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CONSTITUTIVE RELATION OF DISCONTINUOUS REINFORCED METAL-MATRIX COMPOSITES*

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ABSTRACT: A micromechanical model is developed to simulate the mechanical behaviors of discontinuous reinforced composites. The analysis for a representative unit cell is based on the assumption of a periodic array of aligned reinforcements. The minimum energy principle is used to determine the unknown coefficients of the displacement field of the unit cell. The constitutive behavior of composites is studied to obtain the relationship between the main variables of matrix and reinforcements. It is concluded that the flow strength of composites is strongly influenced by volume fraction, aspect ratio of reinforcement, and the strain hardening exponent of matrix. An analytical constitutive relation of composites is obtained. The predicted results are in agreement with the existing experimental and numerical results.

KEY WORDS: composites, constitutive equation, plastic

1 INTRODUCTION

Discontinuous reinforced composites (DRC), such as aluminum alloys reinforced with SiC particles or whiskers, have been commercially available in large quantities. The materials exhibit a unique combination of specific modulus and strength, as well as good thermal stability. The advantages of these engineering materials over the continuous fiber reinforced composites include improved ductility, reduced anisotropy in mechanical properties and, most importantly, they are considerably less expensive. These properties make DRC attractive for many structural applications.

The elasto-plastic behavior of DRC has been the subject of several investigations, and some analytical models are suggested. A closely related problem was examined by Duva[1] who considered the deformation of infinite solid containing a rigid inclusion. An approximate constitutive relation is derived for a power law viscous material stiffened by rigid spherical inclusions using a differential self-consistent analysis. Lee and Mear[2] and Li et al.[3] used the same method to analyze the stiffening effect of rigid elliptical inclusions on an infinite domain of a power law material. The constitutive relations are established for dilute concentrations of inclusions over a wide range of matrix hardening exponents and inclusion aspect ratios.

Tandon and Weng[4] studied the elasto-plastic response of composites under monotonic, proportional loading by using the secant moduli method. The method proposed combines

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Mori-Tanaka's[5] concept of average stress in elasticity and Hill's[6] discovery of a decreasing constraint power of the matrix in polycrystal plasticity. Qiu and Weng[7] studied the influence of inclusion shape on the overall elastoplastic behavior of a two phase isotropic composites by engaging secant moduli method. Just the same as in Tandon and Weng[4], the method presented in [7] is a mean-field one. The approximation inherent in the mean-field approach for a physically nonlinear problem will limit its applications to the low-volume concentration range, under which the majority of the deformation field in the ductile matrix is uniform.

Motivated by the work of Gurson[8] on yield surface for porous materials, Zhu and Zbib[9] investigated the mechanical properties of composites based on the use of finite unit cell model accounting for the interaction of inclusions in composites with periodic microstructures. The finite unit cell model is clearer geometrically and can describe the microstructure features more accurately. The work of Bao et al.[10] combined the finite element methods with the theoretical investigation. The characteristic parameters in the models were found through FEM analysis. A similar micromechanical model was proposed by Llorca and González[11], in which the perfectly bonded or damaged interface was introduced in axisymmetric unit cell and the fraction of the damaged cells was calculated on the basis of Weibull statistics.

An experimental study and a detailed finite element analysis of the tensile properties of the particle or whisker-reinforced metal-matrix composites are investigated by Christman et al.[12,13] and Llorca et al.[14]. For uniformly distributed reinforcements, an axisymmetric unit cell model containing a single reinforcement was adopted in their numerical analyses. Tvergaard[15] analyzed the tensile properties of ductile metal reinforced by a periodic array of short fibers assuming that the whisker ends are not perfectly aligned but staggered (see also Levy & Papazian[16]).

Although much progress has been obtained in that field, an analytical constitutive relation of discontinuous reinforced composite is still desired. The aim of this paper is to derive an explicit analytical constitutive relation of composite, which is easy to use for scientists and engineers. In the present paper an axisymmetric unit cell is used to analyze the tensile properties of the composites. The unit cell analyzed here is a circular cylindrical matrix with a circular cylindrical reinforcement cut off sharply at the ends, and the perfect bonding between reinforcement and matrix is assumed. A primitive displacement field of unit cell is firstly proposed, and then the constitutive relation of the composites is determined based on the primitive displacement field. The refined constitutive relation of composites will be obtained by the perturbation method.

