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A MICROSCOPIC DAMAGE MODEL CONSIDERING THE CHANGE OF VOID SHAPE AND APPLICATION IN THE VOID CLOSING

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Abstract

A microscopic damage model of ellipsoidal body containing ellipsoidal void for nonlinear matrix materials is developed under a particular coordinate. The change of void shape is considered in this model. The viscous restrained equation obtained from the model is affected by stress Σ_{ij}, void volume fraction f, material strain rate exponent m as well as the void shape. Gurson's equation is modified from the numerical solution. The modified equation is suitable for the case of nonlinear matrix materials and changeable voids. Lastly, the model is used to analyze the closing process of voids.

Key words nonlinear material, ellipsoidal void, void shape, void closing

I. Introduction

Since Gurson[1] built the microscopic model of the limited matrix containing a void, microscopic damage mechanics has greatly been developed. Gurson's model was widely used because of its perfection and practicality. Yamamoto[2] used Gurson's equation to analyze the conditions for shear localization in the ductile fracture of void-containing materials. Gurson's constitutive model was used by Aravas[3] in the analysis of void growth that leads to central bursts during extrusion. On the other hand, some modifications to Gurson's model were made. Tvergaard[4] considered the void interaction by means of the finite element analysis. Strain hardening was indirectly taken into account by Yamamoto[2] through introducing the matrix average flow stress and Wang Tze-chiang[5] considered directly the affection of the strain hardening matrix materials.

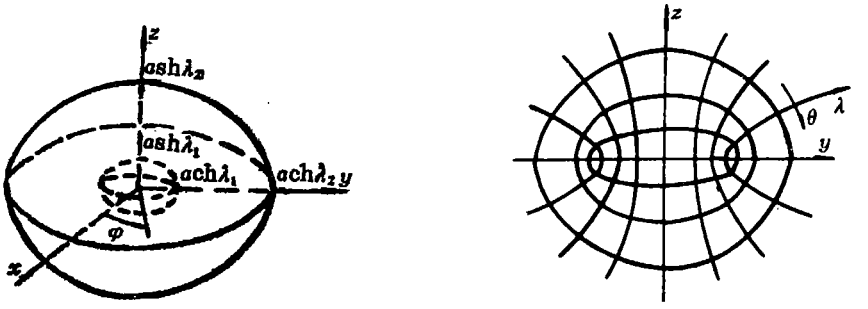
In industral production, the manufacturing process of many parts such as the forge of heavy ingots and the forming of powder metallurgy is to eliminate all kinds of internal cavities in materials. So the study on the void closing in materials becomes significant. We[6] applied Gurson's model to analyze the closing process of voids in rigid-plastic matrix materials. It was found that the entire closing of voids requires the infinite hydrostatic stress or equivalent strain rate. This is because the void shape change was not taken into account in the model. In microscopic damage mechanics, studies were greatly concentrated on void nucleation, growth, coalescence and forming a macro-crack at last. Therefore, the change of the void shape was ignored in Gurson's model and the modified Gurson's model. However, the void shape plays an important part in the void closing,

especially in the entire closing of voids.

In this paper, a microscopic damage model of the limited ellipsoidal body containing ellipsoidal void for nonlinear matrix materials is developed and the change of void shape is taken account of. Then, the void closing of nonlinear materials is analyzed numerically.

II. Microscopic Damage Model Considering the Change of Void Shape

The microscopic damage model of the finite ellipsoidal body containing ellipsoidal void is shown in Fig. 1. In order to take account of the change of the void shape, the model is built in the so-called ellipsoid-rotating hyperboloid coordinates (see Fig. 1).



