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CHARACTERIZATION OF ELASTIC–PLASTIC FIELDS NEAR STATIONARY CRACK TIP AND FRACTURE CRITERION

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Abstract—The effectiveness of the J-Q two parameter solution is clearly displayed in the cases of single-edge shallow cracked panel, double-edge cracked panel and center cracked panel from small scale yielding to large scale yielding. It is also suitable for the bend cracked panel specimen from small scale yielding to medium scale yielding. However, for bend cracked panel from medium scale yielding to large scale yielding, the J-Q two parameter solution will deviate gradually from finite element solution. Especially in large scale yielding, the deviation is remarkable. In this paper, we carry out the finite element calculations for two representative cases of the bend cracked panel and center cracked panel specimens from small scale yielding to large scale yielding. By considering a modified term on the basis of two parameter solution, and properly selecting the second term and modified term by matching with finite element solutions, we obtain a modified two parameter solution, which is very well in conformity with the finite element solutions in a wide range.

The upper bound and lower bound fracture toughness curves predicted by the modified two parameter solution are given. These two curves have covered most experimental data and fully captured the trend of most experimental data.

1. INTRODUCTION

It is well known that the HRR singularity field [1, 2] can characterize the high triaxiality elastic plastic field near the crack tip. The intensity coefficient of the HRR singularity, J-integral, can be taken as a single parameter dominating crack initiation. However, this high triaxiality field is only one of many possible states. Even though crack tip field could be a high triaxiality state during the initial loading stage, it will be changed gradually into low triaxiality state as load increases from small scale yielding to large scale yielding, and the HRR solution will deviate from the finite element solution [3]. The cases of single-edge shallow crack [4], center cracked panel and double-edge cracked panel etc. [5] are examples of the low triaxiality states. In these cases, the single parameter characterization of elastic–plastic field near the crack tip will bring a large error. In view of these reasons, two parameter approaches were developed. Li and Wang [6] first derived the second order asymptotic field under plane strain and Mode I conditions, in which the first term was the HRR singular field, the coefficient of the second term was determined by matching the asymptotic solution with finite element full field solution. The second order asymptotic solution obtained in such a way was a great improvement on the HRR solution. Sharma and Aravas [7] also completed the second order analysis taking account of possible elasticity effects. Recently, Yang *et al.* [8] carried out a higher order asymptotic analysis for Mode I and Mode II cracks, they utilized the higher-order term to establish the size and shape of the zone dominated by the HRR field. Xia *et al.* [9] derived the fifth order asymptotic field under plane strain and Mode I conditions, they gave out the further information about the elastic–plastic field near the crack tip. O'Dowd and Shih [3, 10] developed a two term solution in which the first term was the HRR singularity field, and the second term, including function form and its coefficient, was determined by matching this two term solution with the finite element full field solution. They developed a J-Q two parameter criterion dominating crack initiation. Betegon and Hancock [11] and Ai-Ani and Hancock [4] presented J-T two parameter characterization by carrying out finite element calculations for weakly hardening materials. Here, T was uniform tension stress paralleling to crack faces and associated with the second term of Williams' expansion.

The J-Q two parameter solutions can characterize the elastic–plastic field near crack tip in a wide range. High order asymptotic solution near crack tip [9] also verified the effectiveness of J-Q two parameter criterion. The J-Q two parameter characterization of the elastic–plastic fields near the crack tip is suitable for the cases of the single-edge shallow crack, center cracked panel and double-edge cracked panel etc. from small scale yielding to large scale yielding. For the bend cracked panel from small scale yielding to medium scale yielding, it is also suitable. However, the fine research [3] showed that for bend cracked panel from medium scale yielding to large scale yielding, the *J-Q* two parameter solution will deviate from finite element solution. In this paper, our attention is focused on studying the effectiveness of *J-Q* two parameter characterization from small scale yielding to large scale yielding. We carry out the finite element calculations for two representative cases which are the bend cracked panel (BCP) and center cracked panel (CCP). We construct a three term solution based on the two parameter solution. The second term and the third term (or modified term) are determined by matching with finite element solutions. For various cases, three term solutions obtained are very good simulation to finite element solutions. Furthermore, we present a modified two parameter criterion.