2 SYNOPSIS OF METHODS

In this paper, the composites are idealized as uniformly distributed periodic arrays of aligned unit cylindrical cells, and each cell consists of an elastic cylindrical short fiber surrounded by an elasto-plastically deforming matrix, see Fig.1. Generally, we assume that the macro elasto-plastic constitutive relationship can be expressed with deformation theory according to Ji and Wang[17], as follows

$$E_{ij} = E_{ij}^{e} + E_{ij}^{p} = C_{ijkl}\Sigma_{kl} + \frac{3}{2}\frac{E_e^p}{A(E_e^p)}S_{ij} \tag{1}$$

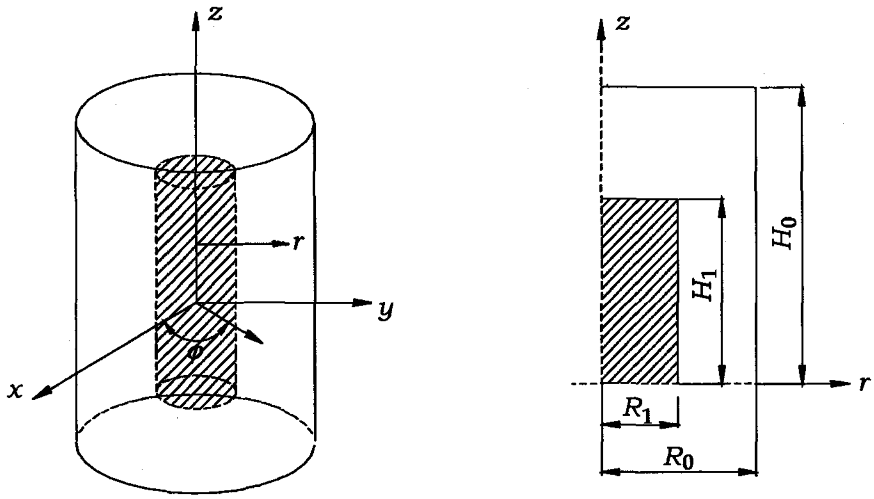

Fig.1 The cell model of discontinuous reinforced composites

where $E_{ij}, E_{ij}^e, E_{ij}^p$ and E_e^p are the total macro strain, macro elastic strain, macro plastic strain, and macro effective strain, respectively, C_{ijkl} is the macro elastic compliance tensor of the unit cell, and S_{ij} is the deviatoric part of the macro stress.

In Eq.(1), function $A(E_e^p)$ describes the work hardening properties of the reinforced composites. It is worth noting that the constitutive relationship of the composites will be completely determined if function $A(E_e^p)$ is given. The macro elastic compliance C_{ijkl} can be obtained according to Mori and Tanaka Method[5]. In this paper, our attention will focus on determining function $A(E_e^p)$ and the stress and plastic strain relation of composites.

Suppose that the matrix material investigated here is elasto-plastic. The tensile stress-strain behavior for the matrix material is supposed to follow the Ramberg-Osgood law

$$\varepsilon = \frac{\sigma}{E} + \alpha\frac{\sigma_0}{E}\Big(\frac{\sigma}{\sigma_0}\Big)^{1/n} \tag{2}$$

where σ_0 is the reference stress, E is Young's modulus and n is the strain hardening exponent. The coefficient α is taken to be 3/7 by Ramberg and Osgood.

If the macro strain is incompressible and large enough, the plastic strain will dominate the whole deformation in the matrix material and the bulk deformation is very small, so the elastic deformation of the matrix material can be neglected without affecting significantly accuracy. Then Eq.(2) can be simplified as

$$\frac{\sigma}{\sigma_0} = \Big(\frac{\varepsilon}{\alpha\varepsilon_0}\Big)^n \tag{3}$$

For the multiaxial stress state, we have

$$s_{ij} = \frac{2}{3}\frac{\sigma_0}{\varepsilon_e}\Big(\frac{\varepsilon_e}{\alpha\varepsilon_0}\Big)^n\varepsilon_{ij} \tag{4}$$

where s_{ij} is the deviatoric part of the stress, and ε_e is the effective strain.

The macro strain energy density can be expressed with volume average of the components of the composites, which is

$$W = \frac{1}{V}\sum_{k=0}^{n}\int_{V_k} w_k \mathrm{d}V \tag{5}$$

where V_0 is the volume of the matrix in the composites, w_0 is the strain energy density of the matrix, V_k is the volume of the kth reinforcement component, w_k is the micro strain energy density of the kth reinforcement component.

