THE EFFECTS OF ROUGHNESS CHANGE ON THE SURFACE LAYER
OF THE ATMOSPHERE (I)—THE VARIATIONS OF WIND
SPEED AND SHEAR STRESS

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Abstract
A general four-layer structure linear theory for predicting the effects of arbitrarily distributed roughness change on the variations of wind speed and shear stress in the surface layer of 3D and 2D atmospheres was presented. The results derived by the theory were agreeable to the previous ones.

Key words atmospheric boundary layer, surface roughness

I. Introduction

When fully-developed atmospheric boundary layer encounters the surface with change in surface roughness (such as from water to land, from lower vegetation to higher vegetation, from suburbs to city and vice versa), the wall shear stress will undergo a variation, and the wind speed and shear stress will, in turn, be subjected to variations. The flow pattern will differ from the original.

To understand the effects of roughness change on the surface layer of the atmosphere is of significance and has many applications. Examples can be found in mass transfer induced by wind\(^1\), wind load of building, and utilization and exploration of wind energy\(^2\) etc. Therefore, the problem has been attracting attention of the scientists in the field of environment, atmosphere, oceanography, meteorology, hydrology, geography and soil physics.

Since the pioneer work of Elliott (1958)\(^3\), as far as the effects of roughness change on the surface layer of the atmosphere is concerned, considerable advances have been made in the aspect of theoretical analysis, observation, experiment and numerical simulation. The author has reviewed the advances in his recent publication\(^4\), so we will no longer go into details herein.

Hitherto most theories have been based on the assumption that throughout the internal boundary layer, the advective velocity is the upwind velocity at the top of the internal boundary layer. This is true over most of the depth of the internal boundary layer, but is invalid near the surface, which leads to wind speed and shear stress error near the surface\(^5\).

Furthermore, most previous theories dealt with simple 2D problems, such as flow over sudden change in surface roughness. No general theory is available for such complex flow as flow over
arbitrarily distributed roughness and 3D problems.

This paper is intended to present a general theory, which overcomes all the shortcoming the previous theories had and is applicable to arbitrarily distributed roughness change and 3D (which can be degenerated to 2D) problems.

The theory, which borrowed the idea of Hunt et al. (1988)\(^6\) for turbulent flow over low hill, divided the surface layer of atmosphere into two regions, each consists of two sublayers. In the outer region, turbulent shear stress perturbations is ignorable. Inertia perturbation balances the pressure gradient perturbation. The outer region is made up by an outer layer and a middle layer. In the outer layer, perturbations are of potential. In the middle layer, wind shear dominates. The inner region is composed of a shear stress layer and an inner surface layer. In the shear stress layer, acceleration balances the shear stress and in the inner surface layer flow is adapted to the changed roughness, and the momentum flux remains constant vertically.

II. Control Equations and Boundary Conditions

Except being indicated in particular, all the variables and parameters used in this paper are made dimensionless with characteristic length \(z_0\) (surface roughness), and characteristic velocity \(u_{*0}=(\tau_0/\rho)^{1/2}\) (friction velocity).

For \(x<0\) (refer to Fig. 1), a fully developed turbulent boundary layer in a neutral atmosphere and a uniform surface roughness are assumed. Horizontal distribution of the velocity is given by

\[
u_0=\frac{1}{\kappa} \ln z \quad (1 \leq z \leq h) \tag{2.1}\]

where \(\kappa=0.4\) is Von Kármán's constant, and \(h\), dimensionless thickness of the surface layer, is usually taken to be the order of 100m/\(z_0\).

Within the scales of \(0<x<L_x, 0<y<L_y\), surface roughness changes, the local roughness is expressed by \(z_t(x,y)\).