2. FUNDAMENTAL RELATIONS

A widely used uniaxial stress–strain relation is the Ramberg–Osgood form

$$\frac{\epsilon}{\epsilon_0}=\frac{\sigma}{\sigma_0}+\alpha\left(\frac{\sigma}{\sigma_0}\right)^n \tag{1}$$

where σ_0 is an effective yield stress, $\epsilon_0=\sigma_0/E$ is the associated elastic strain with E as Young's modulus, α and n are parameters chosen to fit experimental data. n is the strain hardening exponent.

The increment relation of multiaxial stress and strain by J_2 flow theory of plasticity is

$$\mathrm{d}\sigma_{ij}=\frac{E}{1+v}\left\{\delta_{im}\delta_{jn}+\frac{v}{1-v}\delta_{ij}\delta_{mn}-\frac{9\mu\Omega}{(6\mu+2H)\sigma_e^2}S_{ij}S_{mn}\right\}\mathrm{d}\epsilon_{mn} \tag{2}$$

in which, S_{ij} is the deviatoric stress, $\sigma_e=\sqrt{3S_{ij}S_{ij}/2}$ is the effective stress, v is Poisson's ratio, μ is shear modulus and H is tangential modulus of plasticity, which by (1) is

$$H=\frac{\mathrm{d}\sigma_e}{\mathrm{d}\bar{\epsilon}^p}=\frac{\sigma_0}{n\alpha\epsilon_0}\left(\frac{\sigma_e}{\sigma_0}\right)^{1-n} \tag{3}$$

while parameter Ω is as the following

$$\Omega=\begin{cases}1 & \text{on the loading surface and } S_{ij}\mathrm{d}\epsilon_{ij}>0\\ 0 & \text{otherwise.}\end{cases} \tag{4}$$

Generalizing (1) to multiaxial states by J_2 deformational plasticity theory, one obtains the stress–strain relation

$$\epsilon_{ij}=\frac{1+v}{E}s_{ij}+\frac{1-2v}{3E}\sigma_{kk}\delta_{ij}+\frac{3}{2}\alpha\epsilon_0\left(\frac{\sigma_e}{\sigma_0}\right)^{n-1}\frac{s_{ij}}{\sigma_0}. \tag{5}$$

In the basis of the small strain J_2 deformational theory, the asymptotic solution ahead of a stationary crack could be developed. The first order asymptotic solution was given by Hutchinson [1], Rice and Rosengren [2] which was called for short the HRR singularity field:

$$\frac{\sigma_{ij}}{\sigma_0}=\left(\frac{J}{\alpha\epsilon_0\sigma_0 I_n r}\right)^{1/(n+1)}\tilde{\sigma}_{ij1}(\theta) \tag{6}$$

in which r and θ are polar coordinates. The high order asymptotic solutions were given respectively by Li and Wang [6] for second order and Xia *et al.* [9] for fifth order. The form of high order asymptotic solution was as the following

$$\frac{\sigma_{ij}}{\sigma_0}=\left(\frac{J}{\alpha\epsilon_0\sigma_0 I_n r}\right)^{1/(n+1)}\tilde{\sigma}_{ij1}(\theta)+Q\left(\frac{r}{J/\sigma_0}\right)^q\hat{\sigma}_{ij2}(\theta)+\text{higher order terms.} \tag{7}$$

O'Dowd and Shih [3, 10] developed a two term solution, in which the first term was the HRR solution, and the form of the second term was similar with the second term of high order asymptotic solution, but Q, q and angular distribution function $\hat{\sigma}_{ij}(\theta)$ were determined by matching two term solution with finite element solution. O'Dowd and Shih further obtained a *J-Q* two parameter solution

$$\frac{\sigma_{ij}}{\sigma_0}=\left(\frac{J}{\alpha\epsilon_0\sigma_0 I_n r}\right)^{1/(n+1)}\tilde{\sigma}_{ij1}(\theta)+Q\hat{\sigma}_{ij1}(\theta). \tag{8}$$

They proved by finite element calculations that $\hat{\sigma}_{ij1}(\theta)\approx\delta_{ij}$ within $|\theta|\leqslant\pi/4$. Based on (8), they developed a J-Q two parameter criterion dominating crack initiation.