The strain energy density of matrix material w_0 takes the form

$$w_0 = \int s_{ij}(\varepsilon)\mathrm{d}e_{ij} = \frac{\alpha\sigma_0\varepsilon_0}{n+1}\left(\frac{\varepsilon_e}{\alpha\varepsilon_0}\right)^{n+1} \tag{6}$$

According to Duva and Hutchinson[18], the macro deviatoric stress S_{ij} can be obtained by

$$S_{ij} = \frac{\partial W}{\partial E_{ij}} \tag{7}$$

and function $A(E_e^p)$ will be obtained via above equation.

3 CELL MODEL

With the elasticity of matrix being neglected in section 2, the elastic deformation of reinforcement is also neglected in the procedure of deriving the function $A(E_e^p)$. Then the reinforcement becomes a rigid inclusion in the plastic deformation range, i.e. $w_k = 0(k \neq 0)$. The elasticity of matrix and reinforcement will be considered in Eq.(1) by including the macro elasticity of the cell, C_{ijkl}. The reinforcement is in a perfect bonding condition at the fiber-matrix interface, such that the inner boundary condition for displacement field in the matrix, with respect to a reference frame fixed at a point on the reinforcement, is given by

$$u_i = 0 \qquad \text{on } S_1 \tag{8}$$

and the boundary condition at the outer surface is

$$u_i = E_{ij}x_j \qquad \text{on } S_0 \tag{9}$$

where S_1 and S_0 are the inner and outer boundaries of the cell model, respectively, and the macro strain in the analyses satisfies the incompressible condition, which is expressed as, $E_{11} = E_{22} = -E_{33}/2$ and $E_{ij} = 0$ $(i \neq j)$.

Let $(x^1, x^2, x^3) = (x, y, z)$ be the Cartesian coordinate system with the origin being located at the cylinder center and z-axis coinciding with the axis of revolution of cylindrical cell. In cylindrical coordinate $(x^1, x^2, x^3) = (r, \varphi, z)$, and the aspect ratio of cell and fiber reinforcement is defined as

$$\chi = \frac{H_0}{R_0} = \frac{H_1}{R_1} \tag{10}$$

Suppose that the displacement field in the matrix can be expanded as the following series

$$\begin{aligned} u_r &= u_{r0} + \sum_{k=1}^{\infty}\sum_{m=1}^{\infty} a_{km}\, r^m z^k\, \frac{t-t_1}{t_0-t_1}\,\frac{t-t_0}{t_0-t_1} \\ u_z &= u_{z0} + \sum_{k=1}^{\infty}\sum_{m=1}^{\infty} b_{km}\, r^m z^k\, \frac{t-t_1}{t_0-t_1}\,\frac{t-t_0}{t_0-t_1} \\ u_\varphi &= 0 \end{aligned} \tag{11}$$

where

$$\begin{aligned} &t = r \qquad t_0 = R_0 \qquad t_1 = R_1 \qquad \text{for } z/r < H_0/R_0 \\ &t = z \qquad t_0 = H_0 \qquad t_1 = H_1 \qquad \text{for } z/r > H_0/R_0 \end{aligned} \tag{12}$$

and in Eq.(11), u_{r0} and u_{z0} are the primitive displacement of matrix in r and z directions, respectively

$$u_{r0} = E_{11} r \frac{t - t_1}{t_0 - t_1} \qquad u_{z0} = E_{33} z \frac{t - t_1}{t_0 - t_1} \tag{13}$$

We can prove that the displacement field of cell in Eq.(11) satisfies the boundary conditions (8) and (9). In the cylindrical coordinate, the strain field of unit cell can be obtained by the following equations

$$\varepsilon_r = \frac{\partial u_r}{\partial r} \qquad \varepsilon_\varphi = \frac{u_r}{r} \qquad \varepsilon_z = \frac{\partial u_z}{\partial z} \qquad \gamma_{rz} = \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \tag{14}$$

and the deviatoric part of the strain, e_{ij} can be easily obtained

$$e_{ij} = \varepsilon_{ij} - \frac{1}{3}\varepsilon_{kk}\delta_{ij} \tag{15}$$

To obtain the solution of the strain field of the cell for a given aspect ratio, H_0/R_0,volume fraction of reinforcement, f and hardening exponent of matrix, n, the double series in Eq.(11) should be truncated, and the unknown coefficients a_{km} and b_{km} will be determined by minimizing the plastic energy of the unit cell.