(a) (b)

Fig. 1 Microscopic damage model and ellipsoid-rotating hyperboloid coordinates

The coordinates are defined as follows:

$$x=a\mathrm{ch}\lambda\sin\theta\cos\varphi,\ y=a\mathrm{ch}\lambda\sin\theta\sin\varphi,\ z=a\mathrm{sh}\lambda\cos\theta \tag{2.1}$$

$$\lambda:\ [0,\ \infty),\ \theta:\ [0,\ \pi],\ \varphi:\ [0,\ 2\pi]$$

Equation of ellipsoid:

$$\frac{x^2}{a^2\mathrm{ch}^2\lambda}+\frac{y^2}{a^2\mathrm{ch}^2\lambda}+\frac{z^2}{a^2\mathrm{sh}^2\lambda}=1$$

The ellipsoid-rotating hyperboloid coordinates are the orthogonal coordinates. From Fig. 1, it can be seen that the ellipsoid shape is a function of λ and the smaller λ , the flatter the ellipsoid. The ellipsoid becomes a circle in x-y plane when $\lambda=0$. This change of voids is similar to the real process of the void closing.

In this problem, the following boundary condition is only met when an upper bound approach is adopted.

$$v_i|_S=\dot{E}_{ij}x_j \tag{2.2}$$

where v_i is the microscopic velocity field, S is the outer surface of the model, $\dot{E}_{ij}$ is the macroscopic strain rate and x_j is the position of a material point in Cartesian coordinates. Considering axisymmetric deformation, $\dot{E}_{11}=\dot{E}_{22}$ and $\dot{E}_{ij}=0(i\neq j)$, eq. (2.2) after coordinates transform becomes

$$\left.\begin{aligned} v_\lambda&=\frac{a\mathrm{sh}\lambda_2\mathrm{ch}\lambda_2}{\sqrt{\mathrm{ch}^2\lambda_2-\sin^2\theta}}(\dot{E}_{11}\sin^2\theta+\dot{E}_{33}\cos^2\theta)\\ v_\theta&=\frac{a\sin\theta\cos\theta}{\sqrt{\mathrm{ch}^2\lambda_2-\sin^2\theta}}(\dot{E}_{11}\mathrm{ch}^2\lambda_2-\dot{E}_{33}\mathrm{sh}^2\lambda_2)\\ v_\varphi&=0 \end{aligned}\right\} \tag{2.3}$$

where v_λ , v_θ and v_φ are the microscopic velocity field in the ellipsoid-rotating hyperboloid coordinates. When the incompressible condition and eq. (2.3) are met, a kine-possible velocity field can be solved

$$\left.\begin{aligned} v_\lambda &= \frac{a(2\dot{E}_{11}+\dot{E}_{33})}{3\mathrm{ch}\lambda\sqrt{\mathrm{ch}^2\lambda-\sin^2\theta}}(\mathrm{sh}^3\lambda_2+3\mathrm{sh}\lambda_2\cos^2\theta) \\ &\quad+\frac{a(\sin^2\theta-2\cos^2\theta)\mathrm{sh}\lambda}{\mathrm{ch}\lambda\sqrt{\mathrm{ch}^2\lambda-\sin^2\theta}}\left[\dot{E}_{11}+\frac{1}{3}\mathrm{sh}^2\lambda(\dot{E}_{11}-\dot{E}_{33})\right] \\ v_\theta &= \frac{a\sin\theta\cos\theta}{\sqrt{\mathrm{ch}^2\lambda-\sin^2\theta}}(\dot{E}_{11}\mathrm{ch}^2\lambda-\dot{E}_{33}\mathrm{sh}^2\lambda) \\ v_\varphi &= 0 \end{aligned}\right\} \tag{2.4}$$

so that the microscopic strain rates $\dot{\varepsilon}_\lambda$, $\dot{\varepsilon}_\theta$, $\dot{\varepsilon}_\varphi$, $\dot{\gamma}_{\lambda\theta}$ are