In general, the horizontal and lateral scales are much larger than the vertical scale for the surface layer of the atmosphere. It follows that only the vertical gradient and flux are of importance. In addition, the turbulent viscosity is much larger than the molecular viscosity (except in the viscous sublayer close to the wall). In the surface layer, Coriolis force is small and can be neglected. Consequently, the equations controlling the surface layer of the atmosphere are usually as follows\(^7\)

\[
\begin{align*}
    u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= - \frac{\partial p}{\partial x} + \frac{\partial \tau_{13}}{\partial z} \\
    u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} &= - \frac{\partial p}{\partial y} + \frac{\partial \tau_{23}}{\partial z} \\
    u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} &= - \frac{\partial p}{\partial z} \\
    \frac{\partial u}{\partial x} + \frac{\partial z}{\partial y} + \frac{\partial w}{\partial z} &= 0
\end{align*}\tag{2.2a,b,c,d}\]

where \(\tau_{13}\) and \(\tau_{23}\) are turbulent shear stresses. According to the K theory they are expressed as

\[
\tau_{13}=K_m \frac{\partial u}{\partial z}, \quad \tau_{23}=K_m \frac{\partial v}{\partial z} \tag{2.3a,b}\]

Adopting the exchange coefficient recommended by Panofsky and Dutton\(^7\),

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\[ K_m = \kappa u_0 z, \]  

(2.4)

and noting that \( u_+ = r_{13} \) (2.3) become

\[ r_{13} = \kappa^2 z^2 \left( \frac{\partial u}{\partial z} \right) \quad \text{and} \quad r_{15} = \kappa^2 z^2 \frac{\partial u}{\partial z} \frac{\partial v}{\partial z} \]  

(2.5ab)

Equations (2.2) and (2.5) constitute a set of closed control equations.

The boundary conditions are

\[ u = u_0(x), \quad v = w = 0 \quad \text{as} \quad z \to h \]  

(2.6a)

\[ u = v = w = 0 \quad \text{as} \quad z \to z_i \]  

(2.6b)

Assume that after surface roughness changes, the velocities, pressure, shear stresses are expressed in terms of their upstream values and a perturbation as

\[ u = u_0(x) + \Delta u, \quad v = \Delta v, \quad w = \Delta w \]  

(2.7a)

\[ p = p_0 + \Delta p \]  

(2.7b)

\[ r_{13} = 1 + \Delta r_{13}, \quad r_{15} = \Delta r_{15} \]  

(2.7c)

and the perturbations are smaller than their upstream values,

\[ |\Delta u|, \quad |\Delta v|, \quad |\Delta w| \ll |u_0| \]  

(2.8a)

\[ |\Delta p| \ll |p_0| \]  

(2.8b)

\[ |\Delta r_{13}|, \quad |\Delta r_{15}| \ll 1 \]  

(2.8c)

Substituting (2.7) into (2.2) and (2.5), and neglecting the higher order perturbations, we obtain the linearized form of equation (2.2)

\[ u_0 \frac{\partial \Delta u}{\partial x} + \Delta w \frac{\partial u_0}{\partial z} = - \frac{\partial \Delta p}{\partial x} + \frac{\partial \Delta r_{13}}{\partial z} \]  

(2.9a)

\[ u_0 \frac{\partial \Delta v}{\partial x} - \frac{\partial \Delta p}{\partial y} + \frac{\partial \Delta r_{13}}{\partial z} \]  

(2.9b)

\[ u_0 \frac{\partial \Delta w}{\partial x} = - \frac{\partial \Delta p}{\partial z} \]  

(2.9c)

\[ \frac{\partial \Delta u}{\partial x} + \frac{\partial \Delta v}{\partial y} + \frac{\partial \Delta w}{\partial z} = 0 \]  

(2.9d)

\[ \Delta r_{13} = 2 \kappa z^2 \frac{\partial \Delta u}{\partial z} \]  

(2.9e)

\[ \Delta r_{15} = \kappa^2 z^2 \frac{\partial \Delta v}{\partial z} \]  

(2.9f)

In a similar way, boundary condition (2.6) becomes

\[ \Delta u, \quad \Delta v, \quad \Delta w = 0 \quad \text{as} \quad z \to h \]  

(2.10a)

\[ \Delta u = \frac{M}{\kappa}, \quad \Delta v = \Delta w = 0 \quad \text{as} \quad z \to z_i \]  

(2.10b)

where \( M = -1g z_i \) is surface roughness parameter, which reflects the magnitude of the change in surface roughness. \( M \) is usually a function of \( x \) and \( y \), if the surface roughness is arbitrarily distributed.