For the convenience of discussion and calculation, we denote

$$\varepsilon_{ij} = E_e \hat{\varepsilon}_{ij} \qquad e_{ij} = E_e \hat{e}_{ij} \tag{16}$$

where we define $E_e = 2(E_{33} - E_{11})/3$. Substituting above equations into Eq.(5), and considering $w_k = 0 (k \neq 0)$, we have

$$W = \frac{1}{V}\left(\frac{\sigma_0(\alpha\varepsilon_0)^{-n}}{n+1}\right)\int_{V_0} \hat{\varepsilon}_e^{n+1} \mathrm{d}V E_e^{n+1} \tag{17}$$

Then according to Eq.(7)

$$\Sigma_e = S_{33} - S_{11} = \sigma_0 (\alpha\varepsilon_0)^{-n} E_e^n \frac{1}{V}\int_{V_0} \hat{\varepsilon}_e^{n+1} \mathrm{d}V \tag{18}$$

by rearranging above equation, we obtain

$$\frac{\Sigma_e}{\sigma_0} = F(f, n, H_0/R_0)\left(\frac{E_e}{\alpha\varepsilon_0}\right)^n \tag{19}$$

where

$$F(f, n, H_0/R_0) = \frac{1}{V}\int_V \hat{\varepsilon}_e^{(n+1)} \mathrm{d}V \tag{20}$$

and f is the volume fraction of the reinforcement. We call $F(f, n, H_0/R_0)$ the strengthening factor, which is an important parameter to reflect the hardening effect of the reinforcement.

Then the work hardening function $A(E_e^p)$ can be obtained by

$$A(E_e^p) = \sigma_0 F(f, n, H_0/R_0)\left(\frac{E_e}{\alpha\varepsilon_0}\right)^n \tag{21}$$

4 CONSTITUTIVE RELATION OF COMPOSITES

A primitive displacement field of the unit cell which satisfies the boundary condition is selected as an approximation of the real displacement field, i.e.

$$u_r = u_{r0} \qquad u_z = u_{z0} \tag{22}$$

and the primitive strain field of cell can be obtained by substituting above equations into Eq.(14). It is assumed that the bulk deformation of cell is very small, which is a good approximation when the cell endures the incompressible macro displacement constraint loading. The primitive value of the strengthening factor can be obtained according to Eq.(20)

$$F_0(f, n, H_0/R_0) = \frac{1}{V}\int_c^1\int_0^t \left(\frac{54c^2 + 18g^2s^2 - 126ct + 78t^2}{54(-1+c)^2}\right)^{\frac{n+1}{2}} t\mathrm{d}s\mathrm{d}t + \frac{1}{V}\int_c^1\int_0^s \left(\frac{g^2(3c-5s)^2/3 + 0.25t^2}{3(-1+c)^2g^2}\right)^{\frac{n+1}{2}} t\mathrm{d}t\mathrm{d}s \tag{23}$$

where

$$t = \frac{r}{R_0} \qquad s = \frac{z}{H_0} \qquad g = \frac{H_0}{R_0} \qquad c = f^{1/3} = \frac{H_1}{H_0} = \frac{R_1}{R_0} \tag{24}$$

Above integration can be easily obtained by a math software, such as Mathematics. An explicit constitutive relation of the unit cell was obtained by substituting Eq.(23) into Eq.(21), and Eq.(1).

It can be imagined that at a low volume concentration range, the primitive displacement field which satisfies the boundary condition can well approximate the deformation of the unit cell except at the corner of reinforcement. Although the primitive displacement field mode can not accurately describe the deformation of the unit cell, the mean strain and stress field through the volume average is believed to be a reasonable approximation of the real volume average field, which has already been proven to be able to capture the essence of a particle-reinforced plasticity.

For checking the accuracy of the primitive results about the strengthening factor and the validity of the constitutive relation of the composites, the displacement field of the unit cell was refined by adding the perturbation terms into the displacement field equations, then the strengthening factor Eq.(20) and work hardening function Eq.(21) will be re-evaluated.

