dynamics. For the same reason, each particle shall move at a constant velocity u_j, which in turn can be used to identify a particle. We have then

$$z = u_j t + b(u_j) \tag{3}$$

where $b(u_j)$ is the position of particle u_j at time $t=0$, and z is the position of the same particle at time t. It follows that for sufficiently large t, z is proportional to u_j, meaning that the velocity distribution along the length of the jet is nearly linear.

For a well designed shaped charge, $b(u_j)$ is nearly a constant. Thus each particle can be thought of as being emitted from the same point at a certain time. Eq.(3) may also be written as

$$dz/du_j = t + t^* \tag{4}$$

where t^* is a small number.

Examination of X-ray radiographs also reveals that breaking occurs when $t \gg t^*$, and that $|da/dz| \ll 1$ just before necking occurs, a being the radius of the jet elements at that time. Thus we may study the necking and breaking of the jet at a material point with velocity u_j as that of a jet of uniform radius a and uniform velocity gradient du_j/dz. One notes that whereas the kinematic parameter du_j/dz varies with time, $\rho_j du_j/dm$, also a kinematic parameter, does not, thus the latter denoted by Δ^{-1} represents a characteristic property of the jet. Physical argument then leads to the follwing functional relation between the time t_b when breakage occurs and the other pertinent parameters of the jet,

$$t_b/t^* = f(\rho_j \Delta/\sigma t^{*3}) \qquad (5)$$

where σ is the flow stress. For the case $t^* \to o$, we have

$$t_b = c(\rho_j \Delta/\sigma)^{1/3} \qquad (6)$$

where c is a proportional constant, and

$$z_b = u_j t_b + b \qquad (7)$$

Eq.(6) and (7) are found to describe well the breakage of the slower moving portion of the jet as shown in Fig.2.

It can be seen from the same figure that the faster moving part clearly follows a different rule. As explained in [6], this may very well be the result of aerodynamic instability as opposed to necking.

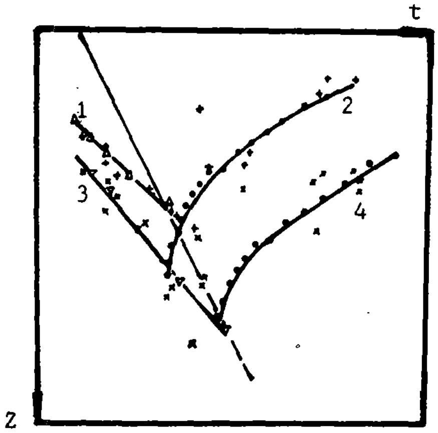


Fig.2 Necking: faster section—1
slower section—2
Breakage: faster section—3
slower section—4

III. PENETRATION OF JET IN TARGET

Since the length of the jet and the depth of penetration are large compared to the radius of the jet, it is sufficiently accurate to assume that the penetration process is stationary. This implies that we may assume that a unique relation exists between velocity u_j and the penetration velocity u. For example, when u_j is sufficiently large, the classical hydrodynamic theory yields the following relation

$$u = u_j/(1+\sqrt{\rho_t/\rho_j}) \qquad (8)$$

where ρ_t is the density of the target. Under other conditions, new u-u_j relations must be sought.

It is known that for steel targets, Eq.(8) is no longer accurate when u_j is less than 3-4 km/s, depending on the particular steel used. Work has been done [7] to modify this $u-u_j$ relation by taking into account of the effect of target strength.

To do this we note that the net force F acting on the target is given by

$$F = 2\pi a^2(u_j - u)^2 \qquad (9)$$

First we regard F as a travelling concentrated load in an infinite elastic medium and by calculating the stress field determine approximately the elastic-plastic boundary along the axis of penetration. This leads to (Fig.3)

$$z_p = -cF/\sigma = c2\pi a^2\rho_j(u_j - u)^2/\sigma \qquad (10)$$

Here σ is the yield stress and c a dimensionless constant dependent on the Mach numbers $M_d = u/c_d$ and $M_s = u/c_s$, where c_d and c_s are respectively the dilatational and shear wave speeds. One obtains at the same time the compression stress σ_{zp} at that point.