Denoting the Fourier transform of any variable \( R(x, y, z) \) as \( \mathcal{R}(k, k_z, z) \), the definition is
\[ R(k_1, k_2, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R(x, y, z) \exp[-i(k_1x + k_2y)] dx dy \]  

(2.11)

where \( k_1 \) and \( k_2 \) are wavenumbers. Taking the Fourier transform of (2.9) with regard to \( x \) and \( y \), we have.

\[
\begin{align*}
  ik_1 u_0[\Delta u] + i\Delta w \frac{du_0}{dz} &= -ik_1[\Delta p] + \frac{\partial[\Delta r_{13}]}{\partial z} \\
  ik_1 u_0[\Delta v] &= -ik_1[\Delta p] + \frac{\partial[\Delta r_{23}]}{\partial z} \\
  ik_1 u_0[\Delta w] &= -\frac{\partial[\Delta p]}{\partial z} \\
  ik_1[\Delta u] + ik_2[\Delta v] + \Delta w \frac{\partial[\Delta w]}{\partial z} &= 0 \\
  [\Delta r_{13}] &= 2\kappa z \frac{\partial[\Delta u]}{\partial z} \\
  [\Delta r_{23}] &= -\kappa z \frac{\partial[\Delta v]}{\partial z}
\end{align*}
\]  

(2.12)

Similarly, taking Fourier transform of boundary condition (2.10) on \( x \) and \( y \) produces.

\[
[\Delta u], [\Delta v], [\Delta w] = 0 \quad \text{as} \quad z \to h
\]  

(2.13a)

\[
[\Delta u] = \frac{[M]}{\kappa}, [\Delta v], [\Delta w] = 0 \quad \text{as} \quad z \to z_1
\]  

(2.13b)

III. Four-Layer Structure Theory

As shown in Fig. 2, the surface layer of the atmosphere is vertically divided into two regions, an inner region close to the wall \((z_1 < z < l)\) and an outer region \((l < z < h)\).

![Fig. 1 Fully-developed atmospheric boundary layer flows over roughness change surface](image)

![Fig. 2 Four-layer Structure](image)
3.1 The outer region

In the outer region the shear stress perturbations are ignorable, and the inertia perturbations balance the pressure gradient perturbation. The region is made up of an outer layer and a middle layer. The perturbations in the outer layer are of potential and wind shears dominate in the middle layer.

Neglecting the shear stress perturbations is the outer region, an equation for $[\Delta w]$ can be derived from equation (2.12),

$$\left\{ \frac{\partial^2}{\partial z^2} - (k_1^2 + k_2^2) - \frac{1}{u_0} \frac{d^2 u_0}{d z^2} \right\}[\Delta w] = 0 \tag{3.1}$$

The variation of $u_0$ in the outer region is rather small, hence,

$$k_1^2 + k_2^2 \gg \frac{1}{u_0} \frac{d^2 u_0}{d z^2}, \tag{3.2}$$

While at the interface of the outer layer and the middle layer we have,

$$k_1^2 + k_2^2 \approx \frac{1}{u_0} \frac{d^2 u_0}{d z^2},$$

which defines the height of the middle layer,

$$h_m = (1n h_m)^{-1/2}(k_1^2 + k_2^2)^{-1/2}. \tag{3.3}$$

Taking (3.2) into account, equation (3.1) becomes,

$$\left\{ \frac{\partial^2}{\partial z^2} - (k_1^2 + k_2^2) \right\}[\Delta w] = 0 \tag{3.4}$$

The approximate solution* of (3.4) is,

$$[\Delta w] = C \exp[-\sqrt{k_1^2 + k_2^2}(z - h_m)] \tag{3.5c}$$

where $C = [\Delta w]_{z=h_m}$ is a function of $k_1$ and $k_2$, and can be determined by matching the solutions in the outer layer and the middle layer.