Based on the primitive displacement field of the unit cell, the coefficients of multi-terms displacement field equations can be obtained by using the perturbation method introduced in [17] according to the minimum energy principle. The perturbation method converts a series of nonlinear problems for determining the unknown coefficients of the displacement field equations into a series of linear equations, which reduces a great deal of calculation. Some representative results are recalculated by using Newton-Raphson method to check the validity of the perturbation method.

In the general case, the minimization of the plastic energy of the unit cell, W, and the integration of function must be performed numerically. Theoretically, one needs to consider a large number of terms of series in Eq.(11) to ensure a good quantitative accuracy. The results with multi-term displacement field presented below are obtained with $k = 1, \cdots, K$ and $m = 1, \cdots, M$, where $K = 6$, and $M = 6$. In the procedure of calculation, convergence studies are carried out and it is indicated that the results are accurate within one percent.

To check the validity of the primitive constitutive relation, the stress-plastic strain curve was calculated compared with the counter part calculated by using the refined constitutive relation with multi-terms displacement field. This calculation is for two cases $f = 0.05, 0.2$, at $H_0/R_0 = 2$, and the results are shown in Fig.2. The material parameters in the calculation are selected from [11]. From Fig.2, it can be seen that the primitive results are very close to the refined results, and the primitive results are the upper bounds of the refined results, as can be imagined. So it is proved again that the primitive displacement field of the unit cell which just satisfies the boundary condition, will be a good approximation to the real deformation of the cell when a displacement constraint acted on the external boundary resulting in a uniform macro strain field. This concept was also used by Qiu and Weng[7] and Gurson[8]. Gurson model used a simple displacement field mode of the cell, and obtained good results, which became a popular meso mechanical model simulating the constitutive behavior of yield surface of the material with voids.

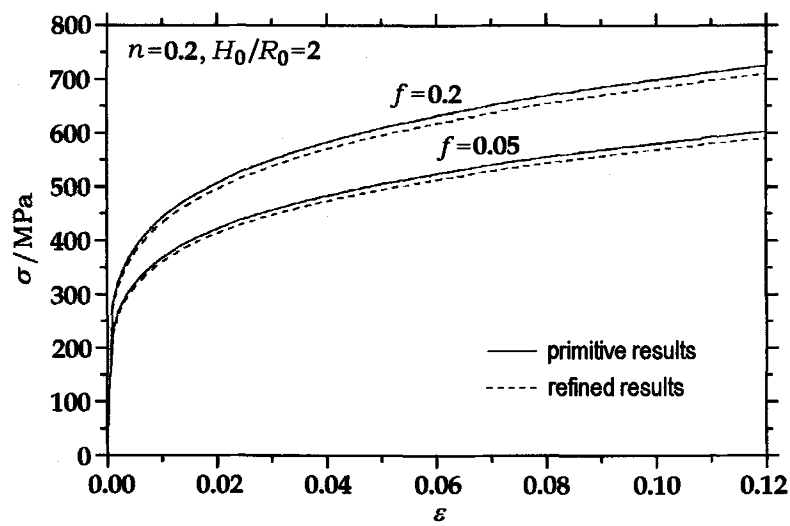

Fig.2 The comparison of the primitive results with the refined results of the uniaxial stress and plastic-strain curve of the unit cell

We can conclude that the approximate constitutive relation of the composites is a good approximation of the refined results. A simple and analytical constitutive relation of the composite is founded basing on the primitive results of the strengthening factor shown in Eq.(23). In the equation, the volume fraction, strain hardening exponent and the aspect ratio of the reinforcement are included. This approximate constitutive relation is very simple and easy to use for scientists and engineers, and the validity of the constitutive relation will be checked by the experimental results and FEM results.

5 RESULTS AND ANALYSIS

The strengthening effect of reinforcement volume fraction on the full stress-strain curve of the composites is firstly studied. As we can see from Fig.2, the larger volume fraction of reinforcement will have greater strengthening effect. The stress and plastic strain curve of a representative volume fraction $f = 0.15$ is calculated and compared with FEM results of Llorca and González[11], which are shown in Fig.3. This curve is computed for a cell whose aspect ratio is equal to unity, and α is selected as unity, the parameters of material are selected from [11]. Llorca and González calculated this kind of ceramic inclusion using finite element code ABAQUS. Present results are in good agreement with the results of Llorca and González. The volume fraction is a main parameter which influences the strengthening hardening effect as we can see from Fig.3. The strength of the reinforced composite is about 20% higher than the unreinforced material in this calculation.