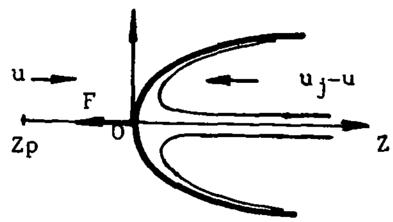


Fig.3

Next, we assume that (1) the velocity field around the stagnation point O and up to z_p is adequately described by the classical hydrodynamic theory, (2) the stress field in the same region can be calculated from this velocity field according to the Prandtl-Reuss flow rule for an ideally rigid-plastic material, (3) the flow is adiabatic and plastic work converts entirely into heat. Then, it can be shown that along the axis of symmetry, the momentum equation and energy equation reduce respectively to

$$d(\rho_t q^2/2 + p + 2\sigma/3) = -\frac{2}{3}\sigma\frac{d^2z}{dq^2}dq/\frac{dz}{dq} \qquad (11)$$

$$\rho_t cqdT = -\sigma dq \qquad (12)$$

where q is the axial veloctiy, c the specific heat per unit mass of the target, T the temperature(absolute). In the above equation q is regarded as the independent variable in the range $0<q<u$ and z is a known function of q. Obviously,

$$p + 2\sigma/3 = \rho_j(u_j - u)^2/2 \qquad \text{at } q = 0 \qquad (13)$$

and

$$T = T_o(\text{room temperature}) \qquad \text{at } z = z_p \qquad (14)$$

We note also that there is a mismatch of the elastic solution and the hydrodynamic flow at z_p, so that according to Eq.(11), a discontinuity has to be introduced there, namely,

$$(\rho_t q^2/2 + p + 2\sigma/3) = \rho_t u/2 + \sigma_{zp} \qquad \text{at } z = z_p \qquad (15)$$

Given σ as a function of pressure p and temperature T, one can then compute in principle $u_j(q_p)$ and $u(q_p)$ by integrating the two first order ordinary differential equations (11) and (12) by using (13) (14), (15) as boundary conditions, and obtain at the same time the temperature and pressure distribution along the z axis ($z_p< z<0$). For an arbitrary q_p, one first calculates z_p/a, which in turn gives $\rho_j(u_j-u)^2$ in accordance with (10). Eqs (14) and (15) then yield p_p (p at z_p) as a function of u. One can then initiate the integration of equations (11) and (12) from $z=z_p$. The value of u must be such that the boundary condition at $q = 0$, namely (13), is satisfied. Thus one obtains $u(q_p)$ by trial and error and finally $u_j(q_p)$.

Computation has been carried out for the planar case and for

$$\sigma = \sigma_0(1 + \mu p) F(\eta) , \qquad \eta = T/T_{Mo}(1 + \gamma p)^{\frac{1}{4}} \qquad (16)$$

and

$$F(\eta) = \begin{cases} 1 & \text{for } \eta < \eta_1 \\ (\eta_2 - \eta)/(\eta_2 - \eta_1) & \text{for } \eta_1 < \eta < \eta_2 \\ 0 & \text{for } \eta_2 < \eta \end{cases} \qquad (17)$$

where σ_0, T_{Mo}, μ, γ, η_1, η_2, are material constants having obvious physical meaning. In this way it is possible to explore the effect of material properties on the u-u_j relation. For metals it appears that next to ρ_t and σ_0, μ may be an important factor whereas thermal softening does not play a significant role. Analysis also shows that if one writes the u-u_j relation in the following form

$$\rho_t u^2/2 + k\sigma_0 = \rho_j(u_j-u)^2/2 \quad (18)$$

then instead of being a constant as was assumed by Eichelberger, k actually increases with u_j. This is in qualitative agreement with our experiment for the axially symmetric case.