3. FINITE ELEMENT ANALYSES

In this paper, we develop a two-dimensional elastic–plastic finite element program. This program, based on the J_2 flow theory of plasticity and adopting the eight-nodal isoparametric element and the tangential stiffness method, was shown its reliability by the calculations of typical examples. Firstly, we calculated the elastic–plastic solution near crack tip under small scale yielding and Mode I condition. The deviations of J-integral values obtained by integrations along the different contours are within 1%. This shows the conservation of J-integral. The difference of both J-integral values calculated by the contour integration and directly by the external K field is also within 1%. Secondly, we calculated the elastic–plastic fields near the crack tip for single-edge cracked panel under tension. The result was in good agreement with that presented by Shih and German [12] for the same problem.

We carry out the finite element calculations for BCP specimens with $n=5$ and $n=10$ and for CCP specimens with $n=3$ and $n=10$ from small scale yielding to large scale yielding. During the calculations, the ratio of the crack length (a) to the specimen width (W) is taken as 0.5 for various cases, and the other material parameters are taken as $E/\sigma_0=500$, $\nu=0.3$, $\alpha=1$. The adopted finite element mesh is shown in Fig. 1.

4. RESULTS AND ANALYSES

The stress distributions near crack tip for bend cracked panel (BCP) with $n=5$ and $n=10$ in different loading stages from medium scale yielding to large scale yielding are shown in Fig. 2. From Fig. 2, HRR singularity solution [see eq. (6)] has remarkably deviated from the finite element solution, and the J-Q two parameter solution [see eq. (8)] is in conformity with finite element solution in a certain range. However, as load increases, or as $r/(J/\sigma_0)$ increases, the J-Q two parameter solutions within annulus range ($1\leqslant r\sigma_0/J\leqslant 6$) will deviate gradually from finite element solution. Especially in the large scale yielding, the deviation of J-Q two parameter solution from finite element solution is quite large. The J-Q two parameter solution overestimates the elastic–plastic stress field within the annulus range from medium scale yielding to large scale yielding.

Figure 3 plots the stress distributions near the crack tip for center cracked panel (CCP) with $n=3$ and $n=10$ in three loading stages from medium scale yielding to large scale yielding. From Fig. 3, HRR solution has a larger deviation from finite element solution than that of BCP specimen, but the difference of two parameter solution to finite element solution is quite small. When $n=10$, two parameter solution is very well in conformity with finite element solution. When $n=3$, this difference will grow slowly with the increases of load or $r/(J/\sigma_0)$. When $n=3$, the J-Q two parameter solution underestimates the elastic–plastic field within the annulus range from medium scale yielding to large scale yielding.

In order to characterize the elastic–plastic field within the annulus range from small scale yielding to large scale yielding, we try to construct a three term solution (modified two parameter solution) based on the two term solution. From the theory of ordinary differential equation (ODE), the eigenfunction for the second order linear ODE with variable coefficients, generally speaking, is either an analytical function, or the product of singular functions and analytical functions. In view of the above reasons, we consider a three term solution based on the two parameter solution

$$\frac{\sigma_{ij}(r,\theta)}{\sigma_0}=\left(\frac{J}{\alpha\epsilon_0\sigma_0 I_n r}\right)^{1/1+n}\tilde{\sigma}_{ij1}(\theta)+Q\hat{\sigma}_{ij1}(\theta)+k_2(r\sigma_0/J-1)\hat{\sigma}_{ij2}(\theta) \tag{9}$$

in which the first term of right hand side is the HRR singularity solution, Q, $\hat{\sigma}_{ij1}(\theta)$, k_2, and $\tilde{\sigma}_{ij2}(\theta)$ in the second and third term are determined by matching with finite element solution. Here, we adopt the weighted residual method in each θ angular direction for determining Q, k_2 as well as the angular distribution functions (weighted function is taken as 1).