From momentum equation, we have

$$[\Delta u] = -i C k_1 (k_1^2 + k_2^2)^{-1/2} \exp[-(k_1^2 + k_2^2)^{1/2}(z - h_m)] \tag{3.5a}$$
$$[\Delta u] = -i C k_2 (k_1^2 + k_2^2)^{-1/2} \exp[-(k_1^2 + k_2^2)^{1/2}(z - h_m)] \tag{3.5b}$$
$$[\Delta p] = i C k_1 (k_1^2 + k_2^2)^{-1/2} \exp[-(k_1^2 + k_2^2)^{1/2}(z - h_m)] \tag{3.5d}$$

In the middle layer, assume that the vertical scale is much smaller than the horizontal scale $L_x$ and lateral scale $L_p$ and the second term is ignorable to compare with the first term in (3.1). The equation for the middle layer is:

$$\left\{ \frac{\partial^2}{\partial z^2} - \frac{1}{u_0} \frac{d^2 u_0}{d z^2} \right\}[\Delta w] = 0 \tag{3.6c}$$

the solution is

$$[\Delta w] = i (k_1^2 + k_2^2)^{1/2} u_0 \left( A + B \right)^{z / u_0^2} \tag{3.7c}$$

* Note that $1 \ll h_m \ll h$, therefore the boundary condition (3.13a) is approximately satisfied in (3.5).
where $A$ and $B$, functions of $k_1$ and $k_2$, are determined by matching conditions.

The following solution can be derived from a momentum equation and continuity equation,

\[
[\Delta u] = -(k_1^s + k_2^s)^{1/2} \frac{u_0^s}{k_1^s} (A + B) \int_0^z \frac{dz'}{u_0^s} - k_1^s (k_1^s + k_2^s)^{1/2} \frac{B}{u_0^s} \tag{3.7a}
\]

\[
[\Delta v] = -k_2^s (k_1^s + k_2^s)^{-1/2} \frac{B}{u_0^s} \tag{3.7b}
\]

\[
[\Delta \rho] = k_1^s (k_1^s + k_2^s)^{-1/2} B \tag{3.7d}
\]

### 3.2 Inner region

Inner region is composed of a shear stress layer and an inner surface layer. In the shear stress $(l, z < l)$, the inertia perturbation balances the shear stress perturbation. While in the inner surface layer, the flow is adapted to the changed roughness, and the momentum flux remains constant. In the inner region, $\eta = z/l$ and a small parameter $\varepsilon = 1/\eta$ is induced then the velocity distribution upstream (2.1) is rewritten as

\[
u_0(\eta) = \frac{1}{\kappa} \ln (1 + \varepsilon \ln \eta) \tag{3.8}
\]

In addition, the solutions to (2.12) are found by expressing the perturbations as asymptotic series in

\[
[\Delta u] = [u^0] + \varepsilon [u^1] + \cdots \tag{3.9a}
\]

\[
[\Delta v] = \varepsilon [v^1] + \cdots \tag{3.9b}
\]

\[
[\Delta w] = \varepsilon [w^1] + \cdots \tag{3.9c}
\]

\[
[\Delta \rho] = [\rho^0] + \varepsilon [\rho^1] + \cdots \tag{3.9d}
\]

\[
[\Delta \tau_{13}] = [\tau_{13}^0] + \varepsilon [\tau_{13}^1] + \cdots \tag{3.9e}
\]

\[
[\Delta \tau_{23}] = \varepsilon [\tau_{23}^1] + \cdots \tag{3.9f}
\]

In the shear stress layer, $||\ln l|| > ||\ln \eta||$. Substituting (3.8) and (3.9) into (2.12), by balancing between the inertia force and shear stress we obtain,

\[
l \cdot \ln l = \frac{2\kappa^2}{|k_1^s|} \tag{3.10}
\]

This relation defines the height of the inner region. From (2.12c), we have

\[
[\rho^0] = [\rho^0(k_1, k_2)] \tag{3.11}
\]