IV. PENETRATION AND PYROLYSIS

For metals almost all of the mechanical work of deformation is converted into heat of which only a very small fraction can be transformed back into mechanical energy. For non-metals the situation can be quite different.

Observati by flash X-ray radiography in our laboratory reveals that when a jet penetrates targets made of certain composites, the hole made by the jet caves in at very high speeds, so that with the exception of the head, the body of the jet soon becomes completely wrapped around by the target material.

An explanation of this new phenomenon was put forward in [8]. According to this explanation, a considerable portion of the work of deformation is used to produce a gaseous state through pyrolysis. Thus, close to the point of penetration, there is a two phase medium subjected to a very high hydrostatic pressure. As the target material passes around the stagnation point, the external pressure is released, and the high pressure gaseous phase causes the target material to expand and wrap around the jet.

Existing data on reaction rate, our own experiment on pyrolysis, and temperature measurement of the target during penetration all indicate that such an explanation is valid. Furthermore we modified on purpose the strenth and pyrolysis properties of the material, and the maximum penetration depth varied in a way as predicted. X-ray radiographs gave independent confirmation of this explanation.

The jet produced by shaped charge usually possesses a positive velocity gradient, with the tip velocity much greater than that of the tail. Therefore it stretches during flight and eventually breaks up into droplets or fragments. As long as the jet remains integral, the caving in of the wall of the hole does not appear to disturb the motion and penetration. There, however, is a dramatic change after it breakes. Experimentally it was found that concurrent with breakage of the jet, the penetration velocity u drops rapidly and the penetration-time relation deviates considerably from the previously closely followed hydrodynamic relation. Systematic flash X-ray radiography study showed that as a result of the contraction of the hole, many fragments were pushed aside or knocked out of the line of sight, and deformed in an apparently random manner. Thus a good portion of the jet did not contribute to penetration. From the measured p-t relation alone, one would get the false impression as if such a material offered greater resistance than steel armour plates.

To count for this phenomenon two stochastic models of penetration have been developed in our laboratery, each with but one adjustable constant in order to fit the theoretical curves to the experimental p-t relation[9]. Presumably these constants are related to the mechanical, thermal and pyrolytic properties of the target.

V. MECHANISM OF INTERFACIAL WAVE FORMATION IN EXPLOSIVE WELDING

Aside from pratical use explosive welding provides a unique case where both the deformational and metallurgical effects of intense loading on metals can be studied in detail.

Fig.4 shows schematically a symmetric configuration where two identical plates are used. Because at and near

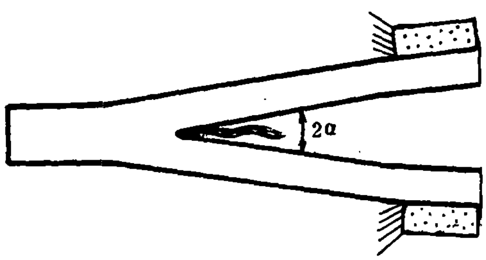
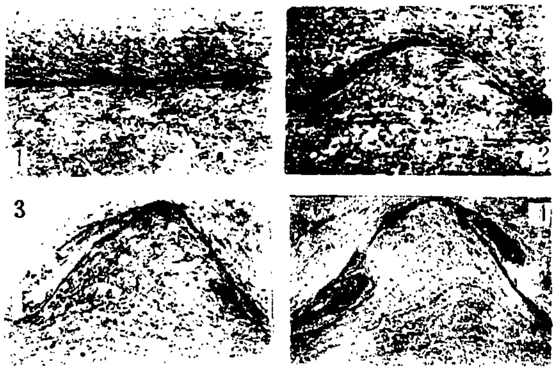


Fig.4

the point of collision the pressure is much higher than the strength of metal,as a first approximation, the flow can be described by the classical hydrodynamic theory when a steady state is reached soon after initiation of detonation. According to this theory, a thin jet(known as the reentrant jet) of thickness equal to $2H\sin^2\alpha/2$ is produced, where α is the so-called dynamical angle of collision and H is the thickness of each of the original plates. This jet serves to clean the surfaces of the two plates so that fresh, unpolluted surfaces come into contact under very high pressure to achieve a solid state bond. Fig.5 is an X-ray radiograph showing that a jet of some kind does exist.