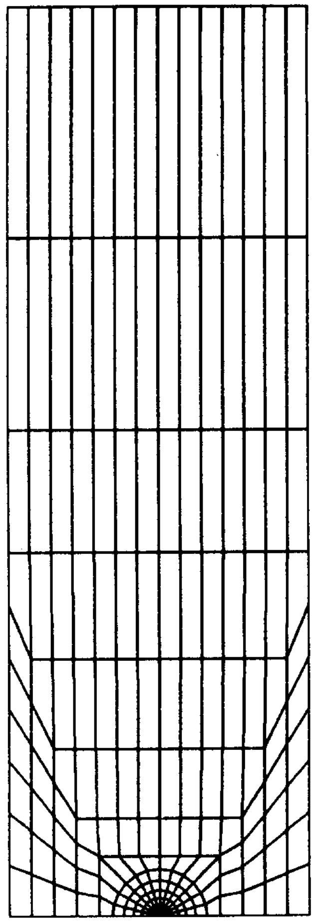

Fig. 1. Finite element mesh.

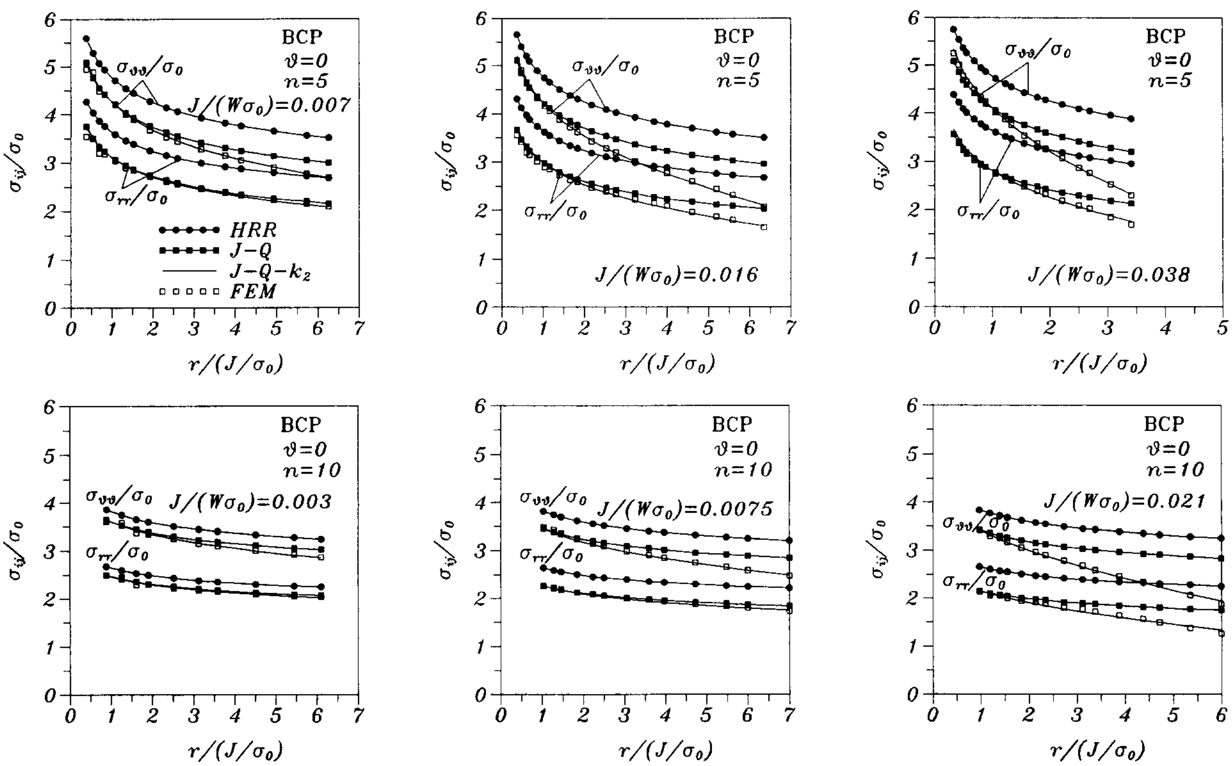

Fig. 2. Solutions of HRR, J-Q, J-Q-k_2 and FEM for BCP specimens.