$$\left.\begin{aligned} \dot{\varepsilon}_\lambda &= \{(\sin^2\theta-2\cos^2\theta)[\dot{E}_{11}+\mathrm{sh}^2\lambda(\mathrm{ch}^2\lambda-(1/3)\mathrm{sh}^2\lambda)\cdot 1.5\dot{E}_e] \\ &\quad -\dot{E}_{ii}\mathrm{sh}\lambda_2\mathrm{sh}\lambda((1/3)\mathrm{sh}^2\lambda_2+\cos^2\theta)\}/\mathrm{ch}^2\lambda(\mathrm{ch}^2\lambda-\sin^2\theta) \\ &\quad -\{\dot{E}_{ii}\mathrm{sh}\lambda_2\mathrm{sh}\lambda((1/3)\mathrm{sh}^2\lambda_2+\cos^2\theta)+(\dot{E}_{11}+0.5\dot{E}_e\mathrm{sh}^2\lambda)\mathrm{sh}^2\lambda \\ &\quad \cdot(\sin^2\theta-2\cos^2\theta)+\sin^2\theta\cos^2\theta(\dot{E}_{11}+1.5\dot{E}_e\mathrm{sh}^2\lambda)\}/(\mathrm{ch}^2\lambda \\ &\quad -\sin^2\theta)^2 \\ \dot{\varepsilon}_\theta &= \{(\dot{E}_{11}+1.5\dot{E}_e\mathrm{sh}^2\lambda)(\mathrm{ch}^2\lambda\cos^2\theta-\mathrm{ch}^2\lambda\sin^2\theta+\sin^4\theta) \\ &\quad +(\dot{E}_{11}+0.5\dot{E}_e\mathrm{sh}^2\lambda)\cdot\mathrm{sh}^2\lambda\cdot(\sin^2\theta-2\cos^2\theta) \\ &\quad +\dot{E}_{ii}((1/3)\mathrm{sh}^2\lambda_2+\cos^2\theta)\mathrm{sh}\lambda_2\mathrm{sh}\lambda\}/(\mathrm{ch}^2\lambda-\sin^2\theta)^2 \\ \dot{\varepsilon}_\varphi &= \{\dot{E}_{ii}((1/3)\mathrm{sh}^2\lambda_2+\cos^2\theta)\mathrm{sh}\lambda_2\mathrm{sh}\lambda+(\dot{E}_{11}+0.5\dot{E}_e\mathrm{sh}^2\lambda) \\ &\quad \cdot\mathrm{sh}^2\lambda(\sin^2\theta-2\cos^2\theta)\}/\mathrm{ch}^2\lambda(\mathrm{ch}^2\lambda-\sin^2\theta) \\ &\quad +\cos^2\theta(\dot{E}_{11}+1.5\dot{E}_e\mathrm{sh}^2\lambda)/(\mathrm{ch}^2\lambda-\sin^2\theta) \\ \dot{\gamma}_{\lambda\theta} &= \{2\dot{E}_{ii}\sin\theta\cos\theta\mathrm{sh}\lambda_2(\cos^2\theta+(1/3)\mathrm{sh}^2\lambda_2)+2(\dot{E}_{11}+0.5\dot{E}_e\mathrm{sh}^2\lambda) \\ &\quad \cdot\sin\theta\cos\theta(\sin^2\theta-2\cos^2\theta)\mathrm{sh}\lambda\}/\mathrm{ch}\lambda(\mathrm{ch}^2\lambda-\sin^2\theta)^2 \\ &\quad +\{6(\dot{E}_{11}+0.5\dot{E}_e\mathrm{sh}^2\lambda)\sin\theta\cos\theta\mathrm{sh}\lambda-2\dot{E}_{ii}\mathrm{sh}\lambda_2\sin\theta\cos\theta\} \\ &\quad /\mathrm{ch}\lambda(\mathrm{ch}^2\lambda-\sin^2\theta)-2(\dot{E}_{11}+1.5\dot{E}_e\mathrm{sh}^2\lambda)\sin\theta \\ &\quad \cdot\cos\theta\mathrm{sh}\lambda\mathrm{ch}\lambda/(\mathrm{ch}^2\lambda-\sin^2\theta)^2+0.5\dot{E}_e\sin\theta\cos\theta \\ &\quad \cdot\mathrm{sh}\lambda\mathrm{ch}\lambda/(\mathrm{cn}^2\lambda-\sin^2\theta) \end{aligned}\right\} \tag{2.5}$$