Fig.5

Fig.6 shows a series of micrographs. It is seen that as the collision speed U relative to the stagnation point is increased, waves of greater and greater distortion appear in succession. The relative wave length λ/H varies in a way shown in Fig.7.



1.$\alpha=13^0$ U=1400m/s 2.$\alpha=13^0$ U=1500m/s
3.$\alpha=13^0$ U=1670m/s 4.$\alpha=13^0$ U=2170m/s

Fig.6

Understanding the mechanism of wave formation is a first step in gaining full knowledge of this high speed impact phenomenon. For this reason much effort has been directed to resolve this problem. Several theories exist so far, but none of them appears to be satisfactory. For example, by taking the velocity component along the salient jet at one particular section down stream of the stagnation point as the velocity field of a parallel incompressible and

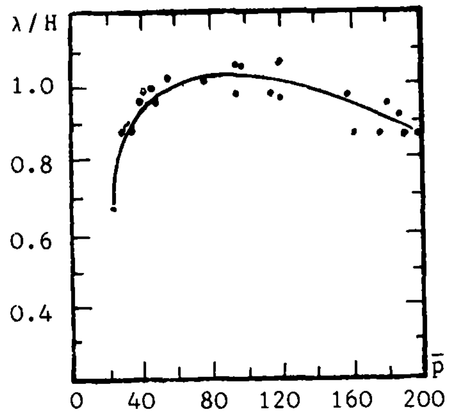


Fig.7

inviscid flow, Utkin[10]showed that, according to Helmholtz instability theory, the most preferred wave length of unstable sinusoidal disturbances is given by the following formula

$$\lambda/H = 128\sin^2\alpha/2 \qquad (19)$$

While this formula fitted well the data he quoted, it is entirely powerless in predicting the dependence on density which we found to be quite significant. This fact alone appears to rule out the validity of the theoretical basis of Utkin's analysis. Our experiment also showed that the dependence on scale(size)is not significant,so that strain rate effects are not important, at least in our range of interest.

Like previous authors we take the classical hydrodynamic flow as our point of departure. This theory gives

$$\pi z/H = -(1-\cos\alpha)\ln(1-\xi)+(1+\cos\alpha)\ln(1+\xi)-e^{i\alpha}\ln(1+\xi e^{i\alpha})-e^{-i\alpha}\ln(1+\xi e^{-i\alpha}) \qquad (20)$$

where $z=x+iy$ and ξ is the complex conjugate of the velocity vector $u+iv$ normalized by the incoming jet velocity U so that $|\xi|<1$.

Next we note that the surface of the plates can not behave like a fluid. We therefore assume that on the reentrant jet side a thin layer of thickness $\bar{h}(=h/H)$ is peeled off such that higher order discontinuity may take place streamwise between this layer and the adjacent material regarded as a fluid. Introducing a small disturbance ζ (antisymmetic with respect to axis of symmetry) representing displacement normal to the free surface on the reentrant jet side.