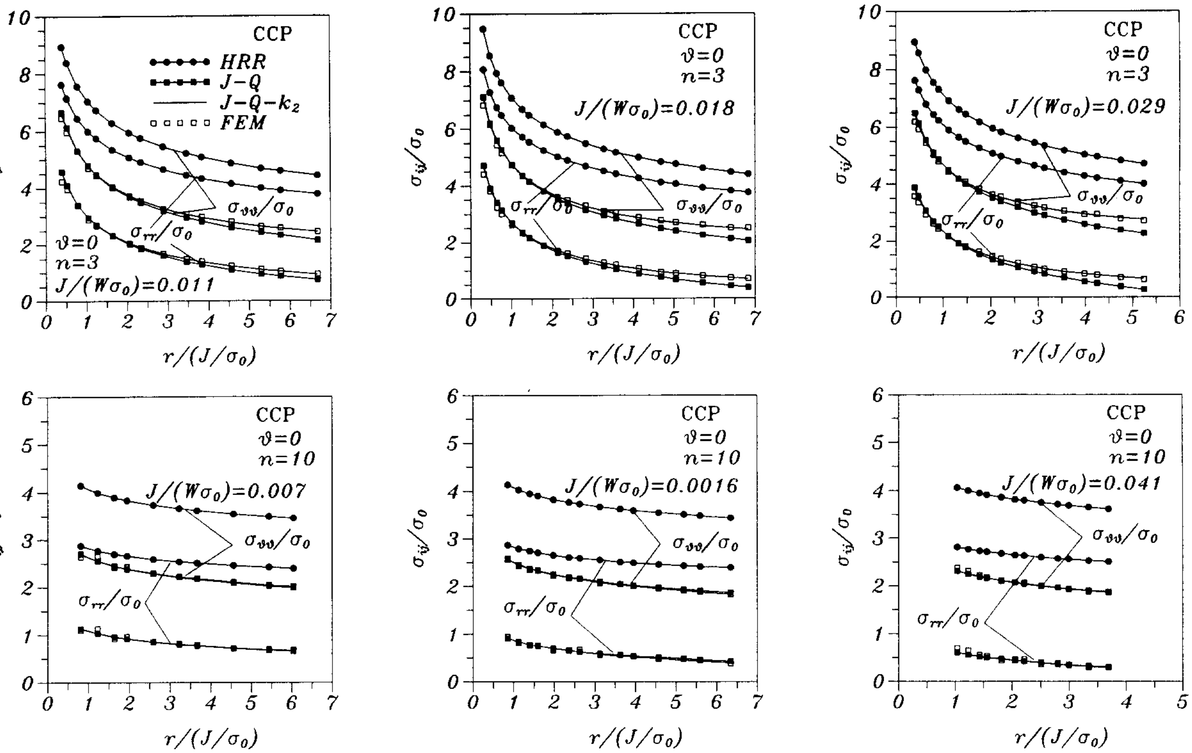

Fig. 3. Solutions of HRR, J-Q, J-Q-k_2 and FEM for CCP specimens.

In Figs 2 and 3, the J-Q-k_2 three parameter solutions are also given. From these figures, three term solutions agree very well with finite element solutions within annulus range. The effectiveness of two parameter solution can also be judged from Fig. 4. Figure 4 plots the angular distribution functions $\hat{\sigma}_{ij1}(\theta)$ and $\hat{\sigma}_{ij2}(\theta)$. Alternatively, the magnitude of $k_2\hat{\sigma}_{ij2}(\theta)(r\sigma_0/J-1)$ modified term (the third term) can be seen clearly from Fig. 4. For BCP specimens, the magnitude of the third term