where $\dot{E}_{ii}=2\dot{E}_{11}+\dot{E}_{33}$ and $\dot{E}_e=2(\dot{E}_{11}-\dot{E}_{33})/3$. The microscopic equivalent strain rate $\dot{\varepsilon}_e$ is

$$\begin{aligned} \dot{\varepsilon}_e^2 &= 2[(\dot{\varepsilon}_\lambda-\dot{\varepsilon}_\theta)^2+(\dot{\varepsilon}_\theta-\dot{\varepsilon}_\varphi)^2+(\dot{\varepsilon}_\varphi-\dot{\varepsilon}_\lambda)^2+1.5\dot{\gamma}_{\lambda\theta}^2]/9 \\ &= A\dot{E}_{ii}^2+B\dot{E}_{ii}\dot{E}_e+C\dot{E}_e^2 \end{aligned} \tag{2.6}$$

where A, B and C are all the functions of λ and θ .

The matrix material is an incompressible, isotropic nonlinear material and the microscopic stress and strain are related to

$$\sigma_e/\sigma_0=(\dot{\varepsilon}_e/\dot{\varepsilon}_0)^m \tag{2.7a}$$

or

$$\sigma_e=K\dot{\varepsilon}_e^m,\quad K=\sigma_0/\dot{\varepsilon}_0^m \tag{2.7b}$$

where the microscopic equivalent stress $\sigma_e=(1.5s_{ij}s_{ij})^{\frac{1}{2}}$, s_{ij} is the microscopic stress deviator, $\dot{\varepsilon}_e=((2/3)\dot{\varepsilon}_{ij}\dot{\varepsilon}_{ij})^{1/2}$ is the microscopic equivalent strain rate, $\dot{\varepsilon}_{ij}$ is the microscopic strain rate deviator, σ_0, $\dot{\varepsilon}_0$ and the strain rate exponent m are the material parameters.

The microscopic dissipation w_0 is defined:

$$w_0=\int_0^{\dot{\varepsilon}_{ij}} \sigma_{ij}d\dot{\varepsilon}_{ij} =\frac{K}{m+1}\dot{\varepsilon}_e^{m+1} \tag{2.8}$$

The macroscopic dissipation W is given by

$$W=\frac{1}{V}\int_{V_m} w_0dV=\frac{K}{V(m+1)}\int_{V_m}\dot{\varepsilon}_e^{m+1}dV \tag{2.9}$$

where V is the entire volume of the model and V_m is the matrix volume. The macroscopic stress can be shown that

$$\Sigma_{ij}=\partial W/\partial\dot{E}_{ij} \tag{2.10}$$

From eq. (2.6), the macroscopic dissipation is

$$\begin{aligned} W&=\frac{K}{(m+1)}\frac{1}{V}\int_{V_m}(A\dot{E}_{11}^2+B\dot{E}_{11}\dot{E}_e+C\dot{E}_e^2)^{\frac{m+1}{2}}dV \\ &=\frac{K\dot{E}_e^{m+1}}{(m+1)}\cdot\frac{1}{V}\int_{V_m}(A\omega^2+B\omega+C)^{\frac{m+1}{2}}dV \\ &=\frac{K\dot{E}_e^{m+1}}{(m+1)}\cdot W^* \end{aligned} \tag{2.11}$$

$$\begin{aligned} W^*&=\frac{1}{V}\int_{V_m}(A\omega^2+B\omega+C)^{\frac{m+1}{2}}dV \\ &=\frac{3}{2\mathrm{ch}^2\lambda_2\mathrm{sh}\lambda_2}\int_0^{\pi}\sin\theta d\theta\int_{\lambda_1}^{\lambda_2}(A\omega^2+B\omega+C)^{\frac{m+1}{2}} \\ &\quad\cdot\mathrm{ch}\lambda\cdot(\mathrm{sh}^2\lambda+\cos^2\theta)d\lambda \end{aligned} \tag{2.12}$$