The first and second order equations for velocities are derived from (2.12a) and (2.12b),

\[
i[u^0] = \text{sign}(k_1) \frac{\partial}{\partial \eta} \left( \eta \frac{\partial [u^0]}{\partial \eta} \right) \tag{3.12a}
\]

\[
i[u^1] = \text{sign}(k_1) \frac{\partial}{\partial \eta} \left( \eta \frac{\partial [u^1]}{\partial \eta} \right) - i\eta \ln \eta [u^0] \frac{[w^1]}{2\kappa^2 \eta} - i\kappa [\rho^0] \tag{3.12b}
\]

\[
i[u^1] = \frac{1}{2} \text{sign}(k_1) \frac{\partial}{\partial \eta} \left( \eta \frac{\partial [u^1]}{\partial \eta} \right) - i\kappa \frac{b}{k_1^s} [\rho^0] \tag{3.12c}
\]

where \text{sign} is the sign function.
The solutions to (3.12) are respectively,

\[ [u^0] =DK_0(2\sqrt{i\eta\text{sign}(k_1)}) \]  

\[ [u^1] =E_0[u^0] - \eta \frac{\partial[u^0]}{\partial \eta} (2-1\eta) \]

\[ -\kappa[p^0] + \frac{3}{2}K_0(2\sqrt{i\eta\text{sign}(k_1)})/3 \]  

\[ [v^1] =E_1K_0(2\sqrt{i2\eta\text{sign}(k_1)}) - \frac{k_1}{K_1}[p^0] \]  

where \( D, E_0, E_1 \), functions of \( k_1 \) and \( k_1 \), are determined by matching conditions, \( K_0 \) is the zero order Bessel function.

From continuity equation,

\[ [w^1] = -2\kappa^2 \int_{\eta_i}^{\eta} [u^0] d\eta' \]  

where \( \eta_i = z_i/l \).

When \( z \) is small, and \( 1n \) and \( 1n \eta \) have the same order, which means the second term in (3.8) is not neglectable, which imply that close to the wall, there exists an inner surface layer. In this layer, flow is adapted to the changed roughness. Similar to the surface layer of the atmospheric boundary layer (constant flux layer), it is believed that in the inner surface layer \( (l=0.1l) \), the momentum flux is a constant, therefore we have

\[ \frac{\partial[r_{1}]}{\partial z} = 0, \quad \frac{\partial[r_{2}]}{\partial z} = 0 \]

Noting (3.9), it follows that

\[ \frac{\partial[r_{1}]}{\partial z} = 0, \quad \frac{\partial[r_{1}]}{\partial z} = 0, \quad \frac{\partial[r_{1}]}{\partial z} = 0 \ (3.14a, b; c) \]

Integrating the above equations, and taking the boundary layer (2.10) into account leads to

\[ [u^0] =F[\ln \frac{z}{z_i} + \frac{M}{\kappa}] \]  

\[ [u^1] =E_0[\ln \frac{z}{z_i}] \]  

\[ [v^1] =E_1[\ln \frac{z}{z_i}] \]

where \( F, H_0, H_1 \), functions of \( k_1 \) and \( k_1 \) are determined by matching conditions.

Finally, from the matching between the solutions in each layer, the following functions are found,

\[ D = \frac{2[M]}{\kappa} b, \quad F = -\frac{M}{\kappa} b \]  

\[ E_0 = \frac{\kappa^2}{[M]}[p^0] - \frac{2}{3} - (2 + 1n \frac{l}{z_i} + \frac{3}{2} 1n 3)b \]  

\[ (3.16a, b) \]  

\[ (3.16c) \]
\[ H_0 = \frac{[M]b}{\kappa} \left( \begin{array}{c} \kappa^2 \left[ [p^0] - \left( 2 + 2\gamma + \frac{\pi}{2}i \text{sign}(k_1) \right) b \right] \end{array} \right) \] (3.16d)

\[ E_1 = 2\kappa k_1 [p^0] C \] (3.16e)

\[ H_1 = -\kappa k_1 [p^0] C \] (3.16f)

\[ A = 0 \] (3.16g)

\[ B = -k_1 (k_1^2 + k_1^2)^{-1/2} d \] (3.16h)

\[ C = ik_1 (k_1^2 + k_1^2)^{-1/2} \frac{1}{u_0(h_m)} \] (3.16i)

\[ [p^0] = -k_1 (k_1^2 + k_1^2)^{-1/2} \] (8.16j)

where

\[ a = \ln \frac{1}{z_1} - 2\gamma, \quad b = \left( a - \frac{\pi}{2} i \text{sign}(k_1) \right)^{-1} \]

\[ c = \left( a - \frac{\pi}{2} i \text{sign}(k_1) - \ln 2 \right)^{-1}, \quad d = u_0 (I) \int_{\eta_0}^{\eta} [u^0] d\eta' \],

and \( \gamma = 0.5772 \) is Euler's constant.