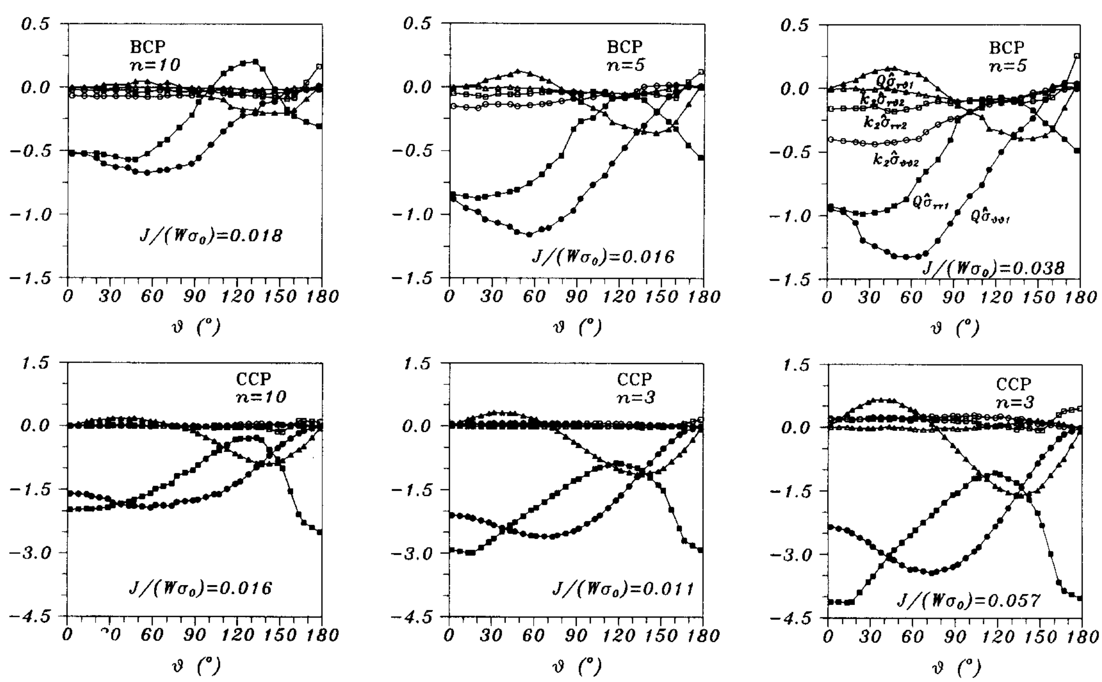

Fig. 4. Angular distribution functions of the second and third term for BCP and CCP specimens.

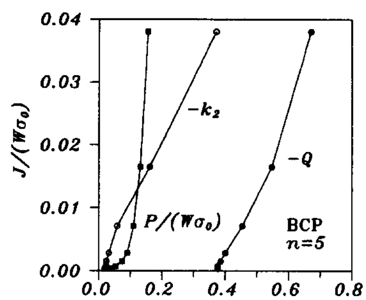

Fig. 5. The relations of external load to three parameters for BCP specimen (n = 5).

is comparable to the second term. Especially in large scale yielding, the third term is equivalently important to the second term. For CCP specimens, the third term is small relative to the second term. When $n = 3$, the third term increases with load increase. From Fig. 4, $|\hat{\sigma}_{r\theta}| \ll |\hat{\sigma}_{rr}|$ or $|\hat{\sigma}_{\theta\theta}|$ within $|\theta| \leqslant \pi/2$ for various cases. The variations of three parameters and external load from small scale yielding to large scale yielding for BCP specimen $n = 5$ are plotted in Fig. 5. From J-P curve in Fig. 5, in large scale yielding state, a small increment of external load $P/(W\sigma_0)$ corresponds to a great increment of deformation, i.e. a great increment of J-integral.

5. FRACTURE TOUGHNESS LOCUS

Kirk *et al.* [13] have measured cleavage fracture toughness for A515 steels at room temperature over a broad range of crack tip constraints. They tested edge-cracked bend bars with thicknesses $B = 10$, 25.4 and 50.8 mm and various ratios of crack length to width. The measured toughness data are plotted against Q in Fig. 6.