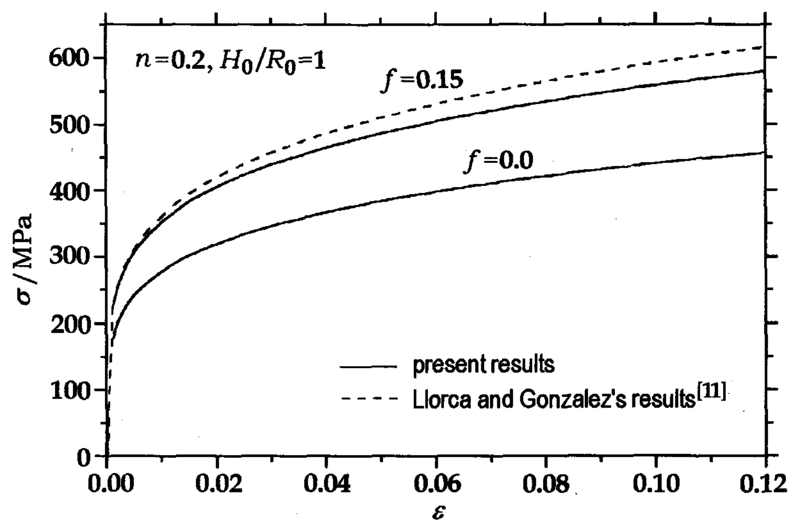

Fig.3 Predicted uniaxial stress and plastic-strain curve of Al-Al_2O_3 composites compared with FEM results of Llorca and González[11]

Secondly, the effect of the reinforcement aspect ratio H_0/R_0 on the mechanical behavior for whisker reinforced composites is studied. Whiskers with large aspect ratio exhibit much greater reinforcing effect than unit cylinders as shown in Fig.4. The results are compared with the results of [10], where α is selected as 3/7. A more complete examination on the aspect ratio dependence of the stress-strain relations is shown in Fig.5. Figure 5 illustrates the effect of the fiber reinforcement aspect ratio H_0/R_0 on the strengthening factor for both prolate and oblate fibers, a large value of $|\lg(H_0/R_0)|$ always indicates a shape of cell far away from the unit cylinder. As can be seen from Fig.5, fibers with large shape index $|\lg(H_0/R_0)|$ (i.e. whisker or disks) exhibit much greater reinforcing effect than unit cylinders. The figure also shows that prolate cylinders are more effective than oblate ones in strengthening the matrix, which is also observed by Lee and Mear[2], Bao et al.[10] and Yang et al.[19].

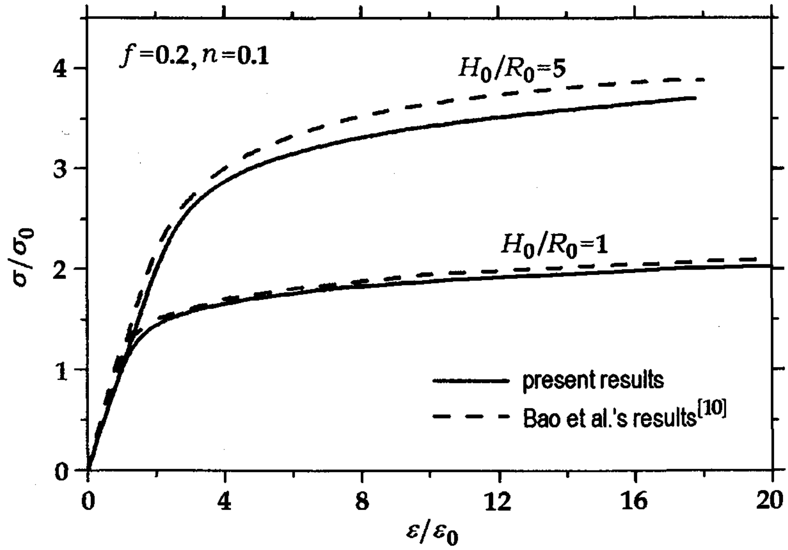

Fig.4 Predicted tensile stress-strain curve of composites compared with the results of Bao et al.[10]