IV. Case Study and Discussion

In order to make comparison with available previous results, the variables and the parameters used in the following examples have their dimensions.

4.1 Step change in surface roughness (2D)

Bradley (1968)\(^{10}\) made observation on the variations of wind speed and shear stress over a step change in surface roughness. For \( x < 0 \), roughness \( z_0 = 0.002 \text{cm} \), and for \( x > 0 \), roughness \( z_1 = 0.25 \text{cm} \). Using a second order turbulent model, Rao et al. (1974)\(^{11}\) investigated numerically the step change in surface roughness on the mean flow and turbulent structure in the neutral surface layer of the atmosphere, and made comparison with Bradley's observation. In order to verify the present theory, we apply the theory to the case both with results of observation and simulation. For the case considered,

\[ [M] = \int_{-\infty}^{\infty} M \exp[-ik_1x] dx = \frac{M}{ik_1} \]

where \( M = \ln z_0/z_1 = -4.85 \).

In Fig. 3 is shown the wind speed comparison between present theory, simulation and observation. The agreement is good. Wall shear stress is compared in Fig. 4. The result predicted by the present theory is slightly higher than Rao's, which happens to be closer to the measured data.

4.2 Two subsequent abrupt changes in surface roughness (2D)

The only results available to compare with is Blom and Wartena (1969)\(^{10}\)'s. For \( x < 0 \) and \( x > 0 \), roughness \( z_0 = 1 \text{cm} \), for \( 0 < x < L \), roughness \( z_1 = 7.39 \text{cm} \), \( L = 185 \text{m} \), \( u_{*0} = 0.3775 \text{m/s} \). The surface roughness parameter is

\[ [M] = \frac{M}{ik_1} (1 - \exp[-ik_1L]) \]

where \( M = -2 \).
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Fig. 3. Velocity profile $x = 6.42m$

Comparison of wind speed between the present theory and Blom and Wartena's is shown in Fig. 5. The agreement is found to be close. In Fig. 6, the variation of wall shear stress is given, there exists difference between the results. From the analysis of Blom and Wartena's, we are aware that the new equilibrium occurs at the distance of 2500m after the roughness change. When the roughness has a step change ($L \rightarrow \infty$), Consequently, when the roughness change is limited. ($L = 185m$), the new equilibrium should happen at the distance longer than the distance given by Blom and Wartena (200m), and therefore, the present theory is believed to give the right wall shear stresses.

4.3 Changes in roughness over limited area

In order to make comparison, we apply the present theory to the Edling and Cermak (1974) experiment. In the experiment, the length and the width of the roughness change area are respectively $L_x = 548.64cm$ and $L_y = 43.18cm$, the roughness parameter is

$$[M] = \frac{M}{k_1 k_2} (\exp[-ik_x L_x] - 1)(\exp[-ik_y L_y] - 1)$$

where $M = -5.05$. Other parameters in the calculation are $u_{*0} = 0.36cm/s$, $z_0 = 0.0012cm$, $z_f = 0.2cm$.

Horizontal velocity comparison is given in Fig. 7. The results of theory and experiment are agreeable. In Fig. 8 is shown the lateral velocity, although there seems to be a phase difference between the experiment and the theoretical prediction, the order of the magnitude is comparable. In the authors' opinion, the results given by the present theory seems to be reliable because the flow should be symmetric about the center-line.

V. Conclusion

The variations of wind speed and shear stress in the surface layer of atmospheric boundary layer over arbitrarily distributed changes in surface roughness are analysed in this paper.

The results show that increased (decreased) surface roughness enhances (weakens) the wall shear stress, and reduces (enlarges) the wind speed.

Due to the existence of the change in surface roughness in the direction perpendicular to the flow, the 3D significant feather deferring from the 2D case is the appearance of the weak lateral flow. Streamline displacement is caused by the changes in surface roughness, which leads to pressure perturbation in the outer region, the pressure perturbation, in turn, drives the lateral flow.
The results given by the present theory are agreeable to the previous theory, observation data as well as experiments.

References