Constraint effects on fracture toughness can be predicted by using three term solution in conjunction with a fracture criterion based on the attainment of a critical stress, $\sigma_{\theta\theta} = \sigma_c$, at a characteristic microstructural distance, $r = r_c$ [14]. Suppose that r_c is within the J-Q annulus ($1 \leqslant r_c\sigma_0/J_c \leqslant 5$). Now impose the RKR fracture criterion on (9) to get

$$\frac{\sigma_c}{\sigma_0} = \left(\frac{J_c}{\alpha\epsilon_0\sigma_0 I_n r_c}\right)^{1/(n+1)} \tilde{\sigma}_{221}(0) + Q + k_2(r_c\sigma_0/J_c - 1). \tag{10}$$

Therefore, we can solve for J_c as a function of Q and k_2 for selected values of σ_c and r_c. With J_c^*, Q^* and k_2^* denoting the corresponding quantities with the remote loading away from the crack tip by K field ($T = 0$), one finds from (10) that

$$\frac{J_c}{J_c^*} = \left\{\frac{\sigma_c/\sigma_0 - Q - k_2(r_c\sigma_0/J_c - 1)}{\sigma_c/\sigma_0}\right\}^{n+1}. \tag{11}$$

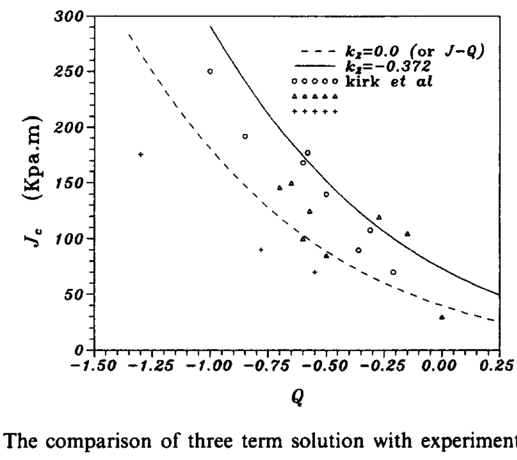

Fig. 6. The comparison of three term solution with experimental results.

Using (11), lower bound toughness curve (dashed line) and upper bound toughness curve (solid line) of $J_c - Q$ are plotted in Fig. 6 for $\sigma_c = 3.5\sigma_0$, $J_c^* = 40$ kPa.m and $n = 5$, corresponding to the maximum value (0) and minimum value (-0.372) of k_2, respectively (strain hardening exponent n of A515 steel is about 5). From Fig. 6, it can be seen that the upper bound and lower bound of predicted toughness curves have covered most experimental data.

6. CONCLUDING REMARKS

The J-Q two parameter solution can characterize the elastic–plastic field within the annulus range near the crack tip from small scale yielding to large scale yielding for single-edge shallow cracked panel, double-edge cracked panel and center cracked panel. For bend cracked panel from small scale yielding to medium scale yielding, it is still valid. However, from medium scale yielding to large scale yielding, J-Q two parameter solutions gradually deviate from the finite element solution as external load increases. This deviation is remarkable for bend cracked panel specimens. By considering a three term solution, and determining the second term and the third term by matching with finite element solution, one obtains a solution which is very well in conformity with the finite element solution over a whole range.

In O'Dowd and Shih's two parameter solution, the second term characterizes a triaxiality stress field within the $|\theta| \leqslant \pi/4$. This triaxiality stress field is unchanged with distance from crack tip. The significance of the second term and third term in three term solution presented by our study is also a characterization of the triaxiality stress field within the $|\theta| \leqslant \pi/4$, but this triaxiality stress field is linearly varying with the distance from crack tip. The third term characterizes the change of triaxiality stress with distance from crack tip.

The third term in three term solution is taken as a modified term of two parameter solution. We present a modified two parameter criterion. The fracture toughness curves predicted by modified two parameter solution have fully captured the trend of most experimental data.

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