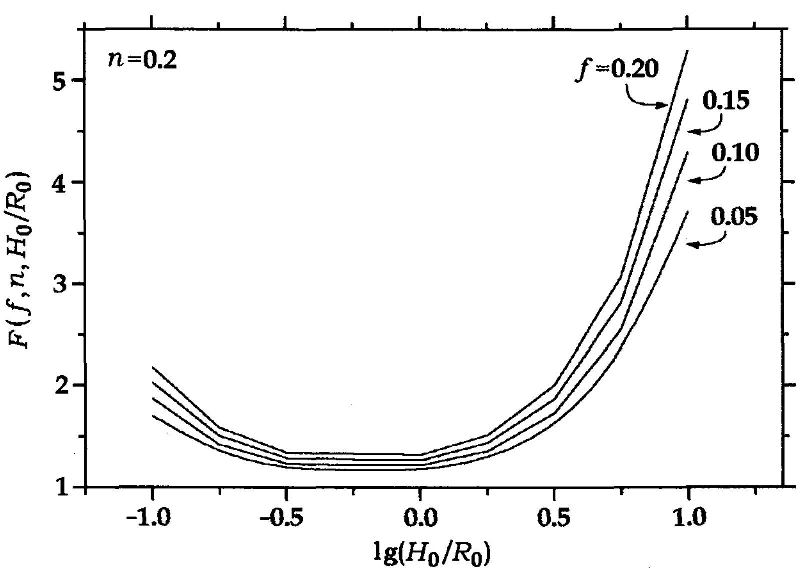

Fig.5 Strengthening effect of the aspect ratio of fiber/particle reinforcement on the strength of composites

Figure 6 describes the predicted stress-plastic strain curve of this paper compared with the experimental results of Nieh and Chellman[20] and the results of Zhu and Zbib[9], which shows that the particle with sharp corner reinforcement can be modeled very well by cylindric unit cell. The spherical cell model can not properly simulate the mechanical behavior of the composites reinforced by particle reinforcement with sharp corner[9]. At low volume fraction, the unit cylindrical particle ($H_0/R_0 = 1$) are about twice as effective as a spherical particle at the same volume fraction[10].

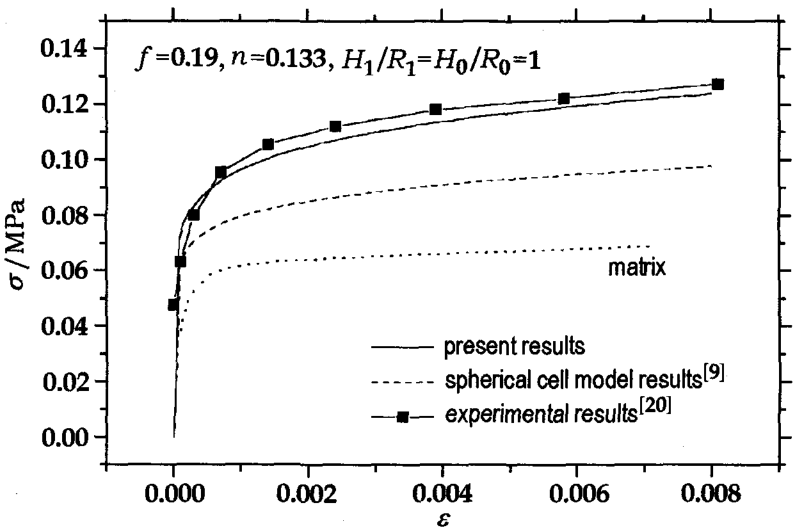

Fig.6 Predicted uniaxial stress plastic-strain curves for particle reinforced composites compared with the experimental results of Nieh and Chellman[20] and results of Zhu and Zbib[9] on 1100 Al-SiC composites containing particles with sharp corner

The predicted stress and strain curve for whisker reinforced composites, where $H_0/R_0 = 5$, $f = 0.13$, and $n = 0.130\,5$, is qualitatively in good agreement with the experimental results of Christman et al.[12], but quantitatively higher by about 10 percent, as shown in Fig.7, with the material parameters being selected from [12]. How to explain this discrepancy? The whisker reinforcements of material tested are randomly distributed and neighbouring fibers are expected to be shifted relative to one another, both in the axial and transverse directions, and maybe with a partial overlap between the fibers ends. The aspect ratio of reinforcement is not strictly equal to 5 in the real material, some references give the aspect