Using eq. (2.10), we can get $\omega=\dot{E}_{11}/\dot{E}_e$

$$\left.\begin{aligned} \Sigma_m&=\frac{\partial W}{\partial\dot{E}_{11}}=\frac{K\dot{E}_e^{m+1}}{m+1}\cdot\frac{\partial W^*}{\partial\dot{E}_{11}}=\frac{K\dot{E}_e^m}{m+1}\cdot\frac{\partial W^*}{\partial\omega} \\ \Sigma_e&=\frac{\partial W}{\partial\dot{E}_e}=\frac{K\dot{E}_e^{m+1}}{m+1}\cdot\frac{\partial W^*}{\partial\dot{E}_e}+K\dot{E}_e^mW^* \\ &=\frac{K\dot{E}_e^m}{m+1}\left[(m+1)W^*-\omega\frac{\partial W^*}{\partial\omega}\right] \\ &=K\dot{E}_e^mW^*-\omega\Sigma_m \end{aligned}\right\} \tag{2.13}$$

The average equivalent stress $\bar{\sigma}_e$ is defined by

$$\begin{aligned} \bar{\sigma}_e\cdot\dot{\bar{\varepsilon}}_e&=\frac{1}{V_m}\int_{V_m}\sigma_e\dot{\varepsilon}_edV=\frac{K}{V_m}\int_{V_m}\dot{\varepsilon}_e^{m+1}dV \\ &=\frac{m+1}{1-f}W=\frac{K\dot{E}_e^{m+1}}{1-f}W^* \end{aligned} \tag{2.14}$$

where the void volume fraction $f=(V-V_m)/V$. Considering $\dot{\bar{\varepsilon}}_e=(\bar{\sigma}_e/K)^{\frac{1}{m}}$ $\bar{\sigma}_e$ can be written

$$\bar{\sigma}_e = K\dot{E}_e^m (W^*/(1-f))^{\frac{m}{m+1}} \tag{2.15}$$

Then,

$$\left.\begin{aligned} T_m &= \frac{\Sigma_m}{\bar{\sigma}_e} = \frac{1}{m+1}\frac{\partial W^*}{\partial \omega} \Big/ \left(\frac{W^*}{1-f}\right)^{\frac{m}{m+1}} \\ T_e &= \Sigma_e/\bar{\sigma}_e = [(1-f)^m W^*]^{\frac{1}{m+1}} - \omega T_m \end{aligned}\right\} \tag{2.16}$$

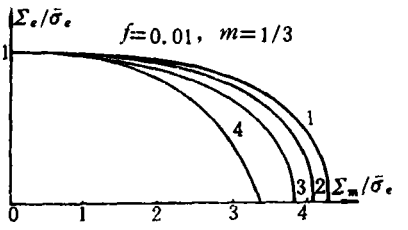
Eq. (2.16) is controlled by the material parameters, the void size and shape.

The long axis of the ellipsoidal void is $a_1 = a\mathrm{ch}\lambda_1$ and the short one is $b_1 = a\mathrm{sh}\lambda_1$. Define the void shape factor $\alpha = b_1/a_1 = \mathrm{th}\lambda_1$, the void volume fraction $f = \mathrm{ch}^2\lambda_1\mathrm{sh}\lambda_1/(\mathrm{ch}^2\lambda_2\mathrm{sh}\lambda_2)$. λ_1 and λ_2 can be solved if α and f are given.