$$\zeta = e^{st-\delta s/U}G(s) \qquad (21)$$

We establish the following integro-differential equation for $G(\bar{s})$[11],

$$\bar{h}\int_{-\infty}^{\bar{s}}\frac{G(s')}{\bar{\kappa}^2}ds'-\bar{h}\frac{dG(\bar{s}')}{d\bar{s}}+\frac{n}{U_0}-\frac{e^{-i\Omega\bar{s}}}{2\pi}\int_{-\infty}^{\infty}e^{i\Omega\bar{s}}\frac{dG(\bar{s}')}{d\bar{s}'}\ln\frac{1-\cos[\theta(\bar{s})+\theta(\bar{s}')]}{1-\cos[\theta(\bar{s})-\theta(\bar{s}')]}d\bar{s}'=0 \quad (22)$$

where $\bar{\kappa}$ is curvature of the free streamline, $\bar{s}$ the distance s measured along this free streamline, both normalized by the minimum curvature κ_0 on the reentrant jet side, $i\Omega=\delta\kappa_0/U$, θ is the angle of inclination of the free streamline as defined by Eq.(20), and n/U_0 is an arbitrary constant, set to one in our numerical calculation.

Once $G(\bar{s})$ is found, one can compute the transverse displacment of the center line downstream of the stagnation point(on the salient jet side). We find that the points along the center line oscillate in phase with circular frequency $\Omega U/\kappa_0$ and the amplitude of oscillation decays monotonically down stream of the stagnation point. Our results also show that transverse displacement reaches maximum values for certain discrete values of Ω . The lowest value of these is found to be approximately 0.3.

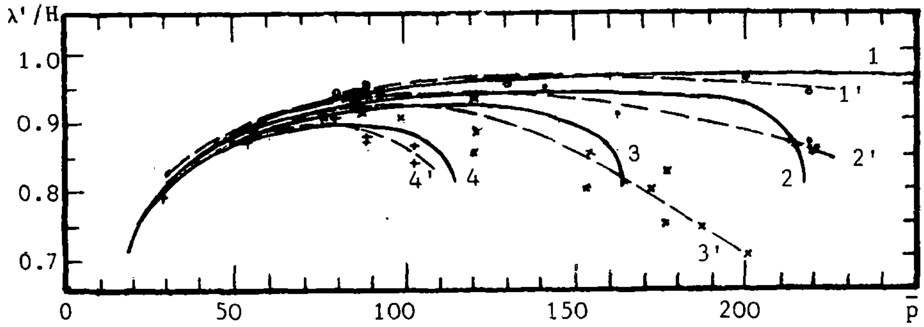
A crucial point in the present theory is that through a certain transition region downstream of the stagnation point, the oscillation becomes frozen. We idealize this transition region by a sharp line at a point along the center line determined essentially by the ratio betweem the hydrodynamic pressure p and the ultimate strength σ_Y. Thus on the entrance side of the transition point the material oscillates and behaves like a fluid while on the exit side each material point takes up the position that particle assumed at the moment when it entered this transition point with velocity U_x because this region behaves like a rigid solid. This consideration then leads to the following formula for the wave length of interfacial wave

$$\lambda=2\pi\kappa_0 U_x/\Omega U_0 \quad (23)$$

Since $\kappa_0\cong H\sin^2\alpha/2$ for sufficiently small α, we have, after inserting an adjustable constant to fit experimental data,

$$\lambda/H=\text{const}\,\frac{2\pi}{\Omega}\,\frac{U_x}{U}\,\sin^2\alpha/2 \quad (24)$$

As far as the dependence on α is concerned,this formula agrees with Utkin's. U_X/U has been computed in such a way as to include the effects of density, strength and compressibility [12]. Fig.8 shows general agreement with experiment for four metals and alloys.



——— Calculated 1-Al, 2-Cu, 3-iron, 4-stainless steel

- - - - Experimental 1'-Al, 2'-Cu, 3'-mild steel, 4'-stainless steel

$\lambda' = \lambda/(\text{const}\ 2\pi/\Omega \sin^2 \alpha/2)$

Fig.8

VI. CLOSING REMARKS

This paper presents results of approximate analysis of several problems in hydro-elasto-plastic dynamics. Although most of them are more qualitative than quantitative, we believe they do add to our understanding of the physics of several previously unclarified phenomena.

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