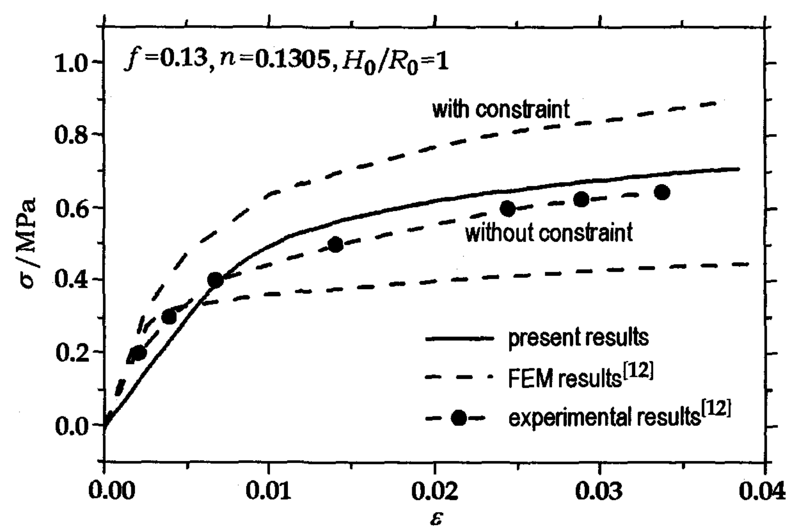

Fig.7 Predicted uniaxial stress-strain curves for 2124 Al-SiC whisker reinforced composites compared with experimental and FEM results by Christman et al.[12]

ratio as $2 \sim 3$[21]. Those are the mechanism that causes the difference between the predicted results and experimental results.

Another composite reinforced with whisker studied by Nieh and Chellman[20] is calculated, at two specific cases $f = 0.09$ and 0.18, and the material parameters are selected from [20], where the aspect ratio of whisker is selected as 5 in the calculation. The results are very close to the experimental results of Nieh and Chellman, shown in Fig.8. It can be seen from the comparison between Fig.6 and Fig.8 that the whiskers have a greater strengthening effect than the particle reinforcement, which is also observed by other authors[9].

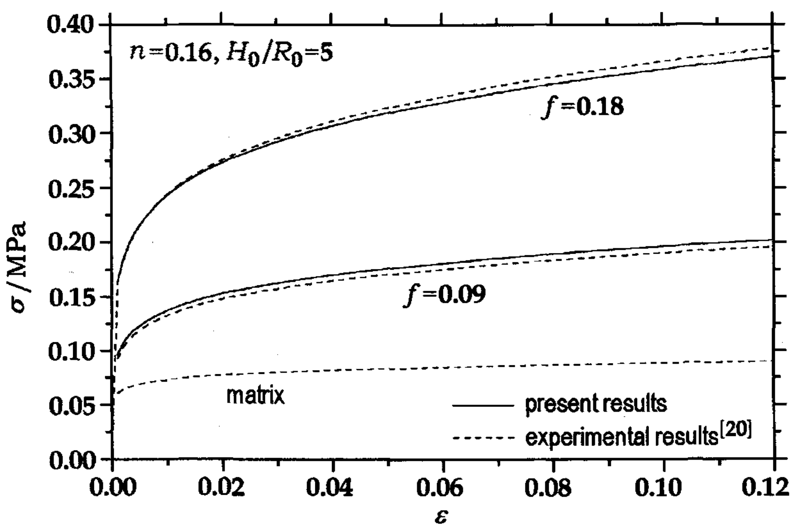

Fig.8 The predicted results of stress-strain curve of whishker reinforced composite compared with the experimental results of Nieh and Chellman[20]

6 DISCUSSION

An analytical and explicit constitutive relation of discontinuous reinforced composites is obtained in this paper. The mechanical behavior of the composites is systematically studied. The main parameters of composites are analyzed, such as the volume fraction of the reinforcement, the strain hardening exponent and the aspect ratio of the reinforcement.

This constitutive relation of composites (23) is a simple and explicit equation and is easy to use. The study of the main mechanical behavior of composites can be completed by simply implementing Eq.(23) and without much calculation.

Since the constitutive relation is basing on the primitive displacement field, and the low volume fraction reinforcement is assumed ($f \leq 0.2$), the use of the constitutive equation of composites in large volume fraction will bring errors sometimes.

Finally, it should be noted that the present results have been theoretically verified, and the models are in a general agreement with the experimental observations. However, the predictive capacity of the model should be strengthened by including other microstructural factors and micromechanisms, which may be important under certain conditions, such as the size effect of reinforcement. This will be done in our future work.

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