When ω is eliminated between eq. (2.16), the restrained equation in T_m and T_e can be got

$$\phi(T_m, T_e, \alpha, f, m) = 0 \tag{2.17}$$

The numerical solution of eq. (2.17) are shown in Fig. 2. These restrained curves are called the yield equation for the matrix materials in obedience to the total strain theory and called the viscous restrained equation for the viscous matrix materials. Besides the void volume fraction f and the material strain rate exponent m, eq. (2.17) also relates to the void shape factor α. Fig. 3 shows the relation between the intersecting point T_m^* of the restrained curves in the T_m coordinate axis and the void shape. It can be found that the smaller the void shape factor, the greater the influence on the restrained equation.



1. $\alpha=0.995$; 2. $\alpha=0.447$;
3. $\alpha=0.1$; 4. $\alpha=0.05$

Fig. 2 Restrained curves of T_m and T_e

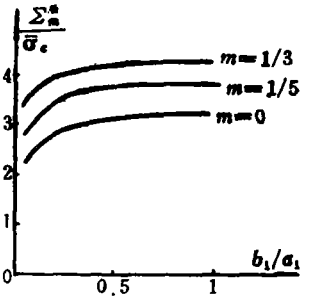


Fig. 3 Void shape influence on restrained equation

For the spherical void with the rigid-plastic von Mises matrix material, $m=0$ and $\alpha=1$, it is easy to be got that eq. (2.17) becomes Gurson's equation. That is

$$\left.\begin{aligned} m=0,\ \alpha=1;\ \phi &= T_e^2 + 2f\mathrm{ch}(3T_m/2) - (1+f^2) \\ \alpha=0;\ \phi &= T_e^2 - 1 \end{aligned}\right\} \tag{2.18}$$

In general case, according to the results of the numerical calculation, eq. (2.17) can be given by

$$\phi = T_e^2 + 2fq_1r_1\mathrm{ch}(3q_2T_m/2) - (1+f^2) = 0 \tag{2.19}$$

$$q_1 = e^{2.6m},\quad q_2 = e^{-1.58m},\quad r_1 = 1 + \mathrm{csch}(15\alpha)$$

where csch (15α) is the hyperbolic cosecant function. Eq. (2.19) is suitable for the case of $m<0.5$ and the small f.

III. The Closing Process of Voids

In the closing process of the internal voids in materials, the size and shape of voids are changed. When voids are closed entirely, the void volume fraction $f=0$ and the void shape factor $\alpha=0$.

From the definition of the void volume fraction, we have

$$\dot{f}=(1-f)\dot{E}_{ii} \tag{3.1}$$

$$\ln[(1-f_0)/(1-f)]=\int_0^t \dot{E}_{ii}dt=E_{ii} \tag{3.2}$$

where f_0 is the initial void volume fraction. When $f=0$, the macroscopic strain condition of the entire closing of voids can be written

$$\ln(1-f_0)=E_{ii} \tag{3.3}$$

The long axis of the outer boundary ellipsoid is a_2 , and the short one is b_2 . We have

$$\left.\begin{aligned} &v_\lambda|_{\lambda=\lambda_1,\theta=\pi/2}=\dot{a}_1,\quad v_\lambda|_{\lambda=\lambda_2,\theta=\pi/2}=\dot{a}_2\\ &v_\lambda|_{\lambda=\lambda_1,\theta=0}=\dot{b}_1,\quad v_\lambda|_{\lambda=\lambda_2,\theta=0}=\dot{b}_2\end{aligned}\right\} \tag{3.4}$$

From eq. (2.4), then

$$\left.\begin{aligned} &\frac{\dot{a}_1}{a_1}=\frac{1}{3}\dot{E}_{ii}\left(\frac{\mathrm{th}^2\lambda_2}{f}+1-\mathrm{th}^2\lambda_1\right)+\frac{1}{2}\dot{E}_e\\ &\dot{a}_2/a_2=\dot{E}_{ii}/3+\dot{E}_e/2\\ &\frac{\dot{b}_1}{b_1}=\frac{\dot{E}_{ii}}{3f}\left(1+\frac{2}{\mathrm{ch}^2\lambda_2}\right)-2\left[\frac{\dot{E}_{ii}}{3\mathrm{ch}^2\lambda_1}-\frac{1}{2}\frac{\dot{E}_e(1-\mathrm{sh}^2\lambda_1)}{\mathrm{ch}^2\lambda_1}\right]\\ &\dot{b}_2/b_2=\dot{E}_{ii}/3-\dot{E}_e\end{aligned}\right\} \tag{3.5}$$

The above equations represent the evolution of the change void shape in the closing process of voids. The macroscopic stress in the void closing can be got from eqs. (2.16) and (3.5). When $b_1=0$, the voids are entirely closed. The stress and strain states of this moment are significant in the engineering application.

From eqs. (2.16) and (3.5), the parameters of the void closing can be reckoned up if the strain rates $\dot{E}_{11}$, $\dot{E}_{33}$ (or $\dot{E}_{ii}$, $\dot{E}_e$), the initial void volume fraction f_0 and the initial size of void a_{10}, b_{10} are given. The results of the numerical calculation are shown in Fig. 4 – 8.

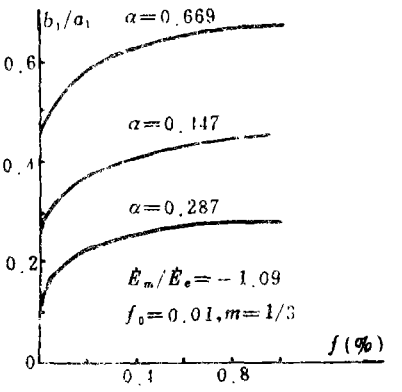


Fig. 4 Relation of α and f

Fig. 4 shows the relation of the void shape factor α and the void volume fraction f. It can be seen that the void shape is flatter and flatter in the closing process of voids.

Fig. 5 and Fig. 6 show the relation of T_e , T_m and f in accordance with different

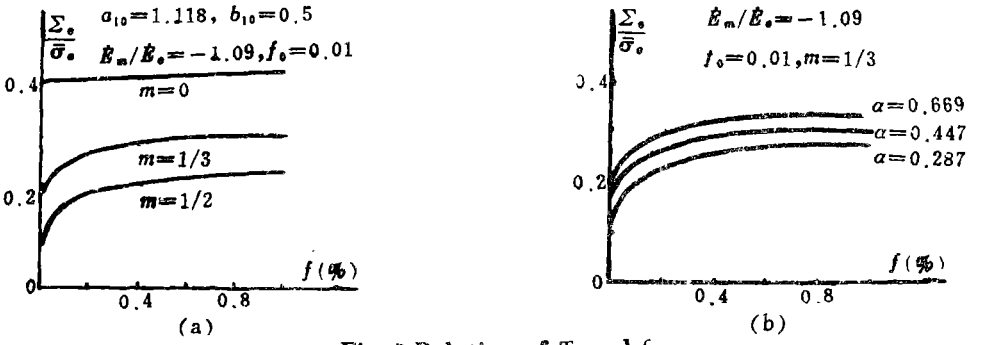


Fig. 5 Relation of T_e and f

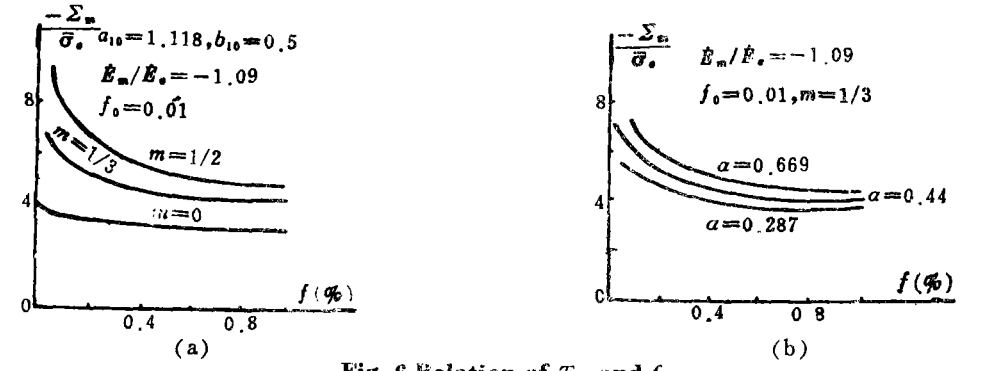


Fig. 6 Relation of T_m and f

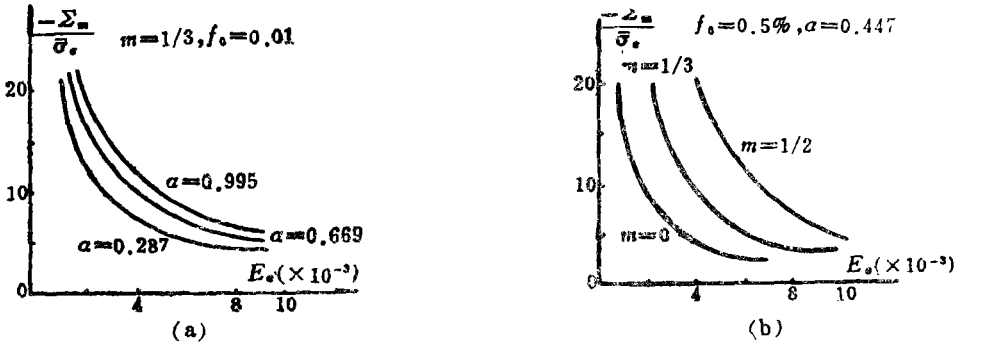


Fig. 7 Relation of T_m and E_e of the entire closing of void

m and α. When f decreases, the hydrostatic stress $|T_m|$ increases and the equivalent stress T_e decreases. The greater α, the greater $|T_m|$ and T_e; The greater m, the greater $|T_m|$ and the smaller T_e.

Fig. 7 shows the relation of the hydrostatic stress T_m and the equivalent strain E_e at the time of the entire closing of voids. In the same E_e, the greater m and α, the greater $|T_m|$; In the same T_m, the greater m and α, the greater E_e. In paper [6], we proved that the hydrostatic stress and the equivalent strain are the main parameters affecting the closing process of the internal voids in materials. It can be known from Fig. 7 that the greater m and α, the more difficult the void closing.

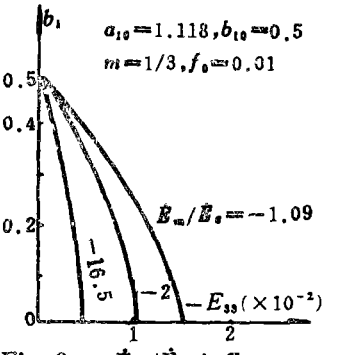


Fig. 8 $\dot{E}_m/\dot{E}_e$ influence on void closing effect

Fig. 8 shows the relation of the void short axis b_1 and the strain in the compressed direction E_{33}. In the same E_{33}, the greater $|\dot{E}_m/\dot{E}_e|$, the better the void closing effect.

IV. Summary

The viscous restrained equation of nonlinear materials containing voids relates to the void shape besides the void volume fraction f and the material strain rate exponent m. Especially for the small void shape factor, the effect of the void shape cannot be ignored.

In the process of the void closing of nonlinear materials, the greater the material strain rate exponent m, the more difficult the void closing and the flatter the void shape, the easier the void closing. The greater $|\dot{E}_m/\dot{E}_e|$, the better the void closing effect.

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